

QUASI-ISOCRONOUS MUON COLLECTION CHANNELS*

C. Yoshikawa[#], C. Ankenbrandt, Muons, Inc., Batavia IL, USA

D. Neuffer, Fermilab, Batavia, IL, USA

Abstract

Intense muon beams have many potential applications, including neutrino factories and muon colliders. However, muons are produced as tertiary beams, resulting in diffuse phase space distributions. To make useful beams, the muons must be rapidly cooled before they decay. An idea conceived recently for the collection and cooling of muon beams, namely, the use of a Quasi-Isochronous Helical Channel (QIHC) to facilitate capture of muons into RF buckets, has been developed further. The resulting distribution could be cooled quickly and coalesced into a single bunch to optimize the luminosity of a muon collider. After a brief elaboration of the QIHC concept, recent developments are described.

INTRODUCTION

A Quasi-Isochronous Helical Channel (QIHC) is being investigated as an alternative to a significant portion of the baseline front end for a neutrino factory or muon collider. The study 2A [1] front end consists of a target solenoid (20 T), a tapered capture solenoid (20 T to 2T, 12 m long), a drift section (99 m), an RF buncher (50 m), an energy-phase rotator (54 m), and a cooling region (80 m). The QIHC might be developed to replace all but the last cooling stage, and this cooling section may be replaced by a Helical Cooling Channel (HCC) [2,3]. The QIHC offers a more natural match into the potentially more efficient HCC.

The QIHC concept takes advantage of the larger RF buckets for particles traveling in nearly isochronous orbits. Critical components of a QIHC system, as presently conceived, include the following: (1) a helical magnetic field that creates helical particle trajectories near a reference orbit of a selected muon momentum, (2) RF cavities that capture particles in stable buckets, and (3) an absorber that reduces the energy of particles that would otherwise be too energetic to be captured. In this paper, we present the analytic theory behind the concept along with simulation results. However, current simulations are based on an existing HCC configuration [3] and hence do not fully exercise all possible parameters; further simulations are to be performed in the near future.

NEARLY ISOCRONOUS CONDITION

The QIHC aims to take advantage of a larger RF bucket area when operating near transition:

$$A_{bucket} \cong \frac{16}{w_{rf}} \sqrt{\frac{eV'_{max} \lambda_{RF} m_{\mu} c^2}{2\pi |\eta_H|}} \left[\frac{1 - \sin(\phi_s)}{1 + \sin(\phi_s)} \right] \quad (1)$$

where

- the term in brackets is an approximation for the moving-bucket factor
- w_{rf} is the RF frequency in radians/second
- V'_{max} is the maximum E-field voltage gradient
- λ_{rf} is the RF wavelength
- m_{μ} is the mass of the muon
- ϕ_s is the synchronous particle RF phase, and

η_H is the slip factor, derived in [2] for an HCC as:

$$\eta_H = \frac{\sqrt{1 + \kappa^2}}{\gamma \beta^3} \left(\frac{\kappa^2}{1 + \kappa^2} \hat{D} - \frac{1}{\gamma^2} \right) \quad (2)$$

where γ becomes γ_T when $\eta_H=0$ and the dispersion factor \hat{D} relates to apparatus quantities and design momentum via:

$$\hat{D}^{-1} = \frac{a}{p} \frac{dp}{da} = \frac{\kappa^2 + (1 - \kappa^2) [(B \sqrt{1 + \kappa^2} / p k) - 1]}{1 + \kappa^2} \frac{(1 + \kappa^2)^{3/2}}{p k^2} \frac{\partial b_{\phi}}{\partial \rho} \quad (3)$$

in which

- p is reference momentum; a is reference radius
- $\kappa = p_{\text{transverse}} / p_z =$ helix pitch
- B is the solenoid B_z
- $k = 2\pi/\lambda$; λ is helix period
- $\frac{\partial b_{\phi}}{\partial \rho}$ being the quadrupole component

DESIGN & SIMULATION

Since the HCC is charge specific [2], this paper focuses on capture of μ^- . Our original intention was to manipulate the parameters that control transition in a HCC after matching into it from a solenoid. However, preliminary studies showed that transition occurs in the match itself for an existing HCC configuration optimized for cooling. Instead of redesigning the match, we decided to start our simulation effort based on an established HCC design and its matching portion [3]. All simulations utilized G4beamline [4].

