Electromagnetic Fields inside a Perfect Conductor

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1 Problem

Discuss how the electric field vanishes inside a perfect conductor according to "all" definitions of this concept, while the interior magnetic field vanishes only under some definitions.

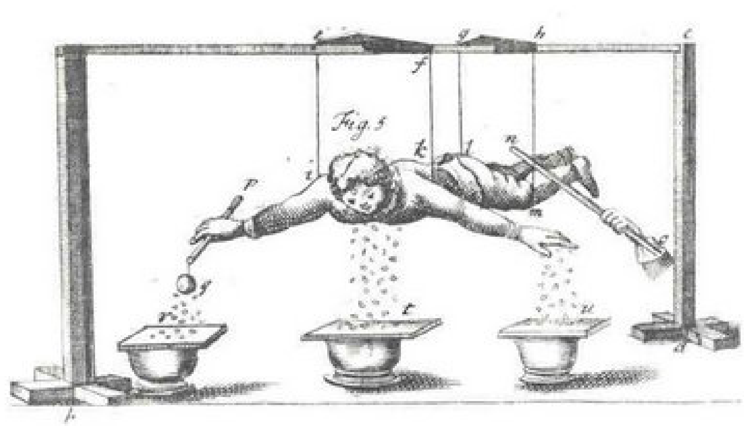
2 Solution

This problem depends on the concept of a conductor, and of a perfect conductor.

2.1 Historical Concepts of Conductors

2.1.1 Gray

The phenomenon of conduction of electricity may have been first reported by Gray in 1731 [1], who described experiments on the "conveyance" and "communication" of electricity (including evidence that people conduct electricity).[1]



2.1.2 Desaguliers

In 1739, Desaguliers [3] (a coworker of Gray) coined the term "conductor", as well as the term "insulator".

2.1.3 Priestley

In his *History of Electricity*, vol. II, p. 3, Priestley [4] mentioned conductors and perfect conductors, defining these by example:

[1]For a historical survey of research into electricity in this era, see, for example, [2].

> All known substances are distributed by electricians into two sorts. Those of one sort are termed electrics, or non-conductors; and those of the other non-electrics, or conductors of electricity.
> Metals of all kinds, together with semimetals, and water are conductors. So also is charcoal, and other substances of a similar nature, as will be shewn at large in the last part of this work. All other substances, whether mineral, vegetable, or animal, are non-conductors. But many of these when they are made very hot, as glass, rosin, baked wood, and, perhaps, all the rest on which the experiment can be made in this state, are conductors of electricity.
> All bodies, however, though in the same state of heat and cold, are not equally perfect electrics, or perfect conductors.

2.1.4 Cavendish

Cavendish (1771) may be the first to have attempted a definition of a conductor, on p. 649 of [5]:[2]

> It appears from experiment, that some bodies suffer the electric fluid to pass with great readiness between their pores; while others will not suffer it to do so without great difficulty; and some hardly suffer it to do so at all. The first sort of bodies are called conductors, the others non-conductors. What this difference in bodies is owing to I do not pretend to explain.
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> It is evident that the electric fluid in conductors may be considered as movable, or answers to the definition given of that term in p. 588. As to the fluid contained in non-conducting substances, though it does not absolutely answer to the definition of immoveables, as it is not absolutely confined from moving, but only does so with great difficulty; yet it may in most cases be looked upon as such without sensible error.
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> Air does in some measure permit the electric fluid to pass through it; though, if it is dry, it lets it pass but very slowly, and not without difficulty; it is therefore to be called a nonconductor. It appears that conductors would readily suffer the fluid to run in and out of them, were it not for the air which surrounds them: for if the end of a conductor is inserted into a vacuum, the fluid runs in and out of it with perfect readiness; but when it is surrounded on all sides by the air, as no fluid can run out of it without running into the air, the fluid will not do so without difficulty.

Cavendish's statement that the "electric fluid" would readily flow from a conductor into vacuum was a speculation, not based on experiment. As such, Cavendish is atypical in supposing that a vacuum is a kind of conductor.

