

# Ph 406: Elementary Particle Physics

## Problem Set 7

K.T. McDonald

kirkmcd@princeton.edu

Princeton University

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### 1. Baryon Magnetic Moments in SU(2)

The requirement that the interaction Hamiltonian be an isoscalar leads to the understanding that the magnetic moment  $\mu$  of spin-1/2 baryons has the form

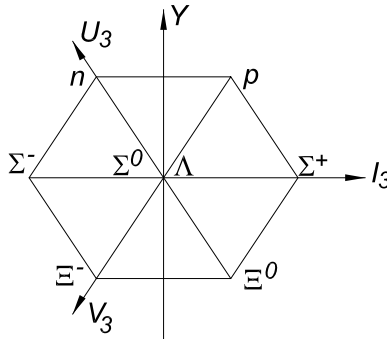
$$\mu = A + BI_3, \quad (1)$$

for particles within an isospin multiplet, where the constants  $A$  and  $B$  differ from multiplet to multiplet.<sup>1</sup> Deduce the one nontrivial relation among magnetic moments of the basic spin-1/2 baryon octet that can be made from eq. (1). Is this prediction testable by the methods used to measure magnetic moments of short-lived baryons?

If the form (1) also holds for the spin-3/2 baryon decuplet, what relations among their magnetic moments are implied?<sup>2</sup> Are these predictions testable?

### 2. Baryon Magnetic Moments in SU(3)

If SU(3) were a good symmetry, then relations like (1) should also hold for so-called  $U$ -spin and  $V$ -spin, which provides relations among magnetic moments in of particles in different isospin multiplets. While SU(3) is not an exact symmetry for baryon “flavor,” Gell-Mann<sup>3</sup> and Okubo<sup>4</sup> had good success in relating baryon masses by supposing the pattern of symmetry breaking was between, but not within,  $U$ -spin multiplets.



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<sup>1</sup>R. Marshak, S. Okubo and G. Sudarshan, *Consequences of Charge Independence for the Magnetic Moments and Masses of  $\Sigma$  Hyperons*, Phys. Rev. **106**, 599 (1957),

[http://physics.princeton.edu/~mcdonald/examples/EP/marshak\\_pr\\_106\\_599\\_57.pdf](http://physics.princeton.edu/~mcdonald/examples/EP/marshak_pr_106_599_57.pdf).

<sup>2</sup>The magnetic moment  $\mu$  of a spin-3/2 particle can be in four states with respect to some  $z$ -axis,  $\mu_z = \mu, \mu/3, -\mu/3$  and  $-\mu$ . Note the appearance of a fractional quantum number.

<sup>3</sup>M. Gell-Mann, *The Eightfold Way: A Theory of Strong Interaction Symmetry*, CTSL-20 (1961),

[http://physics.princeton.edu/~mcdonald/examples/EP/gellmann\\_ctsl-20\\_61.pdf](http://physics.princeton.edu/~mcdonald/examples/EP/gellmann_ctsl-20_61.pdf).

<sup>4</sup>S. Okubo, *Note on Unitary Symmetry in Strong Interactions*, Prog. Theor. Phys. **27**, 949 (1962),

[http://physics.princeton.edu/~mcdonald/examples/EP/okubo\\_ptp\\_27\\_949\\_62.pdf](http://physics.princeton.edu/~mcdonald/examples/EP/okubo_ptp_27_949_62.pdf).

Hence, it seems reasonable to suppose that eq. (1) holds only for  $I$ -spin, and that magnetic moments are the same for all members of a  $U$ -spin multiplet,<sup>5</sup> but differ for different  $U$ -spin multiplets. Show that the magnetic moments of the spin-1/2 baryon octet, and the so-called transition moment  $\langle \Sigma^0 | \mu | \Lambda \rangle$ , can then be expressed in terms of  $\mu_p$  and  $\mu_n$ .

A result from prob. 1 is needed here. Note that the  $U_3 = 0$  states corresponding to the  $I_3 = 0$  states  $\Lambda$  and  $\Sigma^0$  can be obtained from the latter via rotation by  $120^\circ$  in  $SU(3)$  space ( $I_3$ - $Y$  space).