The cooling HCC has B_z on the reference of 4.2 T and the acceptance is approximately $150 \text{ MeV}/c < p < 300 \text{ MeV}/c$. Hence, we modified the portion upstream of the HCC as follows:

1. The tapered solenoid is modified from B_z with 20 T $\rightarrow \sim 2 \text{ T}$ to 20 T $\rightarrow 4.2 \text{ T}$. This shortens the tapered solenoid from about 10 m to 4.5 m.
2. In order to maximize the number of muons that fall into the HCC acceptance, we implement two sequential straight sections:
 - a. 20 m of RF in vacuum at 5 MV/m to capture μ^- 's and π^- 's and allow lower momenta π^- 's to decay into μ^- 's.
 - b. 20 m of RF in material (Be & 100 atm H_2 gas) at 35 MV/m to enlarge RF bucket size and cause otherwise useless higher energy π^- 's to interact

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[#]cary.yoshikawa@muonsinc.com

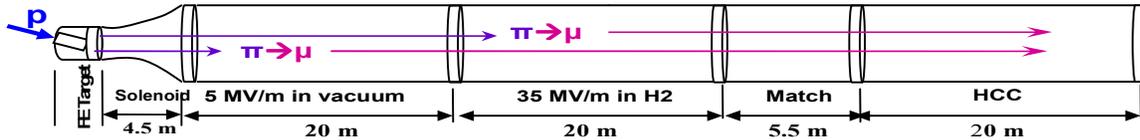


Figure 1: Layout showing modified tapered solenoid ending with $B_z=4.2\text{T}$, 20 m of RF with 5 MV/m in vacuum, 20 m of RF with 35 MV/m in H_2 gas and Be windows with varying thicknesses, the matching section, and the HCC.

Table 1: Layout Parameters

z(m)	Subsystem	Purpose	Physical Dimensions	Fields
0.0 to 4.5	Capture/Tapered Solenoid	Enhance pion/muon capture	$L = 4.5\text{ m}$ $R = 7.5\text{ cm}$ $\rightarrow 35\text{ cm}$	$B_{\text{sol}} = 20\text{ T} \rightarrow 4.2\text{ T}$
4.5 to 24.5	First straight RF Buncher in vacuum	1. Initial capture of π 's & μ 's into RF buckets. 2. Allow lower momenta π 's to decay into μ 's.	$L = 20\text{ m}$ $R = 35\text{ cm}$	$B_{\text{sol}} = 4.2\text{ T}$ 160 RF Cavities: $V'_{\text{max}} = 5\text{ MV/m}$, $f = 162.5\text{ MHz}$ $\phi_s = 186^\circ$: $P(\mu^-) = 150 \rightarrow 162\text{ MeV/c}$
24.5 to 44.5	Second straight RF Buncher in 100 atm H_2 w/ variably thick Be windows.	1. H_2 gas allows higher RF gradient. 2. Be causes higher momenta π 's to interact, enhancing useful μ 's. 3. Transverse cooling.	$L = 20\text{ m}$ $R = 35\text{ cm}$	$B_{\text{sol}} = 4.2\text{ T}$ 160 RF Cavities: $V'_{\text{max}} = 35\text{ MV/m}$, $f = 162.5\text{ MHz}$ $\phi_s = 208 \rightarrow 194^\circ$, $P(\mu^-) = 162 \rightarrow 237\text{ MeV/c}$
44.5 to 50.0	Match into HCC	1. To match between straight solenoid into HCC. 2. Enhance μ capture due to transition occurring in match.	$L = 5.5\text{ m}$ (5.5λ 's) $R = 35\text{ cm}$	$B_{\text{sol}} = 6.3\text{ T} \rightarrow 4.2\text{ T}$ 44 RF Cavities: $V'_{\text{max}} = 35\text{ MV/m}$, $f = 162.5\text{ MHz}$ ϕ_s varied to maintain $P(\mu^-) = 237\text{ MeV/c}$
50.0 to 70.0	HCC	To cool muons in 5D phase space.	$L = 20\text{ m}$ (20λ 's) $R = 35\text{ cm}$	$B_{\text{sol}} = 4.2\text{ T}$ 160 RF Cavities: $V'_{\text{max}} = 35\text{ MV/m}$, $f = 162.5\text{ MHz}$ $\phi_s = -12.6^\circ$ to maintain $P(\mu^-) = 237\text{ MeV/c}$

with material, producing lower energy π 's that decay into μ 's in the useful momenta range.