2.1.5 Poisson

In 1812, Poisson [7] offered a definition similar to that of Cavendish:

> Tous les corps de la nature ne se comportent pas de la même manière par rapport au fluide électrique: les uns, comme les métaux, ne paraissent exercer sur lui aucune espèce d'action; ils lui permettent de se mouvoir librement dans leur intérieur et de les traverser dans tous les sens: pour cette raison on les nomme *corps conducteurs*. D'autres, au contraire, l'air très-sec, par exemple, s'opposent au passage du fluide électrique dans leur intérieur, de sorte qu'ils servent à empêcher le fluide accumulé dans les corps conducteurs de se dissiper dans l'espace.

[2] An unfortunate typographical error was corrected by Maxwell in his edition of Cavendish's *Electrical Researches*, p. 44 of [6].

2.1.6 Ohm

In 1827, Ohm published a treatise [8, 9] containing his famous law, in a form closer to,

$$\mathbf{J} = \sigma \mathbf{E}, \tag{1}$$

where $\mathbf{J}$ is the electric current density and $\mathbf{E}$ is the electric field, both inside the rest frame of a medium with electrical conductivity σ, than to more familiar form, $V = IR$, where V is the potential difference across an electrical resistance R that carries electric current I.

Ohm did not define a conductor so much a provide a model for it, with a flavor that electric current is related to the motion of particles. This view became characteristic of the German school in the mid 1800's, but was not taken up by the English or French until much later.

2.1.7 Green

In 1828, Green spoke of a perfect conductor on p. 22 of [10]:

> It has been long known from experience, that whenever the electric fluid is in a state of equilibrium in any system whatever of perfectly conducting bodies, the whole of the electric fluid will be carried to the surface of those bodies, without the smallest portion of electricity remaining in their interior...

2.1.8 Thomson

W. Thomson wrote in 1848, p. 131 of [11]:[3]

> 8. A very extensive class of bodies in nature, including all the metals, many liquids, &c. are found to possess the property that in all conceivable circumstances of electrical excitation, the resultant force at any point within their substance vanishes. Such bodies are called conductors of electricity, since they are destitute of the property, possessed by non-conductors, of retaining permanently, by resistance to every change, any distribution of electricity arbitrarily imposed; the only kind of distribution which can exist unchanged for an instant being such as satisfies the condition that the resultant force must vanish in the interior.

An interpretation of Thomson's statements is that for a perfect conductor of zero electrical resistance (infinite electrical conductivity σ), Ohm's law (1) implies that the interior electric field $\mathbf{E}$ is zero, to avoid that the current density $\mathbf{J}$ be infinite.

In 1848, Thomson also stated a theorem that the energy of a static system at equilibrium is a minimum (see p. 87 of [12]), and implied that this should hold for both electrostatics and magnetostatics, among other systems. A corollary of this theorem is that a conductor (including a perfect conductor, whatever that may be) should be in a state of minimum energy when at equilibrium.

2.1.9 Maxwell

Maxwell gave a more crisp definition of a conductor in Art. 72 of his *Treatise* [13]:

> A conductor is a body which allows the electricity within it to move from one part of the body to another when acted upon by electromotive force.

[3]Thomson's rambling definition is an early sign that different people may have different conceptions of conductors.

In Maxwell's conception, a conductor must have "electricity within it", and he considered that solids and liquids can be conductors, but not gases in their usual state. Only later did ionized gases/plasmas come to be considered as conductors.

Maxwell immediately continued in Art. 72:[4]

> When the electricity is in equilibrium there can be no electromotive intensity acting with the conductor. Hence $\mathbf{E} = 0$ throughout the whole space occupied by the conductor.

In Art. 840 of his *Treatise* [14] Maxwell stated:

> We have proved in Art. 654, that a closed sheet of perfectly conducting matter of any form, originally free from currents, becomes when exposed to external magnetic force, a current-sheet, the action of which on every point of the interior is such as to make the magnetic force zero.

Maxwell's argument in Arts. 654-655 is essentially that the electric field is related by,

$$\mathbf{E} = -\boldsymbol{\nabla} V - \frac{\partial \mathbf{A}}{\partial t}, \tag{2}$$

in SI units, where V and $\mathbf{A}$ are the electric scalar and vector potentials, respectively. Inside a perfect conductor, where $\mathbf{E} = 0$, the vector potential $\mathbf{A}$ must be independent of time. Hence, for any loop on the surface of the perfect conductor, $\oint \mathbf{A} \cdot d\mathbf{l} = \int \mathbf{B} \cdot d\mathbf{Area}$ is also constant in time. If $\mathbf{B}$ is initially zero inside the perfect conductor, it remains so no matter what changes occur externally.

