Can Dipole Antennas Above a Ground Plane Emit Circularly Polarized Radiation?

Kirk T. McDonald
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544
(September 19, 2008)

1 Problem

Discuss the possible emission of circularly polarized radiation by a dipole antennas (or system of dipole antennas) that is located above a perfectly conducting ground plane.

It can be assumed that the effect of the ground plane is that the fields above the plane are the superposition of the fields from the physical charge and current distributions plus those from image charge and current distributions.

Consider the special cases of antennas that are small compared to a wavelength, and those for which all conductors at a distance from the ground plane that is small compared to a wavelength.

Show that the radiation emitted by any electric dipole antenna (or system of such antennas) close to a conducting ground plane has pure vertical polarization, which it can be arranged that circularly polarization radiation is emitted upwards from an appropriate system of magnetic dipole antennas.

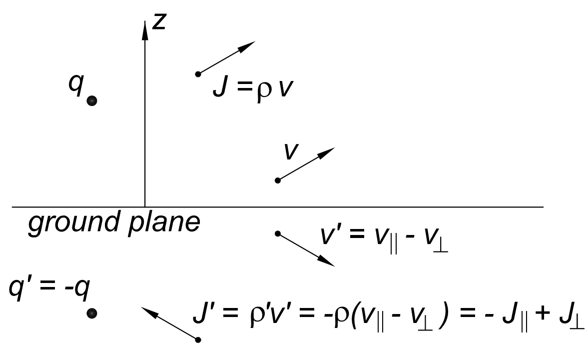
2 Solution

This problem explores the restrictions on the far-field radiation patterns from antennas associated with the reflection symmetry of the conducting ground plane, which implies that the image charge and current distributions ρ' and $\mathbf{J}'$ for $z < 0$ are related to the physical distributions ρ and $\mathbf{J}$ for $z > 0$ according to

$$\rho'(x, y, z < 0) = -\rho(x, y, -z > 0), \tag{1}$$

$$\mathbf{J}'(x, y, z) = -\mathbf{J}_\parallel(x, y, -z) + \mathbf{J}_\perp(x, y, -z), \tag{2}$$

where the ground plane is the x-y plane ($z = 0$), $\mathbf{J}_\perp = \mathbf{J}_z$ is the component of current density perpendicular to the ground plane (and therefore parallel to the z-axis), and $\mathbf{J}_\parallel = \mathbf{J} - \mathbf{J}_\perp$ is the component of current density parallel to the ground plane, as shown in the figure below.



We suppose that the antenna is located close to the origin, so that the far-zone radiation has radial direction $\hat{\mathbf{r}}$ in a spherical coordinate system , (r, θ, ϕ). The electric field is transverse in the far zone,

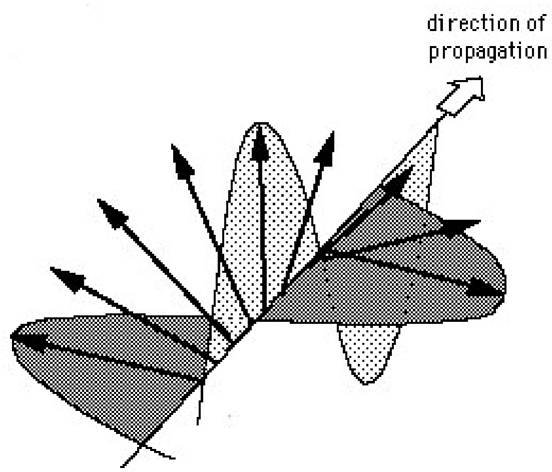
$$\mathbf{E}(r \gg \lambda) = (E_\theta \, \hat{\boldsymbol{\theta}} + E_\phi \, \hat{\boldsymbol{\phi}}) \frac{e^{i(kr-\omega t)}}{r}\,, \tag{3}$$

where the amplitudes E_θ and E_ϕ are complex numbers, $\omega = kc$ is the frequency of the radiation (approximated as being in vacuum), c is the speed of light, and the physical fields are the real part of eq. (3).

The radiation is said to be circularly polarized if $|E_\theta| = |E_\phi|$ and their phases differ by $\pm 90°$,

$$\mathbf{E}(r \gg \lambda) = E_0(\hat{\boldsymbol{\theta}} \pm i\,\hat{\boldsymbol{\phi}}) \frac{e^{i(kr-\omega t)}}{r} \qquad \text{(circularly polarized)}. \tag{4}$$

In this case the electric field vector rotates at angular frequency ω in the transverse plane at any fixed location. We will use the engineering convention (and that of quantum physics) that the wave with $\hat{\boldsymbol{\theta}} + i\,\hat{\boldsymbol{\phi}}$ is said to have right-handed circular polarization,[1] while the wave with $\hat{\boldsymbol{\theta}} - i\,\hat{\boldsymbol{\phi}}$ is said to have left-handed circular polarization. At any moment in time, the electric field vector of a right-handed circularly polarized wave traces out a right-handed helix (screw), as shown in the figure below.