After the modified portion upstream of the HCC, we began to use the established match and HCC. The match has added to it RF, H_2 gas, and Be windows. The HCC itself also incorporates RF and material, but this is an innovation for the matching section. Figure 1 and Table 1 show and describe the layout, respectively.

Figure 2 shows that muons from a MERIT-like targetry [5] exiting the tapered solenoid peak at around 150 MeV/c in momenta, near the bottom of the HCC acceptance. So, the first 20 m straight is timed to capture 150 MeV/c muons and phased to accelerate them to 162 MeV/c at the end of this first straight, with results shown in Figure 3.

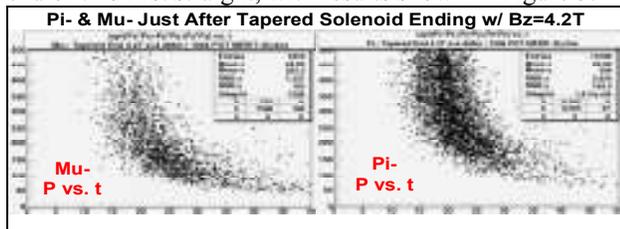


Figure 2: Momentum (MeV/c; vert.) vs. arrival time (nsec; horiz.) for μ^- 's and π^- 's exiting tapered solenoid.

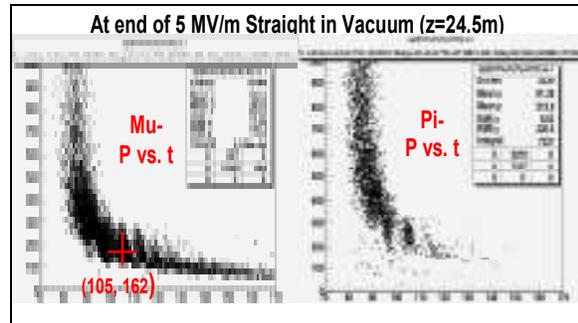


Figure 3: Momentum (MeV/c) vs. arrival time (nsec) for μ^- 's and π^- 's after the first straight solenoid in vacuum.

The second straight section introduces a variable amount of Be, totaling half an interaction length for pions, to cause the otherwise useless higher energy pions to interact and create lower energy pions that decay into muons of the right energy for the HCC acceptance. Figure 4 shows the momenta and longitudinal position at creation of muons and pions; pion creation is readily seen at the start of the second straight. The longitudinal dynamics of μ 's and π 's at the end of the second straight section are displayed in Figure 5.

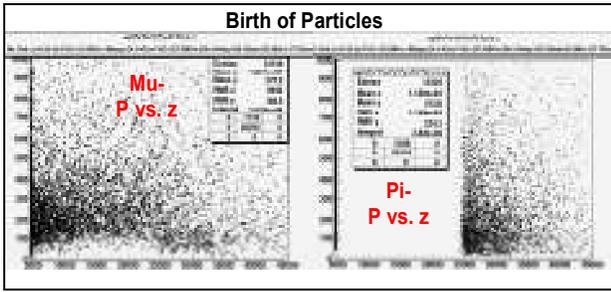


Figure 4: Momentum (MeV/c) vs. z (mm) for μ^- 's and π^- 's at the particle's creation.

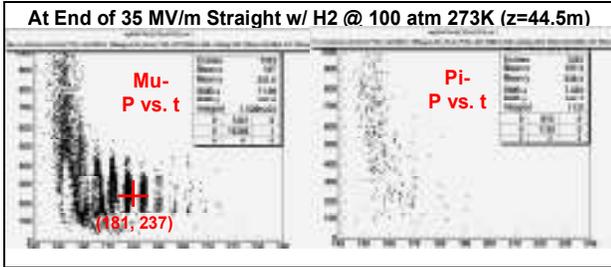


Figure 5: Momentum (MeV/c) vs. z (mm) for μ^- 's and π^- 's at exit of the second straight and entrance of the matching section.

