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PROPERTIES OF FINAL STATE POLARIZATION IN THE REACTION $\gamma n \to p\pi^-$ *

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Abstract

We consider the effect of the Fermi-momentum of the neutron on the polarization of the final state proton in the reaction $\gamma n \to p\pi^-$ in deuterium. Expressions are derived for the deviation of the polarization from the form $\vec{k} \times \vec{p}_p$. In a practical case this effect is shown to be of the order of 1%.

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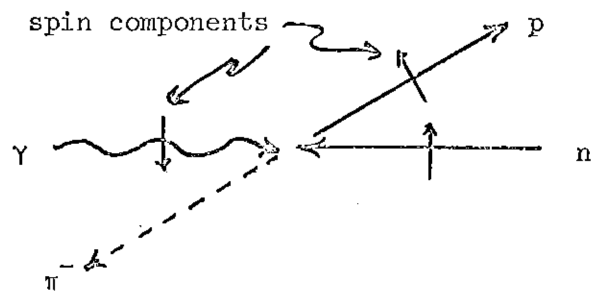
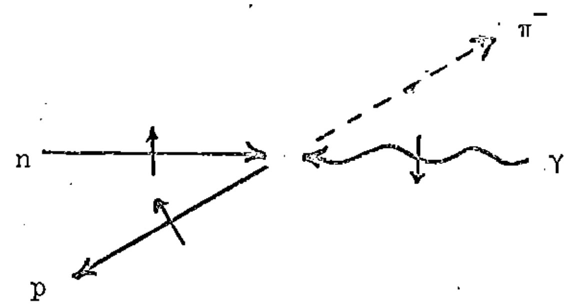
Conservation of parity implies that for unpolarized neutrons, any final state polarization must be perpendicular to the reaction plane in the c.m. frame. An argument in pictures goes like:

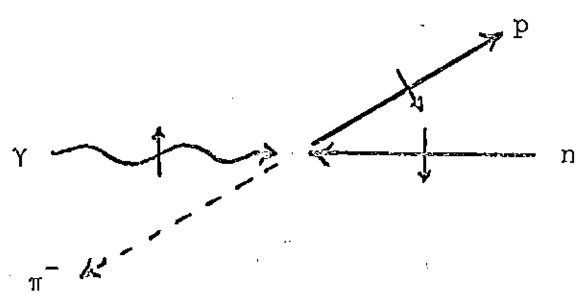
Consider an arbitrary scattering



A parity transformation gives



and a rotation by 180° about an axis perpendicular to the paper gives



Conservation of parity (and unpolarized initial states) implies the first and third pictures occur equally often, and so the proton spin component in the plane averages to zero. This argument doesn't work for the spin component perpendicular to the plane. Result: in the c.m. frame, any polarization must be in the direction $\vec{k} \times \vec{p}$ ($\vec{k}$ = c.m. photon momentum, $\vec{p}$ = c.m. proton momentum).

Conservation of parity implies that for unpolarized photons and neutrons in the reaction $\gamma n \to p\pi^-$, any final state polarization must be perpendicular to the reaction plane in the c.m. frame. To see this, note that the polarization must be a pseudovector. In the c.m. frame the only possible pseudovector is $\vec{k} \times \vec{p}_p$ ($\vec{k}$ = c.m. photon momentum, $\vec{p}_p$ = c.m. proton momentum). In the laboratory frame, however, there are three independent vectors to consider, say $\vec{k}$, $\vec{p}_p$, and $\vec{p}_n$ ($\vec{p}_n$ = neutron momentum). In general, the polarization can be proportional to $\vec{k} \times \vec{p}_p$, $\vec{k} \times \vec{p}_n$, $\vec{p}_p \times \vec{p}_n$ or $\vec{k} \cdot \vec{p}_n \times \vec{p}_n$ multiplied by a linear combination of $\vec{k}$, $\vec{p}_n$, and $\vec{p}_p$. If $\vec{p}_n = 0$, only $\vec{k} \times \vec{p}_p$ remains, a result familiar from consideration of, say, $\gamma p \to p\pi^o$.

To find out what happens to the polarization when we go from the center of mass frame to the lab frame we need a relativistic expression for the polarization of a spin 1/2 particle. We may define a polarization 4-vector, P, such that in the particle's rest frame its time component is zero, and its space part is given by $\vec{P} = \langle\vec{\sigma}\rangle$, where $\vec{\sigma}$ is the Pauli spin operator. Thus $P = (0,\vec{P})$ in the particle's rest frame. The 4-momentum of the particle, p, has components $p = (M,\vec{0})$ in the rest frame. Hence the 4-vector product $pP = 0$ in the rest frame, and, by the principle of relativity, in any other frame.

Returning to polarization in the reaction $\gamma n \to p\pi^-$, we want to consider the c.m. frame of the reaction, where the parity conserving polarization has a known direction. Labelling the c.m. photon momentum by $\vec{k}^*$ (the * indicates c.m. frame) and the proton momentum by $\vec{p}_p^{\,*}$, the favored direction is $\vec{k}^* \times \vec{p}_p^{\,*}$. Decompose the space part of the c.m. polarization, $\vec{P}^*$, into components along the directions $\vec{p}_p^{\,*}$, $\vec{k}^* \times \vec{p}_p^{\,*}$, and $\vec{p}_p^{\,*} \times (\vec{k}^* \times \vec{p}_p^{\,*})$, calling these pieces $\vec{P}_L^*$, $\vec{P}_\perp^*$, and $\vec{P}_{||}^*$ respectively. In the c.m. frame, there is a time component of the polarization given by the requirement of the $pP = 0$ as $P_o^* = \vec{P}^* \cdot \vec{p}_p^{\,*}/E_p^*$ or

$P_o^* = P_L^* p_p^*/E_p^*$. Note that only the longitudinal part of the c.m. polarization is related to the time component.

