ERRORS & DILUTIONS

IN MEASUREMENTS OF CP VIOLATION IN B-$\bar{B}$ SYSTEM

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JUNE 30, 1992

SSC B-PHYSICS MINI WORKSHOP

WE PROPOSE TO MEASURE A CP-VIOLATING ASYMMETRY:

$$A = \frac{\Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f})}{\Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f})}$$

$N \equiv$ TOTAL NUMBER OF $B \to f$ & $\bar{B} \to \bar{f}$ DECAYS

$S \equiv \frac{A}{\sigma_A}$ = STATISTICAL SIGNIFICANCE IN STANDARD DEVIATIONS

$$S = \sqrt{N} \frac{A}{\sqrt{1-A^2}}$$

EXTRA STATISTICAL POWER FOR $A \simeq 1$

FOR SMALL TO MODERATE A, APPROXIMATE:

$$S \approx \sqrt{N}\, A$$

$$\text{or} \quad N \approx \left(\frac{S}{A}\right)^2$$

EXAMPLE: $A = 0.1$
$S = 3\sigma$

$\Rightarrow N \approx 900$ EVENTS REQUIRED.

IN PRACTICE (AT LEAST) 4 EFFECTS 'DILUTE' THE STATISTICAL POWER:

① IF $B = B^0$, MIXING OSCILLATIONS REDUCE THE ASYMMETRY.

② IF $B^0 \to f$ WHERE f IS A CP EIGENSTATE ($\Rightarrow f = \bar{f}$) MUST TAG THE PARTICLE - ANTIPARTICLE CHARACTER OF THE B BY OBSERVATION OF THE SECOND B IN THE EVENT.
IF THE SECOND B IS A B^0, ITS OSCILLATIONS DILUTE THE TAG.

③ IF THE TAG VIA THE SECOND B IS BASED ON A PARTIAL RECONSTRUCTION, THEN MISTAGGING CAN OCCUR.

④ THE SIGNAL OF $B \to f$ MAY BE SUBJECT TO A BACKGROUND THAT IS CP INVARIANT.

① EFFECT OF MIXING OF THE FIRST B

PURE B^0 AT $t=0$ $\Rightarrow$ $|B^0(t)\rangle = e^{-iMt} e^{-\Gamma t/2} \cos \Delta M t/2$

$$|\bar{B}^0(t)\rangle = i\, e^{-iMt} e^{-\Gamma t/2} \sin \Delta M t/2$$

ETC ...

ASSUMING WE KNOW THE PARTICLE - ANTIPARTICLE CHARACTER OF THE B AT TIME t WHEN IT DECAYS:

WRITE $$A(t) = \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to \bar{f})}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to \bar{f})} = \sin 2\varphi \sin x t$$

φ = ANGLE OF UNITARITY TRIANGLE WHEN f = CP EIGENSTATE

$x = \dfrac{\Delta M}{\Gamma}$ = MIXING PARAMETER: $x_d \sim 0.7$ (MEASURED)

$x_s \sim 5\text{-}10$ (ESTIMATED)

CP-VIOLATING INTERFERENCE BETWEEN MIXING AND DECAY VANISHES AT $t=0$!

A. SMALL x (B_d^0): B'S DECAY BEFORE COMPLETING ONE OSCILLATION.

$\Rightarrow$ TIME-RESOLVED EXPERIMENT ESSENTIALLY THE SAME AS TIME INTEGRATED:

$$A \approx \frac{\int \Gamma(B(t)\to f) - \int \Gamma(\bar{B}(t)\to f)}{\int + \int} = \underbrace{\frac{x}{1+x^2}}_{\text{DILUTION FACTOR } D_1} \sin 2\varphi$$

SMALL $x \Leftrightarrow$ NO ASYMMETRY

LITTLE POINT IN DETAILED TIME STUDIES.