In this first study, the design philosophy for the match was to keep the RF bucket as large as possible throughout, which translated into keeping a constant momentum of 237 MeV/c. Since the pitch ($\kappa = P_{\text{trans}}/P_z$) of the path grows from 0 to 1, the path length likewise grows per unit of z, necessitating a growth in ϕ_s as the μ^- plows through more material, causing the RF bucket size to shrink due to the factor in square brackets in equation 1. We also extracted γ_T from the match via the fastest time of flight between transverse planes, which guided us to know when to jump the phase to maintain stability. Figure 6 shows the accelerating phase, ϕ_s , the derived bucket area, and other relevant parameters. Figure 7 contains the final distribution of μ^- 's and π^- 's at the end of the match.

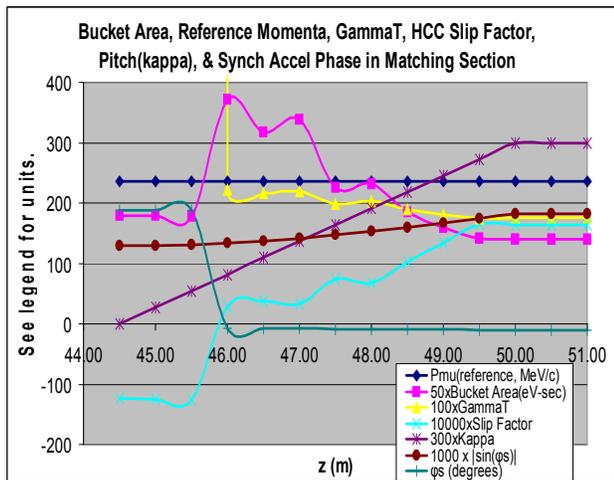


Figure 6: Design parameters in matching section. Accelerating phase ϕ_s is designed to maintain constant momentum of 237 MeV/c.

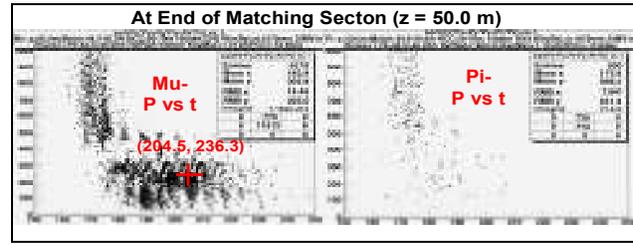


Figure 7: Momentum (MeV/c) vs. z (mm) for μ^- 's and π^- 's at exit of the matching section.

SUMMARY & FUTURE

We have made a preliminary design of a system upstream of the HCC to enhance the number of muons in its acceptance. An innovation has been introduced to use the high energy pions to create useful muons by incorporating material at strategic locations. We have added RF with H_2 gas into the match and performed an initial study that involves crossing transition.

The established matching portion upon which we based our study was designed without RF or material. We believe that capture into RF buckets in the match can be greatly increased by designing it with RF and material from the start. In particular, the size of the RF bucket in equation (1) is driven by ϕ_s , η , and V'_{max} . We have seen that to maintain a fixed momentum, the increase in pitch κ forces ϕ_s to increase, which decreases the RF bucket size, although it is possible to manipulate $p(z)$ to achieve a monotonic growth of bucket area. The slip factor, η_H , provides a degree of freedom to control bucket area growth. Via equations (1) to (3), a given profile of $p(z)$ will define $(\partial b_\phi / \partial r)(z)$ that is necessary to obtain the desired $A_{\text{bucket}}(z)$. If for some unforeseen reason the desired $(\partial b_\phi / \partial r)(z)$ cannot be obtained by current containing coils, the last degree of freedom exercisable is $V'_{\text{max}}(z)$. Hence, we plan to design a matching section that fundamentally integrates RF and material to achieve very large RF buckets for capture and transport into the HCC, which has been demonstrated to be a very efficient cooling scheme for a neutrino factory or muon collider.

Once the match and HCC are designed, the profile of material in the second straight section will be optimized, including other particles created at the target that will increase the rate of muons captured and transported.

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