The present data are, in terms of the nuclear/nucleon magneton  $\mu_N = e\hbar/2m_p c$ ,

$$\langle p | \mu | p \rangle = \mu_p = 2.79\mu_N, \quad (2)$$

$$\mu_n = -1.91\mu_N, \quad (3)$$

$$\mu_\Lambda = -0.613 \pm 0.004\mu_N, \quad (4)$$

$$\mu_{\Sigma^+} = 2.46 \pm 0.01\mu_N, \quad (5)$$

$$\langle \Sigma^0 | \mu | \Lambda \rangle = -1.61 \pm 0.08\mu_N, \quad (6)$$

$$\mu_{\Sigma^-} = -1.16 \pm 0.03\mu_N, \quad (7)$$

$$\mu_{\Xi^0} = -1.25 \pm 0.02\mu_N, \quad (8)$$

$$\mu_{\Xi^-} = -0.651 \pm 0.003\mu_N, \quad (9)$$

### 3. Baryon Magnetic Moments in the $SU(6)$ Constituent-Quark Model

Deduce the quark flavor + spin wavefunctions for the spin-up states of the spin-1/2 baryon octet, and use these to predict their magnetic moments.

In this model we suppose the quarks have Dirac magnetic moments,

$$\mu_q = \frac{Q_q \hbar}{2m_q c}, \quad (10)$$

where  $m_q$  is the (constituent-model) mass of the quark.<sup>6</sup> Use the data given in prob. 2 for  $\mu_p$ ,  $\mu_n$  and  $\mu_\Lambda$  to determine  $m_u$ ,  $m_d$  and  $m_s$ , and give model predictions for the remaining baryon-octet magnetic moments (including the transition moment  $\langle \Sigma^0 | \mu | \Lambda \rangle$ ).

The quarks are fermions, so the total wavefunction should be antisymmetric under quark exchange. There is no orbital angular momentum in the quark wavefunctions for the baryon octet, so the spatial part of the wavefunctions are symmetric under quark exchange. In the  $SU(6)$  symmetry of quark flavor + spin it seems most natural to consider the baryon octet and decuplet as comprising the symmetric multiplet  $\mathbf{56} = (\mathbf{8}, \mathbf{2}) + (\mathbf{10}, \mathbf{4})$ , which seemed initially to contradict that quarks are fermions. Only in the 1970's did it become clear that quarks carry color charge that obeys an  $SU(3)$

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<sup>5</sup>The Dirac magnetic moment a spin-1/2 particle is  $Q\hbar/2Mc$ , so if  $SU(3)$  is a good symmetry all baryon octet masses are the same, and the magnetic moments should be the same within  $U$ -spin multiplets for which the members have the same charge. And, the magnetic moments of the  $U = 1$  and  $U = 0$  multiplets differ only by their sign. This factoid turns out to be equivalent to the requirement that the sum of the baryon octet magnetic moments is zero if  $SU(3)$  is a good symmetry.

<sup>6</sup>If  $SU(3)$  were a good symmetry for quark flavor then all quarks would have the same mass  $m_q$ .

color symmetry in which hadrons are color singlets,<sup>7</sup> which are antisymmetric under exchange.<sup>8</sup> Hence, you are to construct  $SU(18) = SU(3)_{\text{flavor}} \times SU(2)_{\text{spin}} \times SU(3)_{\text{color}}$  wavefunctions in which the  $SU(3)_{\text{flavor}} \times SU(2)_{\text{spin}}$  part is exchange symmetric, and the  $SU(3)_{\text{color}}$  part is antisymmetric (which latter part need not be displayed in your solution).

4. Show that the relative color-force amplitudes for the  $q\bar{q}$  colored-quark states  $\psi_{\text{color}} = (r\bar{r} - g\bar{g})/\sqrt{2}$  and  $(2b\bar{b} - r\bar{r} - g\bar{g})/\sqrt{6}$  due to the exchange of a single color-octet gluon are  $1/3$  (as are the related amplitudes  $\langle r\bar{g} | \text{color} | r\bar{g} \rangle$ , *etc.*, discussed on pp. 262-265, Lecture 14 of the Notes). The implication is that color-octet  $q\bar{q}$  states are not bound.

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<sup>7</sup>H. Fritzsch, M. Gell-Mann and H. Leutwyler, *Advantages of the Color Octet Gluon Picture*, Phys. Lett. **47B**, 365 (1973), [http://physics.princeton.edu/~mcdonald/examples/EP/fritzsch\\_pl\\_b47\\_365\\_73.pdf](http://physics.princeton.edu/~mcdonald/examples/EP/fritzsch_pl_b47_365_73.pdf).

<sup>8</sup>The only antisymmetric  $qqq$  multiplet in color  $SU(3)$ ,  $\mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{8} + \mathbf{8}^* + \mathbf{10}$ , is the  $\mathbf{1}$ . The  $r, g, b$  color singlet wavefunction is  $\psi_{\text{color}} = (rgb - grb + gbr - bgr + brg - rbg)/\sqrt{6}$ .