B. LARGE x (B_s^0): SEVERAL OSCILLATIONS BEFORE DECAY.

$\Rightarrow$ TIME-RESOLVED ASYMMETRY ESSENTIALLY AVERAGES OVER ONE HALF-CYCLE AT A TIME (REVERSING THE FORM OF THE ASYMMETRY EACH HALF CYCLE).

$$A \to \underbrace{\frac{2}{\pi}}_{\text{DILUTION FACTOR } D_1} \sin 2\varphi$$

$\rightarrow$ MUST KNOW WHICH HALF-CYCLE DECAY OCCURRED IN

$\Rightarrow$ MUST KNOW x_s

BUT x_s CANNOT BE MEASURED BY DECAYS $B_s \to f_{CP}$! UNLESS THERE IS CP VIOLATION. IN PARTICULAR, WE DON'T EXPECT CP VIOLATION IN $B_s \to J/\psi\, \phi$

C. GENERAL CASE: $A = D_1(x) \sin 2\varphi$

DILUTION FACTOR $D_1(x) = \frac{x}{1+x^2} \coth\left(\frac{\pi}{2x}\right)$

x	$\frac{x}{1+x^2}$	$D_1(x)$	
0.7	0.47	0.48	
5	0.19	0.63	($2/\pi = 0.64$)

② EFFECT OF OSCILLATIONS OF THE SECOND B

OBSERVE PARTICLE–ANTIPARTICLE CHARACTER OF THE SECOND B TO INFER PARTICLE–ANTIPARTICLE CHARACTER OF THE FIRST B $(B \to f_{CP})$

IF SECOND B IS A B^0, IT MAY HAVE OSCILLATED TO $\bar{B}^0$ BEFORE DECAYING!

INTEGRATED PROBABILITY THAT A B^0 AT $t<0$ DECAYS AS A $\bar{B}^0$ IS

$$P = \frac{x^2}{2(1+x^2)}$$

THE USEFUL NUMBER OF TAGS IS $N\begin{pmatrix}\text{BORN AS } B\\ \text{DECAYS AS } B\end{pmatrix} - N\begin{pmatrix}\text{BORN AS } B\\ \text{DECAYS AS } \bar{B}\end{pmatrix}$

$$= N(1-2p) = N \underbrace{\frac{1}{1+x^2}}_{\text{DILUTION FACTOR } D_2}$$

SECOND B IS

$$B^+ : D_2 = 1$$

$$B^0_d : D_2 = \frac{1}{1+x_d^2} \approx 2/3$$

$$B^0_s : D_2 = \frac{1}{1+x_s^2} \approx 0 \quad (\Rightarrow \text{USELESS AS TAG})$$

PROPORTIONS OF SECOND B:

$$B^\pm : B^0_d : B^0_s \simeq \epsilon : \epsilon : 1-2\epsilon$$

WITH $\epsilon \sim 3/8$ (TO BE MEASURED!)

EFFECTIVE DILUTION FACTOR:

$D_2 = \sqrt{\epsilon}$ FOR $B^\pm$, AS HAVE ONLY ϵ AS MANY EVENTS AS FOR ALL B'S

$$D_2 = \epsilon \cdot 1 + \frac{\epsilon}{1+x_d^2} + \frac{1-2\epsilon}{1+x_s^2} \approx \frac{5\epsilon}{3} \quad \text{FOR ALL B'S}$$

$$\frac{D_2(\text{ALL B'S})}{D_2(B^\pm)} \sim \frac{5}{3}\sqrt{\epsilon} \sim \frac{5}{\sqrt{24}} \sim 1$$

$\Rightarrow$ NO ADVANTAGE TO TAGGING ONLY WITH $B^\pm$.

FOOTNOTE ON TAGGING VIA $B^\pm$ VS. ALL B'S

LET N_0 = # OF RECONSTRUCTED $B \to f_{CP}$ REQUIRED FOR SOME MEASUREMENT, NOT YET TAKING INTO ACCOUNT TAGGING VIA THE SECOND B.

N_1 = # OF $B \to f_{CP}$ REQUIRED SO THAT N_0 OF THEM WILL BE TAGGED VIA A SECOND $B^\pm$

N_2 = # REQUIRED IF TAG VIA ANY SECOND B.

$$\epsilon = \frac{B^\pm}{\text{ALL } B\text{'s}} \approx 3/8$$

$$N_1 = \frac{N_0}{\epsilon (D_2)^2} = \frac{N_0}{\epsilon}$$ SINCE DILUTION FACTOR $D_2 = 1$ FOR $B^\pm$

$$N_2 = \frac{N_0}{(D_2)^2} = \frac{N_0}{\left(\frac{5\epsilon}{3}\right)^2} = \frac{9}{25\epsilon} N_1 \approx \frac{24}{25} N_1$$

THERE IS NO STATISTICAL ADVANTAGE TO USE OF A TAG ON $B^\pm$ ONLY !