Maxwell's argument leaves open the possibility that a perfect conductor could have an initial, nonzero, interior magnetic field.[5]

It seems reasonably clear that in Maxwell's time people regarded the notion of a perfect conductor as an idealization, approximated in Nature only by conductors of large, but not infinite, conductivity. However, in Art. 833 of his *Treatise*, Maxwell speculated that the "magnetic molecules" of permanent magnets might be perfect conductors:

> We have therfore to consider the hypothesis of Ampère, that the magnetism of the molecules is due to an electric current constantly circulating in some closed path within it.
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> By confining the circuits to the molecules, within which nothing is known about resistance, we may assert, without fear of contradiction, that the current, in circulating within the molecule, meets with no resistance.

In Art. 575 of his *Treatise*, Maxwell noted that the "magnetic molecules" might contain angular momentum, and he reported negative results from an experimental search for this effect.[6]

2.1.10 Kamerlingh Onnes

In 1911, Kamerlingh Onnes [18] found that mercury at liquid-helium temperature has immeasurably small electrical resistance, which phenomenon has come to be called superconductivity (first called *supraconductivity*).

Is a superconductor a physical realization of a perfect conductor?

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[4]Maxwell actually used the scalar symbol R for the electromotive intensity = magnitude of the electric field $\mathbf{E}$.

[5]*June 30, 2020.* In 1889, Lippmann [15], without reference to Maxwell, argued that the magnetic flux in the interior of a perfect conductor cannot change with time. See also [16].

[6]In a footnote on p. 345 of [17] (1861), Maxwell reported on an early version of this search.

2.1.11 The Magnetization of Iron is Not a Classical Effect

In the early 1900's, various people [19]-[30] pursued Maxwell's comment that permanent magnetism might be associated with angular momentum. Richardson [19] may have been the first to note the classical argument that if a current loop is due to electric charges e with mass M, the angular momentum $\mathbf{L}$ is related to the magnetic moment $\mathbf{m}$ by,

$$\mathbf{m} = \frac{e}{2M}\mathbf{L}. \tag{3}$$

After some time, the experimental results converged on a value of the angular momentum associated with magnetization of iron being half that predicted by eq. (3); see [31]-[33] for reviews. This discrepancy was explained as a quantum effect associated with electron spin [34] by Dirac [35].

2.1.12 Meissner

In 1933, Meissner and Ochsenfeld [36] observed that superconductors are diamagnetic, with "zero" interior magnetic field (although the field penetrates slightly below the surface of a superconductor). In particular, if a body has an internal magnetic field when warm, this field is expelled from the interior when the body becomes superconducting on being cooled.

This contrasts with the open issue of Maxwell, that a perfect conductor might in principle have an initial, nonzero, interior magnetic field.

2.1.13 Alfvén

In 1942, Alfvén [37] considered the transition of a gas to a plasma in the case that an initial magnetic field existed in the gas volume. The electrical conductivity of a plasma is very large, so one can often usefully approximate a plasma as a perfect conductor.

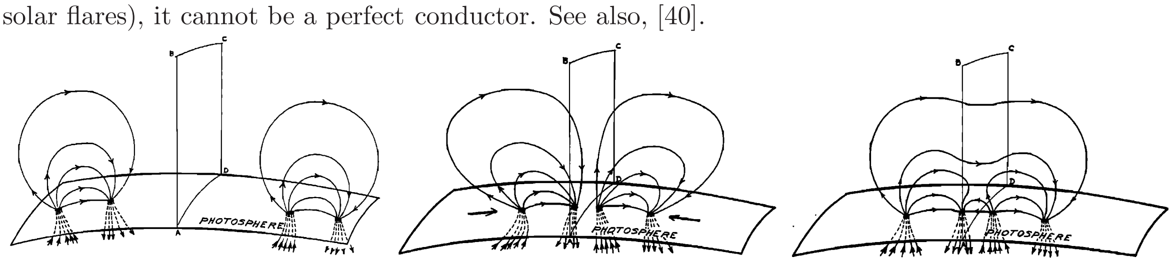
Alfvén gave a briefer version of Maxwell's argument, noting that with $\mathbf{E} = 0$ inside a perfect conductor, Faraday's law tells us,

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = 0, \tag{4}$$

such that the magnetic field in a plasma need not be zero, but can be "fastened" or "frozen in" as a time-independent (but spatially varying) field during/after the creation of the plasma.[7]

[7] A noteworthy extrapolation of Alfén's views was given by Sweet in 1956 [38], when he argued for the phenomenon of "magnetic reconnection", using as an example a perfect conductor from which two bundles of field lines emerged and into which two other bundles penetrated.