## Solutions

1. The two constants  $A$  and  $B$  in eq. (1) provide no constraints unless the isospin is 1 or higher. In the spin-1/2 baryon octet, we learn something nontrivial from this equation only for the isotriplet  $\Sigma^+$ ,  $\Sigma^0$ ,  $\Sigma^-$ ,

$$\mu_{\Sigma^+} = A + B, \quad \mu_{\Sigma^0} = A, \quad \mu_{\Sigma^-} = A - B, \quad (11)$$

and hence,

$$\mu_{\Sigma^0} = \frac{\mu_{\Sigma^+} + \mu_{\Sigma^-}}{2}. \quad (12)$$

Unfortunately, this relation is not testable because the  $\Sigma^0$  decays rapidly to  $\Lambda + \gamma$ , which electromagnetic decay conserves parity, and is independent of the direction of the magnetic moment of the  $\Sigma^0$ .

The spin-1/2 baryon decuplet include the isoquartet  $\Delta^{++}$ ,  $\Delta^+$ ,  $\Delta^0$ ,  $\Delta^-$  and the isotriplet  $\Sigma^{*+}$ ,  $\Sigma^{*0}$ ,  $\Sigma^{*-}$ , for which eq. (1) implies

$$\mu_{\Sigma^{*0}} = \frac{\mu_{\Sigma^{*+}} + \mu_{\Sigma^{*-}}}{2}. \quad (13)$$

and

$$\mu_{\Delta^{++}} = A + \frac{3}{2}B, \quad \mu_{\Delta^+} = A + \frac{1}{2}B, \quad \mu_{\Delta^0} = A - \frac{1}{2}B, \quad \mu_{\Delta^-} = A - \frac{3}{2}B, \quad (14)$$

so we predict,

$$2A = \mu_{\Delta^{++}} + \mu_{\Delta^-} = \mu_{\Delta^+} + \mu_{\Delta^0}, \quad B = \frac{\mu_{\Delta^{++}} - \mu_{\Delta^-}}{3} = \mu_{\Delta^+} - \mu_{\Delta^0}. \quad (15)$$

However, all of these baryons decay quickly by strong interactions, which conserve parity, so their magnetic moments cannot be analyzed.

*The only member of the baryon decuplet whose magnetic moment can be measured is the  $\Omega^-$ , with strangeness  $-3$ , which must decay weakly (parity nonconserving) to violate strangeness. Its magnetic moment has been measured<sup>9</sup> to be  $\mu_{\Omega^-} = 2.02 \pm 0.06\mu_N$ , where  $\mu_N = e\hbar/m_p c$  is the so-called nuclear/nucleon magneton.*

2. From the diagram on p. 1 we see that  $p$  and  $\Sigma^+$  are members of a  $U$ -spin doublet, as also are  $\Sigma^-$  and  $\Xi^-$ . Hence, two predictions of the slightly broken  $SU(3)$  model are

$$\mu_p = \mu_{\Sigma^+}, \quad \mu_{\Sigma^-} = \mu_{\Xi^-}. \quad (16)$$

We also predict that the moments are the same within the  $U$ -spin triplet,

$$\mu_n = \mu_{\Xi^0} = \mu_{|U=1, U_3=0\rangle}. \quad (17)$$

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<sup>9</sup>N.B. Wallace *et al.*, *Precision Measurement of the  $\Omega^-$  Magnetic Moment*, Phys. Rev. Lett. **74**, 3732 (1995), [http://physics.princeton.edu/~mcdonald/examples/EP/wallace\\_pr1\\_74\\_3732\\_95.pdf](http://physics.princeton.edu/~mcdonald/examples/EP/wallace_pr1_74_3732_95.pdf).

The  $U$ -spin states can be obtained from the  $I$ -spin states by a rotation by  $120^\circ$  keeping the axes fixed, which is equivalent to a rotation by  $-120^\circ$  of the axes keeping the states fixed. Hence,

$$\begin{pmatrix} |U=1, U_3=0\rangle \\ |U=0, U_3=0\rangle \end{pmatrix} = \begin{pmatrix} \cos 120^\circ & -\sin 120^\circ \\ \sin 120^\circ & \cos 120^\circ \end{pmatrix} \begin{pmatrix} \Sigma^0 \\ \Lambda \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \Sigma^0 - \sqrt{3}\Lambda \\ \sqrt{3}\Sigma^0 + \Lambda \end{pmatrix} \quad (18)$$

