

Introduction

Intense muon beams have many potential applications, including neutrino factories and muon colliders. However, muons are produced as tertiary beams, resulting in diffuse phase space distributions. To make useful beams, the muons must be rapidly cooled before they decay. An idea conceived recently for the collection and cooling of muon beams is the Quasi-Isochronous Helical Cooling Channel (QHCC), which takes advantage of the larger RF buckets for particles traveling in nearly isochronous orbits. The QHCC also offers a natural match into the HCC, which is recognized as the most efficient cooling scheme for a neutrino factory or muon collider.

RF Bucket Area

$$A_{bucket} \cong \frac{16}{w_{rf}} \sqrt{\frac{eV'_{max} \lambda_{RF} m_{\mu} c^2}{2\pi |\eta_H|}} \left[\frac{1 - \sin(\phi_s)}{1 + \sin(\phi_s)} \right]$$

$$\eta_H = \frac{\sqrt{1 + \kappa^2}}{\beta^3} \left(\frac{\kappa^2}{1 + \kappa^2} \hat{D} - \frac{1}{\gamma^2} \right)$$

$$\hat{D} = \frac{adp}{pda} = \frac{\kappa^2 + (1 - \kappa^2)[\mathcal{B}\sqrt{1 + \kappa^2}/p\theta - 1]}{1 + \kappa^2} \frac{(H\kappa)^2 \partial b_p}{p \hat{\rho}}$$

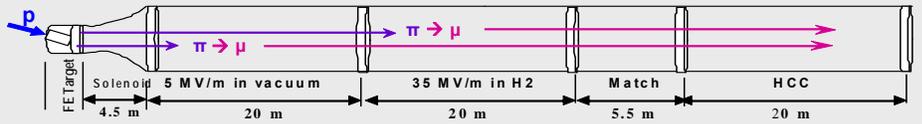


Figure 1: Layout showing modified tapered solenoid ending with $B_z = 4.2$ T, 20 m of RF with 5 MV/m in vacuum, 20 m of RF with 35 MV/m in H_2 gas and Be windows with varying thicknesses, the match, and HCC.

Table 1: Layout Parameters

z(m)	Subsystem	Purpose	Physical Dimensions	Fields
0.0 to 4.5	Capture/Tapered Solenoid	Enhance pion/muon capture	L = 4.5 m R = 7.5 cm → 35 cm	$B_{sol} = 20 \text{ T} \rightarrow 4.2 \text{ T}$
4.5 to 24.5	First straight RF Buncher in vacuum	1. Initial capture of π^+ 's & μ^+ 's into RF buckets. 2. Allow lower momenta π^+ 's to decay into μ^+ 's.	L = 20 m R = 35 cm	$B_{sol} = 4.2 \text{ T}$ 160 RF Cavities: $V'_{max} = 5 \text{ MV/m}, f = 162.5 \text{ MHz}$ $\phi_s = 186^\circ; P(\mu^-) = 150 \rightarrow 162 \text{ MeV/c}$
24.5 to 44.5	Second straight RF Buncher in 100 atm H_2 w/ variably thick Be windows.	1. H_2 gas allows higher RF gradient for enlarged buckets. 2. Be causes higher momenta π^+ 's to interact, enhancing useful μ^+ 's.	L = 20 m R = 35 cm	$B_{sol} = 4.2 \text{ T}$ 160 RF Cavities: $V'_{max} = 35 \text{ MV/m}, f = 162.5 \text{ MHz}$ $\phi_s = 208 \rightarrow 194^\circ; P(\mu^-) = 162 \rightarrow 237 \text{ MeV/c}$
44.5 to 50.0	Match into HCC	1. To match between straight solenoid into HCC. 2. Enhance μ capture due to transition occurring in match.	L = 5.5 m (5.5 λ_s) R = 35 cm	$B_{sol} = 6.3 \text{ T} \rightarrow 4.2 \text{ T}$ 44 RF Cavities: $V'_{max} = 35 \text{ MV/m}, f = 162.5 \text{ MHz}$ ϕ_s varied to maintain $P(\mu^-) = 237 \text{ MeV/c}$
50.0 to 70.0	HCC	To cool muons.	L = 20 m (20 λ_s) R = 35 cm	$B_{sol} = 4.2 \text{ T}$ 160 RF Cavities: $V'_{max} = 35 \text{ MV/m}, f = 162.5 \text{ MHz}$ $\phi_s = 12.6^\circ$ to maintain $P(\mu^-) = 237 \text{ MeV/c}$

Muons & Pions in the Straight Solenoids

Pi- & Mu- Just After Tapered Solenoid Ending w/ $B_z=4.2\text{T}$

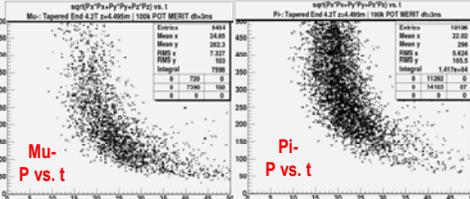


Figure 2: Momentum (MeV/c; vertical) vs. time (nsec; horizontal) for μ^- 's and π^- 's out of tapered solenoid.

