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E-144: Rate Estimate for Optical Synchrotron Radiation

Dave Meyerhofer proposes to diagnose the time jitter between the laser and the electron beam using electro-optic switching of infrared synchrotron radiation. At the recent E-144 group meeting Al Odian presented an estimate of the rate of such synchrotron radiation that was somewhat low for practical use. Here we review the rate estimate and appear to find a larger rate.

1 Radiation During Cyclical Motion

As suggested by Al, I start from SLAC-121 by Matthew Sands (1970), but will convert the formulae to ones involving the QED critical field strength

$$B_{\rm crit} = \frac{m^2c^3}{e\hbar} = 4.41 \times 10^{13} \text{ Gauss}.$$

The total radiation rate is given by eq. (4.4) as [see also eq. (10.112) of *Radiation from Relativistic Electrons* by A.A. Sokolov and I. M.Ternov (American Institute of Physics Translation Series, 1986)] for the case of motion involving many complete cycles in a circular storage ring:

$$P_\gamma = \frac{2}{3}\alpha\frac{c}{\lambda_C}mc^2\left(\frac{\gamma B}{B_{\rm crit}}\right)^2 \approx 10^{24}\frac{\rm eV}{\rm sec}\left(\frac{\gamma B}{B_{\rm crit}}\right)^2.$$

The photon-number spectrum $n(u)$ per second per unit energy u is given by eq. (5.8) as

$$n(u)du = \frac{P_\gamma}{u_c^2}F(u/u_c)du,$$

where the so-called critical energy is given by eq. (5.9) as

$$u_c = \frac{3}{2}mc^2\frac{\gamma^2 B}{B_{\rm crit}}.$$

For the case of interest, the infrared-photon energy u is small compared to the critical energy, so we combine eqs. (5.6) and (5.10) to find [see also eq. (10.115) of Sokolov and Ternov]

$$F(u/u_c) \approx 1.34(u_c/u)^{2/3}, \qquad (u \ll u_c),$$

and hence

$$n(u)du \approx 0.4\alpha\frac{c}{\lambda_C}\frac{1}{\gamma^2 mc^2}\left(\frac{u_c}{u}\right)^{2/3}du.$$

I prefer to convert this rate per second into a rate over path length L in the magnetic field, and to insert the expression for u_c in terms of magnetic field. This leads to

$$n(u)du \approx 0.5\alpha \frac{L}{\lambda_C} \frac{1}{\gamma^{2/3}(mc^2)^{1/3}} \left(\frac{B}{B_{\text{crit}}}\right)^{2/3} \frac{du}{u^{2/3}} \approx 0.001 \frac{LB^{2/3}}{\gamma^{2/3}} \frac{du}{u^{2/3}},$$

for path length L in cm, magnetic field B in Gauss, and photon energy u in eV. Then for a 50-GeV electron, $\gamma^{2/3} \approx 2000$, so

$$n(u)du \approx 5 \times 10^{-7} LB^{2/3} \frac{du}{u^{2/3}}.$$

The very soft bend magnet just upstream of IP1 has $L = 100$ cm and $B = 100$ Gauss. Then we expect

$$n(u)du \approx 10^{-3} \frac{du}{u^{2/3}}.$$

For $du/u^{2/3} = 1$ at $u = 1$ eV, and a beam of 10^{10} electrons we then have 10^7 synchrotron-radiation photons (about 1000 times the estimate of Odian).

The soft bend magnets with $L = 100$ cm and $B = 667$ Gauss yield about 4 times as many infrared photons.

It remains to estimate the rate of synchrotron radiation in other elements of the FFTB, but it already appears that there should be plenty of light.

2 Radiation in a Short Magnet

Details of the soft end of the synchrotron-radiation spectrum have probably not been as well studied as at the hard end. As noted in Appendix B of the E-144 proposal, the radiation spectrum due to a small kick in a single short magnet is not necessarily the same as that for electrons in circular motion for many complete cycles. In particular, we found in a brief argument due to Feynman that the low-frequency component is not as small for a small kick as for cyclical motion. Indeed, I infer that

$$F(u/u_c) \approx u_c/u, \qquad (u \ll u_c),$$

is closer to the truth than the expression on p. 1. With this form, the photon-number spectrum over path L would be

$$n(u)du \approx 0.6\alpha \frac{L}{\lambda_C} \frac{B}{B_{\text{crit}}} \frac{du}{u} \approx 3 \times 10^{-6} LB \frac{du}{u}.$$

For $du/u = 1$ at $u = 1$ eV and $B = 100$ Gauss this rate is about 25 times larger than the estimate based on cyclical motion. For the soft bend at 667 Gauss the rate would be 50 times lárger.

Hence I believe the situation in E-144 is very favorable for an optical synchrotron-radiaton monitor.

The literature on synchrotron radiation in short magnets is rather sparse, and is mainly concerned with photon energies above the critical energy. Coïsson's first paper [1], and the Russian papers [5, 7] discuss the low-energy regime and lend support to my claim that $(u/u_c)F(u/u_c)$ goes to a finite constant at low energies in a short magnet.

3 References

[1] R. Coïsson, *On Synchrotron Radiation in Nonuniform Magnetic Fields*, Opt. Comm. **22** (1977) 135.

[2] R. Coïsson, *Angular-Spectral Distribution and Polarization of Synchrotron Radiation from a "Short" Magnet*, Phys. Rev. A **20** (1979) 524.

[3] R. Bossart *et al.*, *Observation of Visible Synchrotron Radiation Emitted by a High-Energy Proton Beam at the Edge of a Magnetic Field*, Nucl. Instr. and Meth. **164** (1979) 375.

[4] R. Bossart *et al.*, *Proton Beam Profile Measurements with Synchrotron Light*, Nucl. Instr. and Meth. **184** (1981) 349.

[5] V.G. Bagrov *et al.*, *Charged Particle Radiation in Magnetic Systems*, Nucl. Instr. and Meth. **195** (1982) 569.

[6] A. Hoffmann and F. Méot, *Optical Resolution of Beam Cross-Section Measurement by Means of Synchrotron Radiation*, Nucl. Instr. and Meth. **203** (1982) 483.

[7] V.G. Bagrov *et al.*, *Radiation of Relativistic Electrons Moving in an Arc of a Circle*, Phys. Rev. D **28** (1983) 2464.