and so,

$$\mu_{|U=1, U_3=0\rangle} = \langle (\Sigma^0 - \sqrt{3}\Lambda)/2 | \mu | (\Sigma^0 - \sqrt{3}\Lambda)/2 \rangle = \frac{\mu_{\Sigma^0}}{4} - \frac{\sqrt{3}\langle \Sigma^0 | \mu | \Lambda \rangle}{2} + \frac{3\mu_\Lambda}{4}. \quad (19)$$

$$\mu_{|U=0, U_3=0\rangle} = \langle (\sqrt{3}\Sigma^0 + \Lambda)/2 | \mu | (\sqrt{3}\Sigma^0 + \Lambda)/2 \rangle = \frac{3\mu_{\Sigma^0}}{4} + \frac{\sqrt{3}\langle \Sigma^0 | \mu | \Lambda \rangle}{2} + \frac{\mu_\Lambda}{4}. \quad (20)$$

$$\begin{aligned} 0 &= \langle U=1, U_3=0 | \mu | U=0, U_3=0 \rangle = \langle (\Sigma^0 - \sqrt{3}\Lambda)/2 | \mu | (\sqrt{3}\Sigma^0 + \Lambda)/2 \rangle \\ &= \frac{\sqrt{3}\mu_{\Sigma^0}}{4} - \frac{\langle \Sigma^0 | \mu | \Lambda \rangle}{2} - \frac{\sqrt{3}\mu_\Lambda}{4}. \end{aligned} \quad (21)$$

From eq. (21) we have

$$\langle \Sigma^0 | \mu | \Lambda \rangle = \frac{\sqrt{3}(\mu_{\Sigma^0} - \mu_\Lambda)}{2}, \quad (22)$$

so eqs. (17) and (19)-(21) can be rewritten as

$$\mu_n = \mu_{\Xi^0} = \mu_{|U=1, U_3=0\rangle} = \frac{3\mu_\Lambda}{2} - \frac{\mu_{\Sigma^0}}{2}, \quad \mu_{|U=0, U_3=0\rangle} = \frac{3\mu_{\Sigma^0}}{2} - \frac{\mu_\Lambda}{2}. \quad (23)$$

The auxiliary requirements that the baryon octet moments sum to zero and that  $\mu_{|U=1, U_3=0\rangle} = -\mu_{|U=0, U_3=0\rangle}$  both imply that

$$\mu_\Lambda = -\mu_{\Sigma^0} \left( = \frac{\mu_n}{2} \right), \quad (24)$$

and we should recall eq. (12) from prob. 1 that

$$\mu_{\Sigma^-} = 2\mu_{\Sigma^0} - \mu_{\Sigma^+} (= -\mu_p - \mu_n). \quad (25)$$

We now have relations for the baryon octet magnetic moments in terms of  $\mu_p$  and  $\mu_n$ ,

$$\mu_\Lambda = \frac{\mu_n}{2}, \quad \mu_{\Sigma^+} = \mu_p, \quad \mu_{\Sigma^0} = -\frac{\mu_n}{2}, \quad \mu_{\Sigma^-} = -\mu_p - \mu_n, \quad (26)$$

$$\langle \Sigma^0 | \mu | \Lambda \rangle = -\frac{\sqrt{3}\mu_n}{2}, \quad \mu_{\Xi^0} = \mu_n, \quad \mu_{\Xi^-} = -\mu_p - \mu_n. \quad (27)$$

As noted in prob. 1,  $\mu_{\Sigma^0}$  cannot be measured, so the predictions  $\mu_{\Sigma^0} = -\mu_n/2$  and (22) cannot be tested (although, perhaps surprisingly,  $\langle \Sigma^0 | \mu | \Lambda \rangle$  can be measured).<sup>10</sup>

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<sup>10</sup>P.C. Peterson *et al.*, *Measurement of the  $\Sigma^0$ - $\Lambda$  Transition Magnetic Moment*, Phys. Rev. Lett. **57**, 949 (1986), [http://physics.princeton.edu/~mcdonald/examples/EP/peterson\\_pr1\\_57\\_949\\_86.pdf](http://physics.princeton.edu/~mcdonald/examples/EP/peterson_pr1_57_949_86.pdf).