[WE HAVE IGNORED POSSIBLE DIFFERENCES IN DETECTION EFFICIENCY AND MISTAGGING PROBABILITY FOR TAGGING VIA DIFFERENT B'S, BUT EXPECT THESE DIFFERENCES TO BE SMALL.]

③ MISTAGGING OF SECOND B

SUPPOSE ONLY PARTIAL RECONSTRUCTION OF SECOND B:

$b \to c \to s$ ↳ OBSERVE K^+ OR K^-

$b \to c \ell \nu$ ↳ OBSERVE ℓ^+ OR ℓ^-

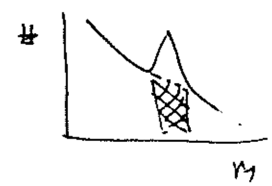
p = PROBABILITY OF ASSIGNING WRONG SIGN

EFFECTIVE FRACTION OF CORRECT TAGS IS $P_{RIGHT} - P_{WRONG}$

$$\Rightarrow D_3 = 1 - 2p$$

④ NON RESONANT BACKGROUND

THE FIRST B IS FULLY RECONSTRUCTED, BUT MAY HAVE BACKGROUND IN THE MASS PLOT



$$b = \frac{\text{BACKGROUND}}{\text{SIGNAL}}$$

IN GENERAL THERE WILL BE NO CP VIOLATION IN THE BACKGROUND EVENTS, SO

$$A \to \underbrace{\frac{1}{1+b}}_{\text{DILUTION FACTOR } D_4} A$$

Summary of Dilution Factors

$$A_{OBS} \sim D_1 D_2 D_3 D_4 \sin 2\varphi$$

Need $N \approx \left(\frac{S}{A_{OBS}}\right)^2$ TAGGED events to measure $\sin 2\varphi$ to S standard deviations

$$\Rightarrow \quad N = \left(\frac{1}{D_1 D_2 D_3 D_4}\right)^2 \left(\frac{S}{\sin 2\varphi}\right)^2$$

Example: for B_d, $D_1 = \frac{x}{1+x^2} \approx \frac{1}{2}$

$$D_2 \approx \frac{5}{3}\epsilon \approx \frac{5}{3}\left(\frac{3}{8}\right) = 5/8 \quad \text{(Tag on all B's)}$$

$$D_3 = 1 - 2p \approx 0.8 \text{ at hadron collider}$$

$$D_4 = \frac{1}{1+b} \sim 1 \text{ for clean mode } (J/\psi\, K^0_S)$$

$$D_1 D_2 D_3 D_4 \sim \frac{1}{4}, \quad N \approx 16\left(\frac{S}{\sin 2\varphi}\right)^2$$

i.e. for $S=3$, $\sin 2\varphi = 0.1$ need $N \sim 14{,}400$!

At e^+e^-, $N \approx \left[\frac{1}{\frac{x}{1+x^2}(1-2p)}\right]^2 \left(\frac{S}{\sin 2\varphi}\right)^2 \approx 6\left(\frac{S}{\sin 2\varphi}\right)^2$

Tagging via Leptons and Kaons

What fraction of $B^0 \to f_{CP}$ decays can be correctly tagged?

$$b \to c \to s \quad \text{(charged } K\text{)}$$

$$b \to c \ell \nu \quad (e \text{ or } \mu)$$

'Wrong-sign' K, e, or μ occur via other stages of decay cascade.

ISAJET study: Tag via highest p_T $K^\pm$, $e^\pm$, or $\mu^\pm$ in rest of event.

No secondary vertex requirement on tagging particle

(Favored if correct tagging probability is larger than secondary vertexing efficiency)

$\Rightarrow$ K's best option.