A subsequent paper by Parker [39] clarified that if the fields lines move with respect to the conductor (as for solar flares), it cannot be a perfect conductor. See also, [40].



Alfvén was primarily interested in electromagnetic waves in a plasma, which could not exist if the electric field were zero and the magnetic field independent of time inside a plasma. That is, Alfvén regarded the notion of "frozen-in" lines of force as only a useful approximation for plasmas, while other people consider this behavior to be an exact property of perfect conductors.[8]

The transition of a gas to a plasma involves ionization of the gas molecules, a process that is not well described by "classical" electrodynamics. Hence, Alfvén's "frozen-in" field lines of plasmas are, strictly speaking, an aspect of quantum electrodynamics. However, once a plasma is created, one can analyze it largely via "classical" concepts (with scattering and recombination of the plasma ions being exceptions to this).

Thus, the question is left open as to whether a perfect conductor in "classical" electrodynamics can have a nonzero, interior magnetic field.

2.1.14 Fiolhais *et al.*

Recalling Thomson's argument of 1848 ([12]; sec. 2.1.8 above) that a (classical) system in equilibrium should be a state of minimum energy, we expect both the electric and magnetic fields associated with an isolated, perfect conductor to be zero, since nonzero fields imply nonzero field energy. Hence, if we suppose that "classical" electrodynamics respects Thomson's minimum-energy condition, a perfect conductor in zero external fields has magnetic field everywhere zero. If the conductor is later subject to external magnetic fields, surface currents must be induced on the perfect conductor so as to cancel the external magnetic field inside the conductor. That is, a perfect conductor in this vision of "classical" electrodynamics is diamagnetic, and has no permanent magnetic moment.

The above argument has been formalized in [42], where a "magnetic" version of Thomson's energy-minimum theorem was deduced.

2.2 Comments

In essentially all conceptions of a perfect conductor, the interior electric field is zero, and the interior magnetic field is time-independent (in the rest frame of the conductor).

In a vision of "classical" electrodynamics (following W. Thomson [12]) where the initial conditions do not include nonzero sources of energy, the interior magnetic field of a perfect conductor is also zero.

In a broader vision of (quantum) electrodynamics, where "initial" conditions can include nonzero energy, the interior magnetic field of a perfect conductor can be nonzero, but time-independent, in the rest frame of the conductor.

A major challenge for "classical" electrodynamics is to provide a description of the observed magnetism of materials.

Maxwell remarked in Art. 831 of his *Treatise*:

[8]Henyey [41] argued that there is a "distinction between a perfect conductor and a superconductor", contrasting the "frozen-in" field lines of a plasma (regarded by Henyey, but not by Alfvén, as an example of a perfect conductor) with the Meissner effect for superconductors. As noted below, this distinction is in a quantum context.

> Whether this matter is or is not electricity, whether it is a continuous fluid interpenetrating the spaces between molecular nuclei, or is itself molecularly grouped; or whether all matter is continuous, and molecular heterogeneousness consists in finite vortical or other relative motions of contiguous parts of a body; it is impossible to decide, and perhaps in vain to speculate, in the present state of science.

Feynman took a stronger stance [43]:

> ...*it is not possible* to understand the magnetic effects of materials in any honest way from the point of view of classical physics. Such magnetic effects are a *completely quantum-mechanical phenomenon*. It is, however, possible to make some phoney classical arguments and to get some idea of what is going on.

Despite such cautions, some people find it convenient to associate permanent magnetism with perfect conductors with "frozen-in" magnetic fields and corresponding nonzero magnetic moments, ignoring the experimental evidence (sec. 2.1.11) that this view is not viable.

The view of this author is that it's better to adopt Thomson's vision of "classical" electrodynamics as obeying an energy-minimum principle, sec. 2.1.8 above, and to acknowledge that a detailed understanding of magnetic materials is beyond the scope of "classical" electrodynamics.