These  $SU(3)$  predictions were first made in 1961 (shortly after Gell-Mann's landmark paper) by a more abstract argument.<sup>11</sup>

The present data are

$$\langle p|\mu|p\rangle = \mu_p = 2.79\mu_N, \quad (28)$$

$$\mu_n = -1.91\mu_N, \quad (29)$$

$$\mu_\Lambda = -0.613 \pm 0.004\mu_N, \quad (30)$$

$$\mu_{\Sigma^+} = 2.46 \pm 0.01\mu_N, \quad (31)$$

$$\langle \Sigma^0|\mu|\Lambda\rangle = -1.61 \pm 0.08\mu_N, \quad (32)$$

$$\mu_{\Sigma^-} = -1.16 \pm 0.03\mu_N, \quad (33)$$

$$\mu_{\Xi^0} = -1.25 \pm 0.02\mu_N, \quad (34)$$

$$\mu_{\Xi^-} = -0.651 \pm 0.003\mu_N, \quad (35)$$

which indicate that the  $SU(3)$  predictions (26)-(27) for the baryon magnetic moments are only roughly correct.

Note also that  $SU(3)$  makes no prediction as to the relation between  $\mu_p$  and  $\mu_n$ .

3. We start to construct the spin-1/2 baryon octet states in the quark model beginning with  $p = uud$ . We display wavefunctions only for the spin up states.

Recall that  $SU(3)$  octets have mixed (exchange) symmetry, so we do not expect the flavor and spin parts of the wavefunction to factorize. Since isospin should remain a good subsymmetry, we note that the  $uu$  combination has  $I = 1$ ,  $I_3 = 1$ , which is part of a flavor symmetric isotriplet.<sup>12</sup> If we accept that physical quark states must have a symmetric flavor-spin wavefunction, with overall antisymmetry being associated with their color-singlet wavefunction, then the flavor symmetric  $uu$  state must have a symmetric spin state, and hence  $S = 1$  (rather than  $S = 0$  as also possible for a spin-1/2 pair).

A spin-up proton can then consist of a  $uu$  with  $S_z = 1$ , *i.e.*,  $u\uparrow u\uparrow$  and  $d$  with  $S_z = -1/2$ , *i.e.*,  $d\downarrow$ , as well as a  $uu$  with  $S_z = 0$ , *i.e.*,  $(u\uparrow u\downarrow + u\downarrow u\uparrow)/\sqrt{2}$ , and  $d$  with  $S_z = 1/2$ , *i.e.*,  $d\uparrow$ . Using the appropriate Clebsch-Gordon coefficients, we have

$$\begin{aligned} p\uparrow &= \sqrt{\frac{2}{3}}|1, 1\rangle|1/2, -1/2\rangle - \sqrt{1/3}|1, 0\rangle|1/2, 1/2\rangle = \sqrt{\frac{2}{3}}u\uparrow u\uparrow d\downarrow - \sqrt{1/3}\frac{u\uparrow u\downarrow + u\downarrow u\uparrow}{\sqrt{2}}d\uparrow \\ &= \frac{2u\uparrow u\uparrow d\downarrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow}{\sqrt{6}}. \end{aligned} \quad (36)$$

To have full symmetry under quark flavor exchange the wave function has 6 more terms that are permutations of the above form,

$$p\uparrow = \frac{2u\uparrow u\uparrow d\downarrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow + 2u\uparrow d\uparrow u\downarrow - u\uparrow d\downarrow u\uparrow - u\downarrow d\uparrow u\uparrow + 2d\uparrow u\uparrow u\downarrow - d\uparrow u\downarrow u\uparrow - d\downarrow u\uparrow u\uparrow}{\sqrt{18}}. \quad (37)$$

<sup>11</sup>S. Coleman and S.L. Glashow, *Electrodynamical Properties of Baryons in the Unitary Symmetry Scheme*, Phys. Rev. Lett. **6**, 423 (1961), [http://physics.princeton.edu/~mcdonald/examples/EP/coleman\\_pr1\\_6\\_423\\_61.pdf](http://physics.princeton.edu/~mcdonald/examples/EP/coleman_pr1_6_423_61.pdf).

<sup>12</sup>In  $SU(3)$  language, we are considering  $(\mathbf{3} \times \mathbf{3}) \times \mathbf{3} = (\mathbf{3}^* + \mathbf{6}) \times \mathbf{3} = \mathbf{1} + \mathbf{8} + \mathbf{8}' + \mathbf{10}$ , and noting the that  $uu$  diquark is part of the symmetric  $\mathbf{6}$  multiplet.

