

# Pulses and $1/f$ Noise

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## 1 Problem

Discuss the relation between pulsed phenomena and  $1/f$  noise.

## 2 Solution

Low-frequency fluctuations with a  $1/f$  frequency spectrum were first noticed by Johnson [1] in thermionic-emission currents in vacuum tubes, and exist in a very wide range on phenomena.<sup>1</sup> A feature common to most of these phenomena is that they involve a quasirandom sequence of pulses on the microscopic scale.

The spectrum of energies associated with a short pulse of characteristic width  $\Delta t$  is roughly flat up to  $E_{\max} = \hbar/\Delta t$ . In general, the pulse process can be associated with quanta (such as phonons or photons) of energy  $\hbar\omega$ , so the number spectrum of these quanta varies as  $1/\omega$  up to  $\omega_{\max} = 1/\Delta t$ .

When a phenomenon consists of a sequence of closely spaced pulses, such that the macroscopic behavior is quasistatic, the microscopic behavior involve fluctuations with the frequency spectrum of the associated quanta, namely  $1/\omega$ , *i.e.*,  $1/f$ .

*While the author considers the above to be fairly “obvious”, the literature on  $1/f$  noise mainly involves much more arcane, and less satisfactory, explanations. The above view was advocated by Handel in 1975 [11, 12] (in the context of Bremsstrahlung emitted by conduction electrons in collisions with lattice ions), although it seems to be regarded with surprising (to this author) skepticism.<sup>2</sup>*

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<sup>1</sup>See, for example, [2, 3, 4, 5, 6, 7, 8, 9, 10].

<sup>2</sup>See, for example, sec. IIIC of [6]. Exceptions are [13, 14]. For a “fractal” view of  $1/f$  noise, see [7].

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