2.3 Boundary Conditions at the Surface of a Perfect Conductor

It follows from Faraday's law, eq. (4), that the tangential component of the electric field is continuous across a surface, so the requirement that the electric field be zero inside a perfect conductor implies that the electric field at the surface of a perfect conductor can only be perpendicular to it. Then, that surface supports a surface charge density $\sigma = \epsilon_0 E_\perp$.

The normal component of the magnetic field $\mathbf{B}$ is continuous across any surface (since $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$), so in the view that the magnetic field is zero inside a perfect conductor, the magnetic field at the surface of a perfect conductor can only have a tangential component.[9]

The perfect electric conductor (PEC) boundary conditions are,

$$\mathbf{E}_{\parallel} = 0, \qquad \mathbf{B}_{\perp} = 0 \qquad \text{(PEC)}. \tag{5}$$

2.4 Perfect Magnetic Conductor

The concept of a perfect magnetic conductor is sometimes discussed.

2.4.1 Both Electric and Magnetic Charges Exist

The context for this would seem to be that magnetic "charges" (monopoles) could exist in Nature, together with electric charges, and $\boldsymbol{\nabla} \cdot \mathbf{B} = 4\pi\rho_m$, where the volume density ρ_m of magnetic charge can be nonzero.

When Heaviside first presented Maxwell's equations in vector notation [44] he considered assumed that in addition to electric charge and current densities, ρ_e and $\mathbf{J}_e$, there existed magnetic charge and current densities, ρ_m and $\mathbf{J}_m$,[10] although there remains no experimental

[9] In the view that the magnetic field can be nonzero, but independent of time, inside a perfect conductor, the magnetic field at the surface of a perfect conductor can only have a normal component that is independent of time. In both views, there can be no time-independent, normal component of the magnetic field at the surface of a perfect conductor.

[10] For a review of Maxwell's equations in the case that magnetic charges exist, see [46].

evidence for the latter.[11,12] Maxwell's equations for microscopic electrodynamics are then (in SI units),[13]

$$\boldsymbol{\nabla} \cdot \epsilon_0 \mathbf{E} = \rho_e, \quad \boldsymbol{\nabla} \cdot \frac{\mathbf{B}}{\mu_0} = \rho_m, \quad -c^2 \boldsymbol{\nabla} \times \epsilon_0 \mathbf{E} = \frac{\partial}{\partial t} \frac{\mathbf{B}}{\mu_0} + \mathbf{J}_m, \quad \boldsymbol{\nabla} \times \frac{\mathbf{B}}{\mu_0} = \frac{\partial \epsilon_0 \mathbf{E}}{\partial t} + \mathbf{J}_e, \tag{6}$$

The surface of a perfect (electric) conductor could support a surface density σ_m of magnetic charge, and the magnetic field $\mathbf{B}$ could have a nonzero normal component at the surface. That surface could also have density of magnetic current density $\mathbf{J}_m$, and Faraday's law would now be,

$$\boldsymbol{\nabla} \times \mathbf{E} = -\mu_0 \mathbf{J}_m - \frac{\partial \mathbf{B}}{\partial t} = 0, \tag{7}$$

and the electric field could have a nonzero tangential component just outside the surface of a perfect electric conductor.

That is, if magnetic charges exist (as well as electric charges), there would be no "boundary conditions" at the surface of a perfect electric conductor.[14]

Similarly, if both electric and magnetic charges exist, there would be no "boundary conditions" at the surface of a perfect magnetic conductor.

However, the magnetic field in the interior of a perfect magnetic conductor must be zero, and following the view of Thomson, sec. 2.1.8 above, the electric field in the interior of a perfect magnetic conductor must also be zero.

2.4.2 Only Magnetic Charges Exist

If only magnetic charges existed, and not electric charges, then Maxwell's equations for microscopic electrodynamics would be (in SI units),

$$\boldsymbol{\nabla} \cdot \epsilon_0 \mathbf{E} = 0, \quad \boldsymbol{\nabla} \cdot \frac{\mathbf{B}}{\mu_0} = \rho_m, \quad -c^2 \boldsymbol{\nabla} \times \epsilon_0 \mathbf{E} = \frac{\partial}{\partial t} \frac{\mathbf{B}}{\mu_0} + \mathbf{J}_m, \quad \boldsymbol{\nabla} \times \frac{\mathbf{B}}{\mu_0} = \frac{\partial \epsilon_0 \mathbf{E}}{\partial t}. \tag{8}$$

The only conductors would be those that support the flow of magnetic charge, *i.e.*, magnetic currents. A perfect magnetic conductor would have zero interior magnetic and (in most views) zero interior electric field.[15]

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[11]Heaviside seems to have regarded magnetic charges as "fictitious", as indicated on p. 25 of [47].