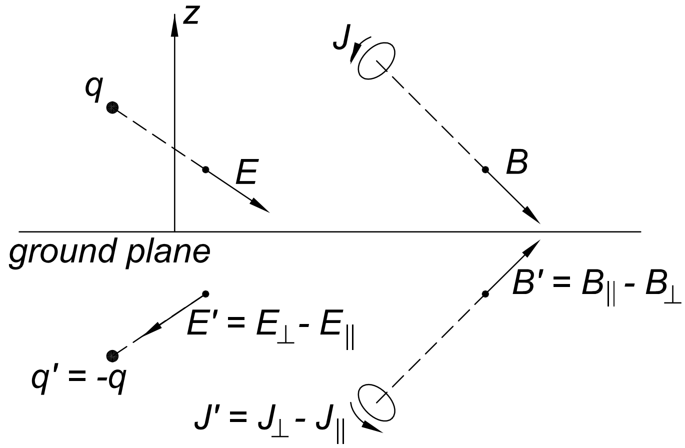
2.1 No Antenna System Above a Ground Plane Can Emit Circularly Polarized Radiation Parallel to the Plane

A consequence of the relations (1)-(2) between the physical charge and current distributions above the ground plane and the image distributions is that

$$\begin{aligned} \mathbf{E}'(r, \theta, \phi) &= \mathbf{E}_\perp(r, \pi - \theta, \phi) - \mathbf{E}_\parallel(r, \pi - \theta, \phi), & (5) \\ \mathbf{B}'(r, \theta, \phi) &= \mathbf{B}_\parallel(r, \pi - \theta, \phi) - \mathbf{B}_\perp(r, \pi - \theta, \phi), & (6) \end{aligned}$$

[1]The convention in the optics community, and that of most classical physics textbooks, is that the wave with $\hat{\boldsymbol{\theta}} + i\,\hat{\boldsymbol{\phi}}$ is said to have left-handed circular polarization.

where the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ are those due to the physical charge and current distributions at $z > 0$ in the absence of the ground plane, as shown in the figure below.



Thus, the total electric field in the far zone of an antenna above a ground plane can be written

$$\begin{aligned} \mathbf{E}_{\rm total}(r,\theta,\phi) &= \mathbf{E}(r,\theta,\phi) + \mathbf{E}'(r,\theta,\phi) \\ &= \mathbf{E}_\perp(r,\theta,\phi) + \mathbf{E}_\perp(r,\pi-\theta,\phi) + \mathbf{E}_\parallel(r,\theta,\phi) - \mathbf{E}_\parallel(r,\pi-\theta,\phi). \end{aligned} \tag{7}$$

In general, we do not know the relation between the fields $\mathbf{E}(r,\theta,\phi)$ and $\mathbf{E}(r,\pi-\theta,\phi)$, but for the case of radiation parallel to the ground plane ($\theta = \pi/2$), we find from eq. (7) that

$$\mathbf{E}_{\rm total}(r,\pi/2,\phi) = 2\mathbf{E}_\perp(r,\pi/2,\phi). \tag{8}$$

Thus, the radiation parallel to the ground plane from any antenna system above that plane is linearly polarized perpendicular to the plane.

That is, **no antenna system above a ground plane can emit circularly polarized radiation parallel to that plane**.

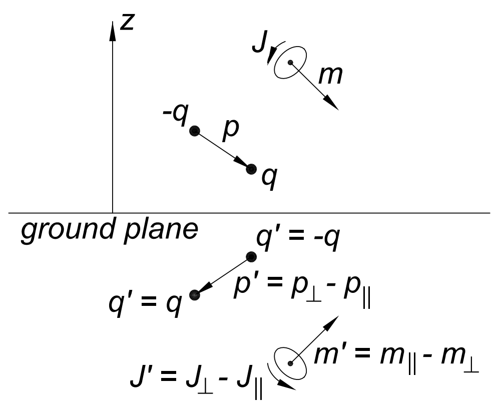
2.2 Systems of Small Dipole Antennas

In the rest of this note we only consider antenna systems composed electric dipole and small magnetic dipole antennas (all radiating at the same angular frequency ω), each of which is small compared to a wavelength. Then, each antenna is characterized by its electric dipole moment $\mathbf{p}\,e^{-i\omega t}$ or by its magnetic dipole moment $\mathbf{m}\,e^{-i\omega t}$, where the vectors $\mathbf{p}$ and $\mathbf{m}$ can have complex components.