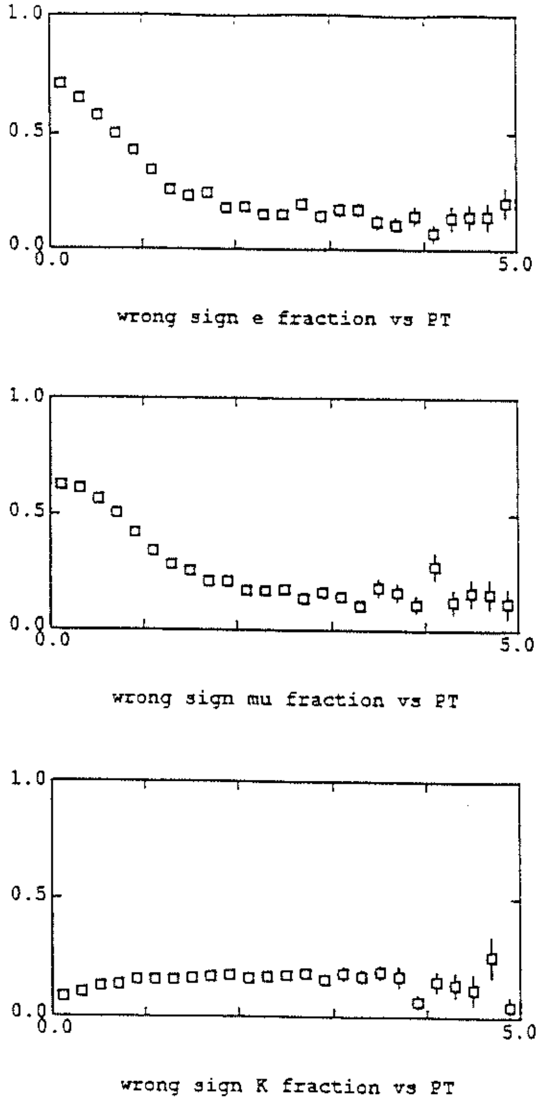


Figure 4: The fraction of leptons (or Kaons) that have the wrong sign as a function of P_t .

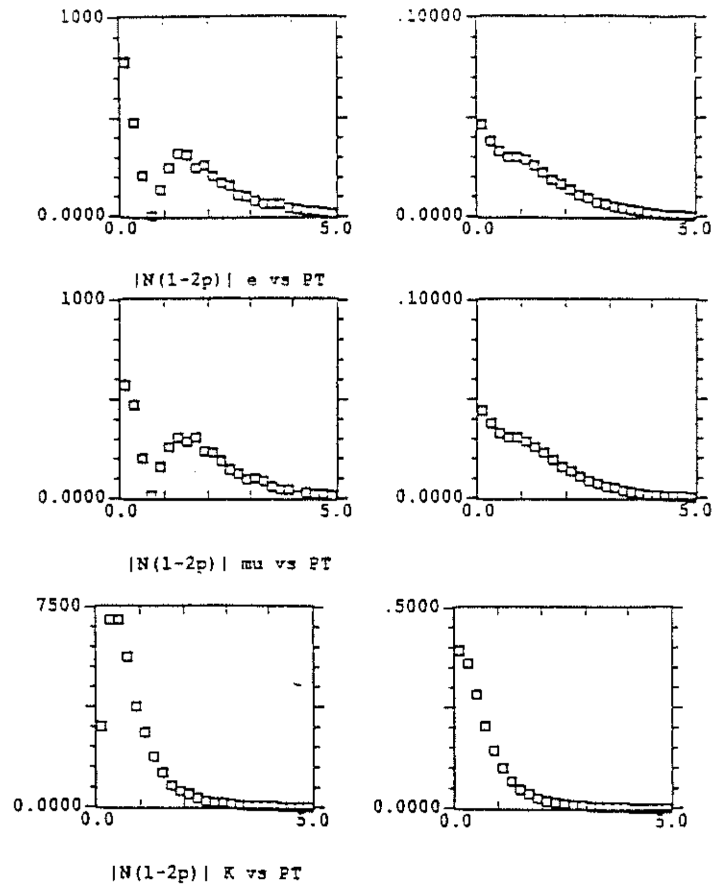


Figure 5: Differential and Integral tagging efficiencies of four types of tags as a function of transverse momentum. Left hand plots: the number $N|1-2p|$ of useful tagged events; right-hand plots: the total efficiency of the tag as a function of the minimum-transverse-momentum requirement. The four tags are, from top to bottom, electron, muon, combined electron and Kaon, and Kaon.