[12]If the interaction of magnetic charges with magnetic moments due to electrical currents is to conserve energy, the magnetic charges must be at the end of "strings" of magnetic flux, as first postulated by Dirac [48, 49].

[13]Maxwell called $\partial \mathbf{D}/\partial t$ the *electric displacement current* (density), p. 14 of [50]. Heaviside called $\partial(\mathbf{B}/\mu_0)/\partial t$ the *magnetic displacement current* (density), or just the *magnetic current*, in [51].

The notion of a magnetic current was used in some early discussions of vector diffraction theory, by Love [52], and by Macdonald, sec. 14 of [53] and p. 95 of [54]. See also p. 93 of [55] and p. 69 of [56].

As discussed in [46], if magnetic charges exist, there can be a density $\mathbf{M}_m$ of magnetic dipoles due to pairs of opposite magnetic charges, and one should define the auxiliary field $\mathbf{H}_m = \mathbf{B}/\mu_0 + \mathbf{M}_m$. This contrasts with the usual field $\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}_e$, where $\mathbf{M}_e$ is the density of magnetic dipoles due to electric current loops. Then, the magnetic displacement current would be $\partial \mathbf{H}_m/\partial t$.

[14]The extensive success in comparisons of experiment with analyses of electromagnetic problems using the perfect-conductor boundary conditions is yet more evidence that magnetic charges are not common, if they exist at all, in Nature.

[15]Ohm's law would have the form $\mathbf{J}_m = \varsigma_m \mathbf{B}$, where ς_m is the magnetic conductivity, which would be infinite for a perfect magnetic conductor.

It follows from the fourth Maxwell equation of (8), that the tangential component of the magnetic field is continuous across a surface, so the requirement that the magnetic field be zero inside a perfect magnetic conductor implies that the magnetic field at the surface of a perfect magnetic conductor can only be perpendicular to it. Then, that surface supports a surface magnetic charge density $\sigma_m = B_\perp/\mu_0$.

The normal component of the electric field $\mathbf{E}$ would be continuous across any surface (since $\boldsymbol{\nabla} \cdot \mathbf{E} = 0$), so in the view that the electric field is zero inside a perfect magnetic conductor, the electric field at the surface of a perfect conductor can only have a tangential component.

The boundary conditions for fields just outside a perfect magnetic conductor (PMC) would be,

$$\mathbf{B}_\parallel = 0, \qquad \mathbf{E}_\perp = 0 \qquad (\text{PMC, if only magnetic charges}). \tag{9}$$

Of course, if only magnetic charges existed, would could call these charges "electric", and also rename the electric field as the "magnetic" field, and the magnetic field as the " electric" field, such that the case of only magnetic charges is in effect the same as the case of only electric charges.[16] As such, the conditions (9), under the assumption that only magnetic charges exist, don't have independent physical content from the perfect-electric-conductor boundary conditions (5), and eq. (9) is seldom discussed in the literature.

2.4.3 Only Electric Charges Exist

If magnetic charges do exist in Nature, they are so rare as to have never been observed. Many electrical engineers assume that magnetic charges do not exist, and use the term *perfect magnetic conductor* to mean a material whose only charges are electric, and which obeys the conditions,

$$\mathbf{D}_{\text{interior}} = 0 = \mathbf{H}_{\text{interior}} \qquad (\text{PMC, but only electric charges}). \tag{10}$$

Fields just outside the surface of such a material obey the boundary conditions,

$$\mathbf{D}_\perp = 0 = \mathbf{H}_\parallel \qquad (\text{PMC, but only electric charges}), \tag{11}$$

This usage may have originated on p. 107 of [56], where the boundary conditions were given for $\mathbf{E}$ and $\mathbf{H}$.[17] It also appeared on p. 34 of [58], where only the conditions on $\mathbf{H}$ were mentioned. An early textbook discussion of magnetic currents and "perfect magnetic conductors" is in sec. 7.1 of [60], but here (and in subsequent electrical-engineering literature) the boundary conditions are written in terms of $\mathbf{D}$ and $\mathbf{H}$ rather than $\mathbf{E}$ and $\mathbf{H}$. See also chap. 7 of [61]. The attitude seems to be that although magnetic charges do not exist, there can exist magnetic surface currents (which would seem to require moving magnetic

[16]See, for example, p. 274 of [57] for discussion of duality transformations of electric and magnetic fields.