For the rest of this problem, it suffices to consider the abbreviated form (36), with the understanding that permutations are included in the full form. This abbreviated form can be written in a factorized form (although the full form is not factorizable),

$$p\uparrow = uud \frac{2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{\sqrt{6}}. \quad (38)$$

The 6 states on the perimeter of the octet all have two like quarks, so their wavefunctions are all of the form (38).

$$n\uparrow = ddu \frac{2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{\sqrt{6}}, \quad (39)$$

$$\Sigma^+\uparrow = uus \frac{2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{\sqrt{6}}, \quad (40)$$

$$\Sigma^-\uparrow = dds \frac{2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{\sqrt{6}}, \quad (41)$$

$$\Xi^0\uparrow = ssu \frac{2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{\sqrt{6}}, \quad (42)$$

$$\Xi^-\uparrow = ssd \frac{2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{\sqrt{6}}. \quad (43)$$

Of the two states at the center of the octet, the  $\Sigma^0$  is the isospin partner of the  $\Sigma^+$  and  $\Sigma^-$ , so its wavefunction can be obtained from that of the  $\Sigma^+$  by lowering the  $u$  quarks to  $d$  quarks,

$$\Sigma^0\uparrow = (uds + dus) \frac{2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{\sqrt{12}}. \quad (44)$$

The  $\lambda$  is orthogonal to the  $\Sigma^0$ , so the  $ud$  pair in the  $\Lambda$  has  $I = 0$  and  $S = 0$  so as to be overall exchange symmetric,

$$\Lambda\uparrow = (uds - dus) \frac{\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{2}. \quad (45)$$

The magnetic moment of (spin-up) baryons in the constituent quark model is just the sum of the magnetic moments of the (constituent) quarks, taking into account whether the quarks have spin up or down.

$$\begin{aligned} \mu_p &= \langle p\uparrow | \mu | p\uparrow \rangle = \frac{\langle 2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow |}{\sqrt{6}} (\mu_u + \mu_u + \mu_d) \frac{|2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow \rangle}{\sqrt{6}} \\ &= \frac{4(\mu_u + \mu_u - \mu_d) + (\mu_u - \mu_u + \mu_d) + (-\mu_u + \mu_u + \mu_d)}{6} \\ &= \frac{4}{3}\mu_u - \frac{1}{3}\mu_d, \end{aligned} \quad (46)$$

$$\mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u, \quad (47)$$

$$\mu_\Lambda = \mu_s, \quad (48)$$

$$\mu_{\Sigma^+} = \frac{4}{3}\mu_u - \frac{1}{3}\mu_s, \quad (49)$$

$$\mu_{\Sigma^0} = \frac{2}{3}\mu_u + \frac{2}{3}\mu_d - \frac{1}{3}\mu_s, \quad (50)$$

$$\langle \Sigma^0 | \mu | \Lambda \rangle = \sqrt{\frac{1}{3}}\mu_d - \sqrt{\frac{1}{3}}\mu_u, \quad (51)$$

$$\mu_{\Sigma^-} = \frac{4}{3}\mu_d - \frac{1}{3}\mu_s, \quad (52)$$

$$\mu_{\Xi^0} = \frac{4}{3}\mu_s - \frac{1}{3}\mu_u, \quad (53)$$

$$\mu_{\Xi^-} = \frac{4}{3}\mu_s - \frac{1}{3}\mu_d, \quad (54)$$

If quark-flavor SU(3) were an exact symmetry then all quark masses would be the same, and the Dirac magnetic moments of the quarks would be

$$\mu_u = \frac{2e\hbar}{6m_q}, \quad \mu_d = -\frac{e\hbar}{6m_q}, \quad \mu_s = -\frac{e\hbar}{6m_q}, \quad (55)$$

and the moments (47)-(54) would given in terms of  $\mu_p = e\hbar/2m_q (= 2.79\mu_N)$  as

$$\begin{aligned} \mu_n = \mu_{\Sigma^0} = -\frac{2}{3}\mu_p = -1.86\mu_N, \quad \mu_\Lambda = -\frac{1}{3}\mu_p = -0.93\mu_N, \quad \mu_{\Sigma^+} = \mu_p = 2.79\mu_N, \\ \mu_{\Sigma^-} = \mu_{\Xi^-} = -\frac{2}{9}\mu_p = -0.62\mu_N, \quad \mu_{\Xi^0} = -\mu_p = -2.79\mu_N, \\ \langle \Sigma^0 | \mu | \Lambda \rangle = -\frac{\sqrt{3}}{3}\mu_p = -1.61\mu_N. \end{aligned} \quad (56)$$

Of these predictions, the data given in prob. 2 are in reasonable agreement with  $\mu_n$ ,  $\mu_{\Sigma^+}$ ,  $\mu_{\Xi^-}$  and  $\langle \Sigma^0 | \mu | \Lambda \rangle$ .