At end of 5 MV/m Straight in Vacuum ($z=24.5\text{m}$)

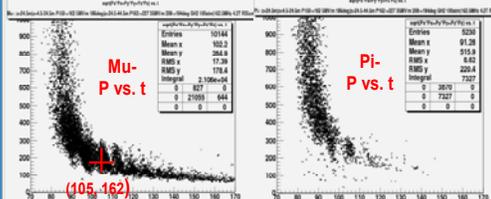


Figure 3: Momentum (MeV/c) vs. time (nsec) for μ^- 's and π^- 's after the first straight in vacuum.

Birth of Particles

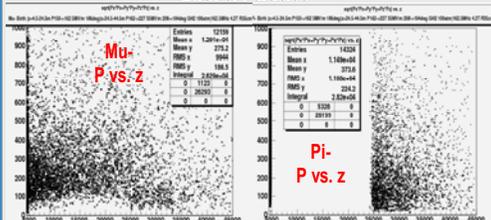


Figure 4: Momentum (MeV/c) vs. z (mm) for μ^- 's and π^- 's at the particle's creation.

At End of 35 MV/m Straight w/ H_2 @ 100 atm 273K ($z=44.5\text{m}$)

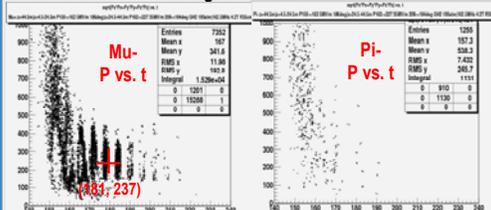


Figure 5: Momentum (MeV/c) vs. z (mm) for μ^- 's and π^- 's at exit of the second straight and entrance of the matching section.

Matching Section

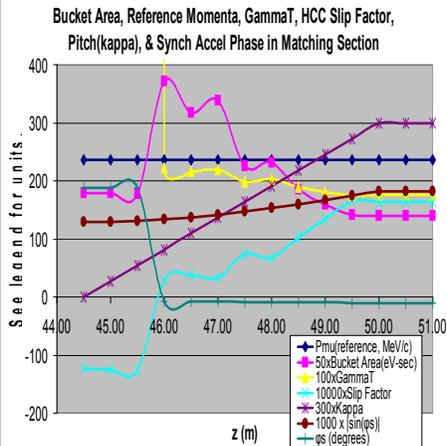


Figure 6: Design parameters in matching section. Accelerating phase ϕ_s designed to maintain constant momentum of 237 MeV/c

At End of Matching Section ($z = 50.0 \text{ m}$)

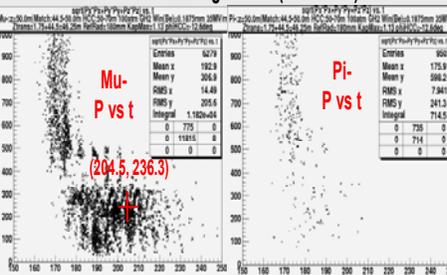


Figure 7: Momentum (MeV/c) vs. z (mm) for μ^- 's and π^- 's at exit of the matching section.

Summary

- We have made a preliminary design of a system upstream of the HCC to enhance the number of muons in its acceptance.
- An innovation has been introduced to use the high energy pions to create useful muons by incorporating material at strategic locations.
- We have added RF with H_2 gas into an HCC matching section and performed an initial study that involves crossing transition.

Acknowledgements

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Optimization with Only ϕ_s

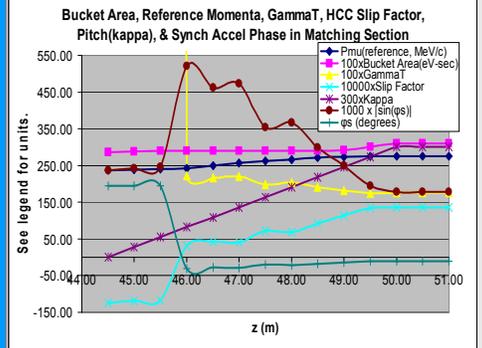


Figure 9: Example of design parameters in matching section that uses same γ_T of established HCC match, but manipulates reference momenta to allow monotonic growth of bucket area.

Future

- The present study started from an HCC designed without RF or material.
- Capture into RF buckets in the match should be greatly increased by designing it with RF and material from the start:
- ϕ_s : We have seen that the increase in pitch κ forces ϕ_s to increase with z , which decreases the RF bucket when other parameters are fixed. It may still be possible to have a monotonically increasing bucket area along z by adjusting the reference momenta (equivalently $\phi_s(z)$), to an unrealistic degree of control ($dp \sim 0.01 \text{ MeV/c}$) as illustrated in Figure 9, motivating need for other parameters to control bucket area.
 - η_H : The slip factor, η_H (or db_p/dp), provides a degree of freedom to control bucket growth.
 - V'_{max} : If for some unforeseen reason the desired bucket area cannot be achieved by current containing coils, the last degree of freedom exercisable is $V'_{max}(z)$ at price of running sub-optimal earlier in the channel.
- Upstream of QHCC,
- Placement and profile of material in the second straight section will be optimized for using high energy pions to create useful muons.
 - Quantify enhanced muon rates from other particles produced at the target that interact with the material to produce useful muons.

References

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