Now let us transform the polarization into the lab frame. The velocity $\vec{\beta}$ of the c.m. frame with respect to the lab frame is

$$\vec{\beta} = \frac{\vec{k} + \vec{p}_n}{k + E_n} = \frac{\vec{p}_p + \vec{p}_\pi}{E_p + E_\pi}$$

where all the momenta and energies are lab quantities. The c.m. polarization $(P_o^*, \vec{P}^*)$ is related to the lab polarization $(P_o, \vec{P})$ by a Lorentz transformation by $-\vec{\beta}$:

$$P_o = \gamma(P_o^* + \vec{\beta} \cdot \vec{P}^*)$$

$$\vec{P} = \vec{P}^* + (\gamma-1)(\hat{\beta} \cdot \vec{P}^*)\,\hat{\beta} + \gamma\,\vec{\beta}\,P_o^*$$

where $\hat{\beta} = \vec{\beta}/\beta$.

First, consider only the parity conserving polarization, $\vec{P}_\perp^*$. For it, $P_o^* = 0$ so

$$\vec{P}_\perp = \vec{P}_\perp^* + (\gamma-1)(\hat{\beta} \cdot \vec{P}_\perp^*)\,\hat{\beta} \quad .$$

Consider the magnitude of $\vec{P}_\perp^*$ to be 1 for now. Then

$$\vec{P}_\perp^* = \vec{k}^* \times \vec{p}_p^* / |k^* \times \vec{p}_p^*|$$

using

$$\vec{k}^* = \gamma(\vec{k} - k\vec{\beta})$$

$$\vec{p}_p^* = \gamma(\vec{p}_p - E_p\vec{\beta})$$

$$\vec{P}_\perp^* = (\vec{k} \times \vec{p}_p + \vec{\beta} \times (E_p\vec{k} - k\vec{p}_p))/|\text{numerator}| \quad .$$

Thus

$$\vec{P}_I = \frac{\vec{k} \times \vec{p}_p + \vec{\beta} \times (E_p\vec{k} - k\vec{p}_p) + (\gamma-1)(\hat{\beta} \cdot \vec{k} \times \vec{p}_p)\,\hat{\beta}}{|\vec{k} \times \vec{p}_p + \vec{\beta} \times (E_p\vec{k} - k\vec{p}_p)|}$$

$$= \frac{E_n(\vec{k} \times \vec{p}_p) + E_p(\vec{p}_n \times \vec{k}) + k(\vec{p}_p \times \vec{p}_n) + (\gamma-1)(\vec{p}_n \cdot \vec{k} \times \vec{p}_p)(\vec{k} \times \vec{p}_n)(E_n + k)/|\vec{k} + \vec{p}_n|}{|E_n(\vec{k} \times \vec{p}_p) + E_p(\vec{p}_n \times \vec{k}) + k(\vec{p}_p \times \vec{p}_n)|}$$

Recall that if the neutron were at rest in the lab we would expect $\vec{P}_\perp \sim \vec{k} \times \vec{p}_p$. Putting $\vec{p}_n = 0$, the last expression for $\vec{P}_\perp$ reduces to $\vec{k} \times \vec{p}_p$ as desired.

We note that the lab polarization $\vec{P}_\perp$ is not transverse to the proton momentum $\vec{p}_p$. As a result, the time component of the lab polarization is non-zero. Further, the magnitude of $\vec{P}_\perp$ is different from $\vec{P}^*_\perp$.

In a similar fashion the lab components of $P_{||}$ and P_L may be calculated.

The polarization is found by measuring the asymmetry in scattering of the proton off carbon. The differential cross-section is proportional to

$$1 + A\vec{P}' \cdot \hat{n}$$

where A is the analyzing power of carbon, $\hat{n}$ is the normal to the pC scattering plane and $\vec{P}'$ is the proton polarization in the c.m. frame of the proton-carbon system. This frame differs from the lab frame by a Lorentz transformation in the direction of the proton. This transformation leaves $\vec{P}' \cdot \hat{n}$ invariant, so we may substitute $\vec{P}_{lab}$ for $\vec{P}'$ without error.

To find the magnitude of $\vec{P}^*_\perp$, write $1 + AP^*_\perp\vec{P}_\perp \cdot \hat{n}$ using $\vec{P}_\perp$ as above, and solve for $P^*_\perp$ using the likelihood method. To get a measure of the parity violating polarization, write

$$1 + AP_v \hat{n} \cdot \vec{p}_p \times \vec{p}_\perp / |\vec{p}_p \times \vec{p}_\perp|$$

and solve for P_v. P_v is some linear combination of $P^*_{||}$ and P^*_L.

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A calculation for a practical case shows the difference between $\vec{P}_\perp$ and $\vec{k} \times \vec{p}_p$ to be small. I took $E_k = 1000$ MeV, $P_n = 50$ MeV/c, $T_p = 300$ MeV and $\vec{P}_n$ in a plane perpendicular to $\vec{k}$. The table gives $|\vec{P}_\perp|$ and the angle between it and $\vec{k} \times \vec{p}_p$ for several directions of $\vec{p}_n$. It appears that neglecting the relativistic behavior of the polarization would cause less than a 1% error.

Angle of $\vec{p}_n$	$\lvert\vec{P}_\perp\rvert$	Angle Between $\vec{P}_\perp$ and $\vec{k} \times p_p$
0°	1.0	0°
30°	1.001	2.88°
60°	1.002	4.83°
90°	1.002	5.37°
120°	1.001	4.51
150°	1.000	2.55°