[17]The term *perfect magnetic conductor* was used by Heaviside, p. 536 of [59], although he considered that the conditions (10)-(11) were for $\mathbf{D}$ and $\mathbf{B}$ rather than for $\mathbf{E}$ and $\mathbf{H}$.

charges).[18,19]

While the concept a "perfect magnetic conductor" as used in the engineering community is formally inconsistent, its usage is generally restricted to surfaces that are in vacuum, with a goal of describing the electromagnetic fields on one side of that surface in terms of "fictitious" sources on that surface, in the spirit of Huygens [65] and Kirchhoff [66]. With this restriction, use of "perfect magnetic conductors" can lead to (approximately) valid results for examples in which only electric charges exist.[20]

An example of recent usage is in sec. I of [68]. An engineered material that approximates the surface of a perfect magnetic conductor is reported in [69].

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[18] As mentioned in footnote 11, a magnetic-displacement-current density $\mathbf{J}_m = \partial(\mathbf{B}/\mu_0)/\partial t$ can be defined when only electric charges exist, but there is no surface-current density associated with this volume density, just as there is no surface displacement-current density associated with the volume displacement-current density $\partial\mathbf{D}/\partial t$.

[19] Discussions of "perfect magnetic conductors" often mention *duality* (a term inspired by Minkowski (1908), eq. (35) of [62]; for a history of this concept, see Appendix D.1 of [46]), as in sec. 3.2 of [58], sec. 7.1c of [60], sec. 7.2 of [61] and sec. 2.4 of [63], but these discussions are somewhat misguided as to the character of electromagnetic duality based on electric and magnetic charges.

If magnetic charges did exist, then duality would hold, but the duality relations (reviewed in Appendix D of [46]) are different than those discussed in [58, 60, 61, 63] in that one should include the transformations of charge densities, $\rho_e \to \rho_m/c, \qquad \rho_m/c \to -\rho_e$.

If magnetic charges don't exist, there is only a "pseudoduality", in that while electric charges cannot produce the same effects as magnetic charges, electric currents can produce the effects of magnetic dipoles due to pairs of opposite magnetic charges (outside the dipole itself). That is, one can deduce the fields due to magnetic dipoles $\mathbf{m}_e$ (associated with electric currents) by a "duality" transformation, that includes the form $\mathbf{p}_e \to \mathbf{m_e}/c$, of the fields due to electric dipoles $\mathbf{p}_e$ (associated with pairs of opposite electric charges). Thus, the field of an electric dipole, $\mathbf{E} = [3(\mathbf{p}_e \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}_e]/4\pi\epsilon_0 r^3$ has the "duality" transformation $[3(\mathbf{m}_e \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}_e]/4\pi\epsilon_0 c r^3 = c\mathbf{B}$, such that the field of a magnetic dipole $\mathbf{m}_e$ is $\mathbf{B} = [3(\mathbf{m}_e \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}_e]/4\pi\epsilon_0 c^2 r^3 = \mu_0[3(\mathbf{m}_e \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}_e]/4\pi r^3$ (outside the dipole). For the fields of an oscillating magnetic dipole obtained via a"duality" transformation, see sec. 2.4.2 of [63] and [64].

[20] Many applications of the "perfect magnetic conductor" boundary conditions are to examples in which electric fringe fields are neglected. For example, if the open faces of a parallel-plate capacitor are considered to be "perfect magnetic conductors", then $\mathbf{E}$ must be parallel to these faces, which corresponds to the idealization of no fringe fields. See sec. 14.2.2 of [67].

[5] H. Cavendish, *An Attempt to explain some of the principal Phenomena of Electricity, by Means of an elastic Fluid*, Phil. Trans. Roy. Soc. London **61**, 584 (1771), http://physics.princeton.edu/~mcdonald/examples/EM/cavendish_ptrsl_61_584_71.pdf

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