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Alternative Analyses of CP-Violating Asymmetries

In Sec. 6.2 of Ref. [1] we noted that our proposed method of analysis of CP violation in the neutral B system would yield a null result if we integrate over time and if the B-$\bar{B}$ pair was produced in a C(odd) state. As the latter condition holds for B's produced at the $\Upsilon(4S)$ resonance at an e^+e^- collider, this analysis would be inappropriate there. A clever alternative procedure has been proposed[2] that maximizes the analyzing power at an e^+e^- collider. Here we examine whether this procedure would be effective at a hadron collider.

Both B's of a produced B-$\bar{B}$ pair must be observed in a CP analysis. We label B_1 as the (neutral) B that decays to the CP eigenstate f, and B_2 as the (charged or neutral) B that decays to a state $g \neq \bar{g}$ that permits us to determine whether B_2 was a particle or antiparticle at the moment of its decay. We can accumulate four time distributions, where one B decays at time t_a and the other at time t_b with $t_a < t_b$:

$$
\begin{aligned}
I: &\ \Gamma_{B_1\to f}(t_b)\Gamma_{B_2\to g}(t_a),\\
II: &\ \Gamma_{B_1\to f}(t_a)\Gamma_{B_2\to g}(t_b),\\
III: &\ \Gamma_{B_1\to f}(t_b)\Gamma_{B_2\to \bar{g}}(t_a),\\
IV: &\ \Gamma_{B_1\to f}(t_a)\Gamma_{B_2\to \bar{g}}(t_b).
\end{aligned}
$$

The four distributions can be combined to form asymmetries in various ways:

$$A_1(t_a,t_b) \equiv \frac{III+IV-I-II}{I+II+III+IV}.$$

Another asymmetry is

$$A_2(t_a,t_b) \equiv \frac{II+III-I-IV}{I+II+III+IV},$$

as considered in Ref. [2]. A third might be defined as

$$A_3(t_a,t_b) \equiv \frac{I+III-II-IV}{I+II+III+IV}.$$

For the case that mesons 1 and 2 are of the same type the four time distributions take the form

$$
\begin{aligned}
\Gamma_I(t_a,t_b) &\propto e^{-(t_a+t_b)}[1 \pm \sin 2\varphi \sin x(t_a \pm t_b)],\\
\Gamma_{II}(t_a,t_b) &\propto e^{-(t_a+t_b)}[1 + \sin 2\varphi \sin x(t_a \pm t_b)],\\
\Gamma_{III}(t_a,t_b) &\propto e^{-(t_a+t_b)}[1 \mp \sin 2\varphi \sin x(t_a \pm t_b)],\\
\Gamma_{IV}(t_a,t_b) &\propto e^{-(t_a+t_b)}[1 - \sin 2\varphi \sin x(t_a \pm t_b)],
\end{aligned}
$$

where φ is the CP-violating phase in the decay amplitude for $B_1 \to f$, $x = \Delta M/\Gamma$ is the mixing parameter for neutral B-meson, and the lower sign in the distributions holds for C(odd) states $|B_1\rangle|\bar{B}_2\rangle - |\bar{B}_1\rangle|B_2\rangle$. In the above, time is measured in units of the lifetime $1/\Gamma$.

Inserting the time distributions into the forms for the asymmetries we have

$$
\begin{aligned}
A_1 &= \begin{cases} 0 & C(\text{odd}) \\ \sin 2\varphi \sin x(t_a+t_b) & C(\text{even}) \end{cases},\\
A_2 &= \begin{cases} \sin 2\varphi \sin x(t_a-t_b) & C(\text{odd}) \\ 0 & C(\text{even}) \end{cases},\\
A_3 &= 0.
\end{aligned}
$$

Clearly the asymmetry A_2 will be useful at an e^+e^- collider where only C(odd) states are produced.

As we have noted elsewhere, in B_d decays where $x_d \approx 0.7$ there are about nine lifetimes per oscillation, and so a time-resolved analysis is actually little different than a time-integrated one. Hence it is relevant to consider the time-integrated forms of the asymmetries.