When this antenna system is above a conducting ground plane the radiation fields are modified by the effect of the image charge and current densities, which can be summarized in terms of image dipoles,

$$\mathbf{p}' = \mathbf{p}_\perp - \mathbf{p}_\parallel, \qquad \mathbf{m}' = \mathbf{m}_\parallel - \mathbf{m}_\perp, \tag{9}$$

as shown in the figure below.



The antennas can be at different positions, which introduces a phase factor

$$kd = kx \sin\theta \cos\phi + ky \sin\theta \sin\phi + kz \cos\theta, \tag{10}$$

in the far fields at angles (θ, ϕ) due to a dipole at (x, y, z). The phase factor for an image dipole antenna is then

$$kd' = kx \sin\theta \cos\phi + ky \sin\theta \sin\phi - kz \cos\theta. \tag{11}$$

The total far-zone electric field for a system of electric dipole antennas $\{\mathbf{p}_j\}$ and magnetic dipole antennas $\{\mathbf{m}_l\}$ (in the absence of the ground plane) is

$$\begin{aligned}\mathbf{E}(r \gg \lambda) \;&=\; -k^2 \frac{e^{i(kr-\omega t)}}{r} \hat{\mathbf{r}} \times \left[\left(\hat{\mathbf{r}} \times \sum_j \mathbf{p}_j e^{ikd_j} + (\mathbf{p}_{j,\perp} - \mathbf{p}_{j,\|}) e^{ikd'_j} \right) \right. \\ &\qquad \left. + \sum_l \mathbf{m}_l e^{ikd_l} + (\mathbf{m}_{l,\|} - \mathbf{m}_{l,\perp}) e^{ikd'_l} \right], \end{aligned} \tag{12}$$

and the magnetic field in the far zone is

$$\begin{aligned}\mathbf{B}(r \gg \lambda) \;&=\; k^2 \frac{e^{i(kr-\omega t)}}{r} \hat{\mathbf{r}} \times \left[\sum_j \mathbf{p}_j e^{ikd_j} + (\mathbf{p}_{j,\perp} - \mathbf{p}_{j,\|}) e^{ikd'_j} \right. \\ &\qquad \left. - \left(\hat{\mathbf{r}} \times \sum_l \mathbf{m}_l e^{ikd_l} + (\mathbf{m}_{l,\|} - \mathbf{m}_{l,\perp}) e^{ikd'_l} \right) \right]. \end{aligned} \tag{13}$$

2.2.1 Dipole Antennas Close to the Ground Plane

If all the antennas are close to the ground plane then the products kz that appear in the phase factors (10)-(11) are negligible, and the phase factors kd and kd' are the same for an antenna and its image. In this case the far-zone electric field (12) simplifies to

$$\mathbf{E}(r \gg \lambda) = -2k^2 \frac{e^{i(kr-\omega t)}}{r} \hat{\mathbf{r}} \times \left[\left(\hat{\mathbf{r}} \times \sum_j \mathbf{p}_{j,\perp} e^{ikd_j} \right) + \sum_l \mathbf{m}_{l,\|} e^{ikd_l} \right]. \tag{14}$$

One consequence of eq. (14) is that **no systems of electric dipole antennas that are all close to a ground plane emits radiation (of any polarization) in the vertical**

(z) **direction**, because only the vertical component $\mathbf{p}_{j,\perp}$ of the electric dipoles contributes to the radiation. We saw above that such a system cannot emit circularly polarized radiation parallel to the ground plane. *It is an open question in the author's mind whether circularly polarized radiation could be emitted at some intermediate direction by a system of electric dipole antennas all close to the ground plane.*

Circularly polarized radiation can be emitted in the vertical direction by, for example, a pair of small loop antennas with common centers, and whose loops lie in orthogonal vertical planes, when the loops are driven 90° out of phase. This configuration is sometimes called an egg beater antenna. The physical magnetic dipole moment of this system can be written

$$\mathbf{m} = m_0(\hat{\mathbf{x}} \pm i\,\hat{\mathbf{y}}), \tag{15}$$

and the far-zone electric field in the vertical direction when these loops are close to a ground plane is

$$\mathbf{E}(r \gg \lambda, 0, 0) = \pm 2im_0k^2 \frac{e^{i(kr-\omega t)}}{r}(\hat{\mathbf{x}} \pm i\,\hat{\mathbf{y}}). \tag{16}$$

2.2.2 Dipole Antennas Above, but Not Necessarily Close to, a Ground Plane

When electric dipole antennas are above, but not necessarily close to, a ground plane the phase factors (10)-(11) can lead to constructive interference in the vertical direction, and to circularly polarized radiation in that direction. For an example, see sec. 2.4 of [1].