In a model of broken flavor symmetry, we suppose that the masses of the  $u$ ,  $d$  and  $s$  quarks are different, such that the magnetic moments of the  $p$ ,  $n$  and  $\Lambda$  are fit exactly. From eqs. (46)-(48) and the data from prob. 2, we have that

$$\mu_u = \frac{4\mu_p + \mu_n}{5} = 1.85\mu_N, \quad \frac{2}{3m_u} = \frac{1.85}{m_p}, \quad m_u = 339 \text{ MeV}, \quad (57)$$

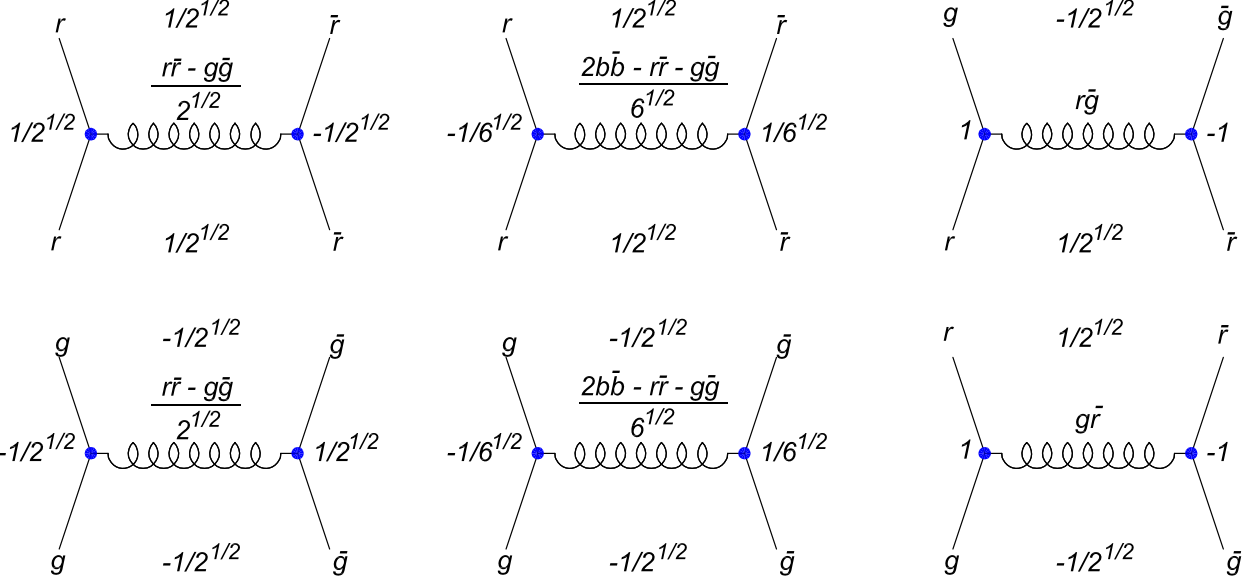
$$\mu_d = \frac{\mu_p + 4\mu_n}{5} = -0.97\mu_N, \quad \frac{1}{3m_d} = \frac{0.97}{m_p}, \quad m_d = 323 \text{ MeV}, \quad (58)$$

$$\mu_s = \mu_\Lambda = -0.613\mu_N, \quad \frac{1}{3m_s} = \frac{0.613}{m_p}, \quad m_s = 511 \text{ MeV}, \quad (59)$$

and the moments (49)-(54) are now predicted to be  $\mu_{\Sigma^+} = 2.67\mu_N$ ,  $\mu_{\Sigma^-} = -1.09\mu_N$ ,  $\langle \Sigma^0 | \mu | \Lambda \rangle = -1.63\mu_N$ ,  $\mu_{\Xi^0} = -1.75\mu_N$ ,  $\mu_{\Xi^-} = -0.49\mu_N$ . Except for  $\mu_{\Xi^-}$ , these predictions (based on more fitted parameters) are better than those of eq. (56). *However, with time the successes of the constituent quark model have come to be regarded as somewhat accidental, with the “bare” masses of the  $u$  and  $d$  quarks being near zero.*



