If a Pig Had Wings...

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1 Problem

If a pig had wings, how far could it fly?

An approximate analysis will suffice.

2 Solution

If a pig had wings, these must provide lift equal to the weight of the pig:

$$\text{lift} = m_{\text{pig}}\, g, \tag{1}$$

where $g \approx 10$ m/s^2 is the acceleration due to gravity. Now,

$$\text{lift} = k\, \text{drag}, \tag{2}$$

with the lift-to-drag ratio k near 10 for a well-designed pig. Using dimensional analysis, we estimate the drag force as

$$\text{drag} \propto \text{area}_{\text{wing}}\, \rho_{\text{air}}\, v_{\text{pig}}^2, \tag{3}$$

where ρ_{air} is the mass density of air, and v is the speed of the pig in flight. Combining (1-3),

$$\text{area}_{\text{wing}} \approx \frac{m_{\text{pig}}\, g}{k\, \rho_{\text{air}}\, v_{\text{pig}}^2}. \tag{4}$$

As $v_{\text{pig}} \to 0,\ \text{area}_{\text{wing}} \to \infty$.

So, we must consider how fast the pig can fly. The faster it flies, the more energy it must provide.

$$\text{Power} = F\, v = \text{drag}\, v_{\text{pig}} \approx \text{area}_{\text{wing}}\, \rho_{\text{air}}\, v_{\text{pig}}^3, \tag{5}$$

so

$$\text{area}_{\text{wing}} \approx \frac{1\ \text{pigpower}}{\rho_{\text{air}}\, v_{\text{pig}}^3}. \tag{6}$$

Equating the two expressions, (4) and (6), for the wing area, we find

$$v_{\text{pig}} \approx \frac{k(1\ \text{pigpower})}{m_{\text{pig}}\, g}. \tag{7}$$

Suppose $m_{\text{pig}} = 200$ kg, $k = 10$, and 1 pigpower = 2/7.6 horsepower = 200 W. Then,

$$v_{\text{pig}} \approx \frac{10 \cdot 200}{200 \cdot 10} = 1\ \text{m/s}. \tag{8}$$

To estimate the wing area, we note that $\rho_{\rm air} \approx 0.001 \rho_{\rm water} = 1$ kg/m^3. Then, from (6),

$$\text{area}_{\rm wing} \approx \frac{200}{1 \cdot (1)^3} = 200 \text{ m}^2. \tag{9}$$

The wings must, of course, be massless for the above analysis to hold.

How far can a pig fly?

Suppose the pig can dedicate his(her) breakfast calories to flying. Say 25,000 calories $= 4,000 \cdot 25,000 = 10^8$ J. The pig uses up this energy at the rate of 200 W, so it could fly for time $t = 10^8/200 = 5 \times 10^5$ s $= 5$ days. However, this seems too long for a pig to exert maximum power. I downrate the pig's flying time to 3 hours $\approx 10,000$ s. Then, the flight distance is

$$d = v_{\rm pig}\, t = 1 \cdot 10,000 = 10,000 \text{ m} = 10 \text{ km}. \tag{10}$$

If the lift-to-drag ratio k had been estimated as 1 rather than 10, then $v_{\rm pig} = 0.1$ m/s, and $d = 1$ km.

It has been demonstrated that humans can power a large wingspan aircraft, such as Paul MacReady's Gossamer Condor [1], at low speeds for distances of a few tens of kilometers.

3 Acknowledgment

This problem is a variant on J.A. Wheeler's classic exam question: How far can a goose fly?

References

[1] http://www.donaldmonroe.com/