Can an antenna above a ground plane emit circularly polarized radiation in any direction other than the vertical?

In the absence of a ground plane, circularly polarized radiation can be emitted in all directions in a plane perpendicular to a given direction $\hat{\mathbf{a}}$ *by a combination of an electric dipole antenna with moment* $\mathbf{p} = p_0\,\hat{\mathbf{a}}$ *and a magnetic dipole antenna with moment* $\mathbf{m} = \pm ip_0\,\hat{\mathbf{a}}$ *whose magnitude (in Gaussian units) is the same as that of the electric dipole, and whose phase differs by* $\pm 90°$.[2] *In this case, the far-zone electric field of waves emitted in direction has* $\hat{\mathbf{r}}$ *has direction* $\mathbf{E} \propto \hat{\mathbf{r}} \times [(\hat{\mathbf{r}} \times \hat{\mathbf{a}}) \pm i\,\hat{\mathbf{a}}] = (\hat{\mathbf{r}} \cdot \hat{\mathbf{a}})\hat{\mathbf{r}} - \hat{\mathbf{a}} \pm i\,\hat{\mathbf{r}} \times \hat{\mathbf{a}}$. *Thus, when* $\hat{\mathbf{r}}$ *is perpendicular to* $\hat{\mathbf{a}}$, *the electric field has direction* $-\hat{\mathbf{a}} \pm i\,\hat{\mathbf{r}} \times \hat{\mathbf{a}}$, *so that the radiation emitted in this direction is circularly polarized.*

If this antenna system is at height z_a *above a ground plane (that is perpendicular to the* z*-axis), and the direction* $\hat{\mathbf{a}}$ *makes angle* θ_a *to the* z *axis in the* x-z *plane, then we can write* $\hat{\mathbf{a}} = \sin\theta_a\,\hat{\mathbf{x}} + \cos\theta_a\,\hat{\mathbf{z}}$, *and from eq. (9) the image moments are*

$$\mathbf{p}' = p_0(-\sin\theta_a\,\hat{\mathbf{x}} + \cos\theta_a\,\hat{\mathbf{z}}) \equiv p_0\,\hat{\mathbf{a}}'_p, \qquad \mathbf{m}' = \pm i\,p_0(\sin\theta_a\,\hat{\mathbf{x}} - \cos\theta_a\,\hat{\mathbf{z}}) \equiv \pm i\,p_0\,\hat{\mathbf{a}}'_m. \tag{17}$$

Also, the waves emitted at angle θ *to the* z*-axis from the image antenna at distance* z_a *below the ground plane have a phase factor of* $e^{-2ikz_a cos\theta}$, *where* θ *is the angle between the direction of the waves and the* z *axis. The far-zone electric field of this antenna above ground then obeys*

$$\begin{aligned} \mathbf{E} &\propto \hat{\mathbf{r}} \times [(\hat{\mathbf{r}} \times \hat{\mathbf{a}}) \pm i\,\hat{\mathbf{a}}] + [(\hat{\mathbf{r}} \times \hat{\mathbf{a}}'_p) \pm i\,\hat{\mathbf{a}}'_m]e^{-2ikz_a cos\theta} \\ &= \hat{\mathbf{r}}[\hat{\mathbf{r}} \cdot (\hat{\mathbf{a}} + \hat{\mathbf{a}}'_p e^{-2ikz_a cos\theta})] - (\hat{\mathbf{a}} + \hat{\mathbf{a}}'_p e^{-2ikz_a cos\theta}) \\ &\quad \pm i\,\hat{\mathbf{r}} \times (\hat{\mathbf{a}} + \hat{\mathbf{a}}'_m e^{-2ikz_a cos\theta}). \end{aligned} \tag{18}$$

[2] Thanks to David Jefferies for pointing this out.

It appears that there are no values of angles θ and θ_a for which the electric field (18) is circularly polarized¿

I believe that tilted eggbeater and turnstile antennas do not emit circularly polarized radiation in any direction if the antennas are above a ground plane. So, I conjecture that the only direction in which antennas above a ground plane can emit circularly polarized radiation is the vertical.

Acknowledgment

Thanks to Jacques Audet for suggesting this problem.

References

[1] K.T. McDonald, *Radiation of Turnstile Antennas Above a Conducting Ground Plane* (Sep. 18, 2008), http://physics.princeton.edu/~mcdonald/examples/turnstile.pdf