Because of the time ordering in the definition of the distributions I-IV, the form of the integrals is

$$\int_0^\infty dt_a \int_{t_a}^\infty dt_b \Gamma_I(t_a,t_b),$$

etc. On evaluating these integrals for the case that meson B_1 is of the same type as B_2, we find

$$A_1 = \begin{cases} 0 & C(\text{odd}) \\ 2x\sin 2\varphi/(1+x^2)^2 & C(\text{even}) \end{cases},$$

while

$$A_2 = \begin{cases} x\sin 2\varphi/(1+x^2) & C(\text{odd}) \\ 0 & C(\text{even}) \end{cases}.$$

At a hadron collider the B-$\bar{B}$ pairs are produced in C(even) and C(odd) states with equal probability, so the question arises as to which asymmetry is to be preferred to attain maximum sensitivity to the CP-violating factor $\sin 2\varphi$. Note that the nonzero cases of the asymmetries are affected by the dilution due to mixing in different ways:

$$A_1(C(\text{even})) = \frac{2}{1+x^2} A_2(C(\text{odd})).$$

For the case of B_d-$\bar{B}_d$ production where $x_d \approx 0.7$, the factor $2/(1+x^2) \approx 4/3$, so asymmetry A_1 is slightly to be preferred over A_2.

However, at a hadron collider a B_d meson can be produced along with any of a $\bar{B}_u$, $\bar{B}_d$, or $\bar{B}_s$. Table 1 lists the coefficients K of $\sin 2\varphi$ for the various possibilities of B-$\bar{B}$ production for the two asymmetries. On weighting by the relative production rates we estimate that A is about 1.5 times as large as A_2 at a hadron collider, so clearly should be used.

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Table 1: The coefficient K in time-integrated CP-violating asymmetries of the form $A = K \sin 2\varphi$ for various possibilities for B_d-$\bar{B}$ production at a hadron collider. The Weighted coefficient is obtained supposing $x_d = 0.7$, $x_s \gg x_d$, and that $\bar{B}_u$, $\bar{B}_d$, and $\bar{B}_s$ mesons are produced along with a B_d in the proportion 0.375 : 0.375 : 0.25. We have assumed that the lifetimes of all three flavors of B mesons are the same.

Asymmetry	B_d-$\bar{B}_d$	B_d-$\bar{B}_u$	B_d-$\bar{B}_s$	Weighted
A_1	$\frac{x_d}{(1+x_d^2)^2}$	$\frac{x_d}{1+x_d^2}$	$\frac{x_d}{1+x_d^2}\frac{1}{x_s^2}$	≈ 0.25
A_2	$\frac{x_d}{2(1+x_d^2)}$	$\frac{x_d}{2(1+x_d^2)}\frac{1-x_d^2/2}{1+x_d^2/4}$	$\frac{x_d}{1+x_d^2}\frac{1}{x_s^2}$	≈ 0.16

In Ref. [1] we considered the asymmetry

$$A(t_a, t_b) = \frac{\Gamma_{B_1\to f}(t_a)\Gamma_{B_2\to g}(t_b) - \Gamma_{B_1\to f}(t_a)\Gamma_{B_2\to g}(t_b)}{\Gamma_{B_1\to f}(t_a)\Gamma_{B_2\to g}(t_b) + \Gamma_{B_1\to f}(t_a)\Gamma_{B_2\to g}(t_b)} = \sin 2\varphi \sin(x_1 t_a \pm x_2 t_b),$$

where there was no restriction on t_a and t_b, and the minus sign holds for C(odd) states. This asymmetry is not quite the same as A_1 or A_2, but the time integrated version of this is identical to the time integrated version of A_1. That is, in the time integrated version of A_1 we effectively lose sight of the time ordering of t_a and t_b.

For a final comparison, the coefficient K that holds for use of asymmetry A_2 at an e^+e^- collider is 0.5. This means that the average dilution due to mixing at an e^+e^- collider is one half of that at a hadron collider. Equivalently, we will need four times as many tagged. reconstructed B_d-$\bar{B}$ decays at a hadron collider as at an e^+e^- collider to achieve the same sensitivity to $\sin 2\varphi$. Stated yet another way, the smallest value of $\sin 2\varphi$ that can be resolved to three standard deviations with N events at a hadron collider is $12/\sqrt{N}$, while at an e^+e^- collider this would be $6/\sqrt{N}$.

1 References

[1] BCD Collaboration, *Addendum to the Proposal for a B-Physics Experiment at TEV I: The μBCD* submitted to Fermilab (Jan. 7, 1991).

[2] *The Physics Program of a High-Luminosity Asymmetric B Factory at SLAC*, SLAC-353, LBL-27856 CALT-68-1588, UC-414 (Oct. 1989).