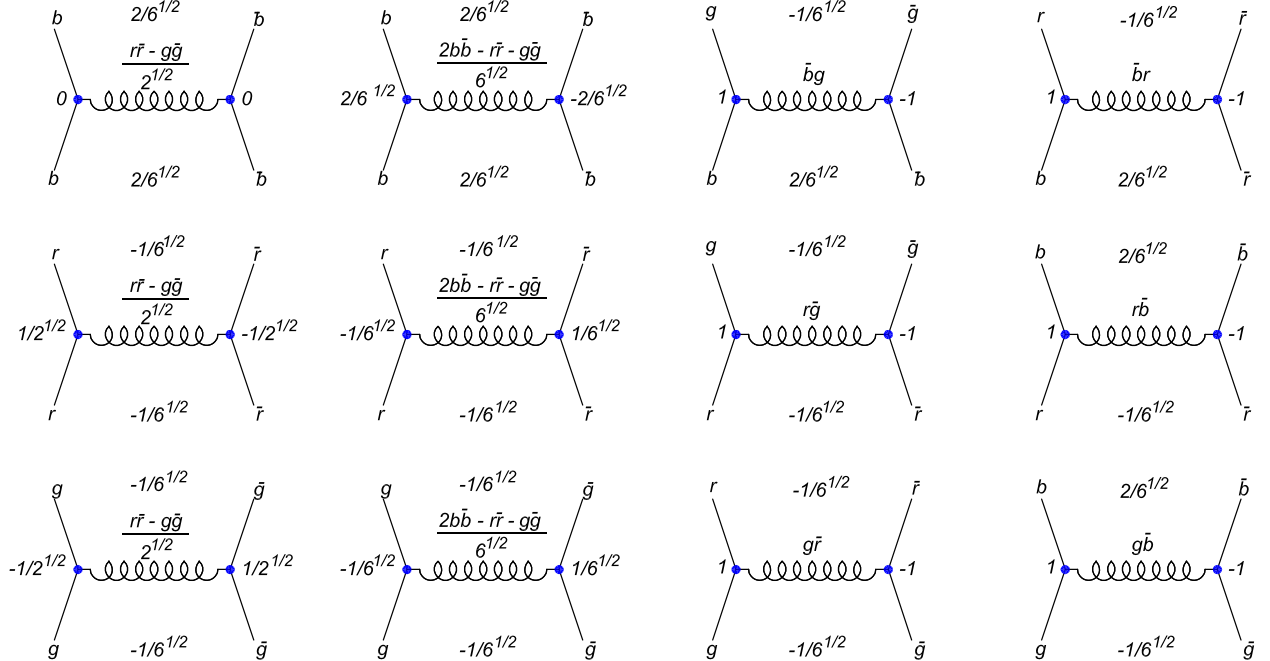
4. The color-force amplitude for the  $q\bar{q}$  state  $\psi_{\text{color}} = (r\bar{r} - g\bar{g})/\sqrt{2}$  has contributions from the six single-gluon-exchange graphs (in which time flows upwards),



where the factors at the bottom and top of each graph are the amplitudes of the initial and final states of the graphs in the state  $(r\bar{r} - g\bar{g})/\sqrt{2}$ , and the vertex factors can be read off from the color content of the exchanged gluon with the convention that the vertex factor for  $r_{\text{quark}} \rightarrow r_{\text{quark}} + r\bar{r}_{\text{gluon}}$  has the opposite sign to that for  $r\bar{r}_{\text{gluon}} + \bar{r}_{\text{quark}} \rightarrow \bar{r}_{\text{quark}}$ . Then,

$$\langle (r\bar{r} - g\bar{g})/\sqrt{2} | \text{gluon} | (r\bar{r} - g\bar{g})/\sqrt{2} \rangle = -\frac{1}{4} - \frac{1}{12} + \frac{1}{2} - \frac{1}{4} - \frac{1}{12} + \frac{1}{2} = \frac{1}{3}. \quad (60)$$

Similarly, the color-force amplitude for the  $q\bar{q}$  state  $\psi_{\text{color}} = (2b\bar{b} - r\bar{r} - g\bar{g})/\sqrt{6}$  has contributions from the 12 single-gluon-exchange graphs (one of which is null),



Thus,

$$\begin{aligned} & \langle (2b\bar{b} - r\bar{r} - g\bar{g})/\sqrt{6} | \text{gluon} | (2b\bar{b} - r\bar{r} - g\bar{g})/\sqrt{6} \rangle \\ &= 0 - \frac{16}{36} + \frac{2}{6} + \frac{2}{6} - \frac{1}{12} - \frac{1}{36} - \frac{1}{6} + \frac{2}{6} - \frac{1}{12} - \frac{1}{36} - \frac{1}{6} + \frac{2}{6} = \frac{1}{3}. \end{aligned} \quad (61)$$

*This problem has been a “brute force” approach, via consideration of all relevant diagrams. More compact approaches are available, along the lines of prob. 4, set 5. See, for example, sec. 8.1 of C. Quigg, Gauge Theories of the Strong, Weak, and Electromagnetic Interactions, 2nd ed. (Princeton U. Press, 2013). Not being a theorist, I have found the “brute force” approach more instructive.*