The Clock Paradox and Accelerometers[1]

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1 Problem

Discuss how the "twins", A and B, in Einstein's "clock paradox"[2] might assess each other's "age"[3] during the "round trip" of B, while A remains at rest in an inertial frame, using only measurements, and calculations based thereon, which they could make without the aid of observers located elsewhere.[4]

This note was inspired by e-discussions with Mike Fontenot.

2 Solution

When considering two inertial frames, A and B, that have relative velocity $\mathbf{v}$, observers in frame A can make two different comments about the rates of clocks in frame B. If a set of observers in frame A watch a single clock in frame B as it passes them by, they find that the rate of that clock in frame B is slower than the rate of the clocks in frame A by the factor $1/\gamma = \sqrt{1 - v^2/c^2}$. On the other hand, if a single observer in frame A watches a set of clocks in frame B that pass him by, he will find that the readings on the B clocks, when next to him, increase at a rate faster than the rate of his (A) clock by the factor γ. Most discussions

[1]It is generally easier to write a paper on the "clock paradox" than to understand one written by someone else.

[2]This "paradox" was first discussed by Einstein in sec. 4 of [4] (1905), who considered clocks A and B with the result that at the end of twin B's trip, his clock (proper time) had advanced by,

$$\Delta\tau_{\rm B} = \int_{\rm trip\ B} d\,\tau_{\rm B} = \int_{\rm trip\ B} d\,t\,\sqrt{1 - v_{\rm B}^2/c^2} \quad < \quad \int_{\rm trip\ A} d\,\tau_{\rm A} = \int_{\rm trip\ A} d\,t = \Delta t_{\rm A} = \Delta\tau_{\rm A}, \tag{1}$$

where time t and clock B's velocity $v_{\rm B}$ are measured with respect to the inertial frame of clock A, and c is the speed of light in vacuum.

The two participants, A and B, were first considered as people (but not explicitly as twins) by Langevin [5] (1911), and changes in their clocks were associated with "aging". Einstein elaborated on this "paradox" on p. 12 of [6] (1911), considering A and B as "living organisms". In his dialogue on relativity [15] (1918), Einstein again emphasized A and B as clocks. The first mention of twins (*Zwillingsbrüdern*) may have been by Weyl, p. 157 of [17] (1919).

The literature [4]-[342] on the "twin paradox" exhibits a dramatic variety of views, of which the most frequently published was that of Dingle (who opposed Einstein's theory of relativity; see [236]).

[3]In this note we consider "age" to mean the reading (proper time) on a clock, for which Einstein's famous result (1) is that an accelerated clock "ages" less than a nonaccelerated one. This goes against "conventional wisdom" that "life in the fast lane" will cause physiological "aging". For discussion of the relation of the "clock paradox" to physiology, see [101, 108, 114, 325].

[4]That the "ages" the twins assign one another during B's trip are dependent on conventions they choose is noted, for example, in [238]. Even for inertial observers, such as twin A, there exists an ongoing debate as to whether the notion of distant simultaneity ("age" of a distant object) is a matter of convention [20, 22, 25, 27, 157, 170, 198, 211, 213, 219, 222, 234, 235, 237, 240, 242, 243, 246, 255, 258, 260, 266, 268, 298, 310, 337].

in special relativity emphasize the first type of observation, but in the clock paradox, this type of observation is more relevant for the nonaccelerated twin A than for the accelerated twin B. For the latter, the second type of measurement is more relevant. This asymmetry leads to the result that both twins agree that the accelerated twin “aged” less.

The solution given here gives a perspective on why this asymmetry exists.

The solution builds on suggestions of Fremlin [93] and Darwin [95] (1957) that the twins communicate with one another during B’s trip via signals sent at the speed of light.[5] We add that the twins have accelerometers[6] which measure the (3-vector) proper acceleration of the device (relative to its instantaneous rest frame).[7,8]

2.1 Use of Twin A’s Accelerometer

The accelerometer held by twin A always reads zero, since this twin remains at rest in an inertial frame.

2.2 Use of Twin B’s Accelerometer

In contrast, the accelerometer of twin B reports a nonzero reading $\alpha_{\rm B}(\tau_{\rm B})$ of B’s proper acceleration as a function of the proper time $\tau_{\rm B}$ on the clock carried by B. Twin B can then use the measurement of $\alpha_{\rm B}(\tau_{\rm B})$ to integrate his equation of motion, using special relativity to deduce his velocity $\mathbf{v}_{\rm B}(\tau_{\rm B})$, position $\mathbf{x}_{\rm B}(\tau_{\rm B})$, and time $t_{\rm B}(\tau_{\rm B})$ relative to the inertial frame of twin A, in which he (twin B) started his trip from rest at, say, $\mathbf{x}_{\rm B}(\tau_{\rm B}(0)) = 0$ at, say, time $\tau_{\rm B}(0) = t_{\rm A}(0) = 0$.

Twin B’s calculations are simplest if his trip is entirely along a straight line, say the x-axis. Then, the 4-vector $x_{\rm B}^\mu$ has components $(ct_{\rm B}, x_{\rm B}, 0, 0)$ in the inertial frame of twin A, while twin B’s proper time interval is related by $d\tau_{\rm B} = d\,t_{\rm B}/\gamma_{\rm B}$, where $v_{\rm B} = d\,x_{\rm B}/d\,t_{\rm B}$ and $\gamma_{\rm B} = 1/\sqrt{(1 - v_{\rm B}^2/c^2)}$. The 4-velocity $u_{\rm B}^\mu$ and 4-acceleration $a_{\rm B}^\mu$ of twin B are related by,

$$u_{\rm B}^\mu = \frac{dx_{\rm B}^\mu}{d\tau_{\rm B}} = \gamma_{\rm B}(c, v_{\rm B}, 0, 0), \qquad u_{\rm B}^\mu\, u_{{\rm B},\mu} = c^2, \tag{2}$$

$$a_{\rm B}^\mu = \frac{du_{\rm B}^\mu}{d\tau_{\rm B}} = \left(c\frac{d\gamma_{\rm B}}{d\tau_{\rm B}}, \frac{d(\gamma_{\rm B} v_{\rm B})}{d\tau_{\rm B}}, 0, 0\right) = \gamma_{\rm B}^3 \frac{dv_{\rm B}}{d\tau_{\rm B}}\left(\frac{v_{\rm B}}{c}, 1, 0, 0\right), \tag{3}$$

$$a_{\rm B}^\mu\, a_{{\rm B},\mu} = -\alpha_{\rm B}^2 = -\gamma_{\rm B}^4 \left(\frac{dv_{\rm B}}{d\tau_{\rm B}}\right)^2 = -\frac{1}{(1 - v_{\rm B}^2/c^2)^2}\left(\frac{dv_{\rm B}}{d\tau_{\rm B}}\right)^2, \tag{4}$$

[5]The earliest proposal for synchronization of clocks via light signals may be that of Poincaré (1904) [3]. Fremlin and Darwin extended discussions by Milne and Whitrow (1933-35) [33, 36, 37] for inertial observers, and that by Page (1936) [38] for uniformly accelerated observers.

[6]While a version of an accelerometer was demonstrated by Atwood [1] (1784), they we not common until recently. Now, every smartphone has one.

Accelerometers were mentioned briefly by McMillan (1957) [92], by not used in the way considered here.

[7]The acceleration due to gravity is also measured by an accelerometer, so in this problem we suppose that gravity can be neglected. That is, we restrict our discussion of the “twin paradox” to special relativity.

[8]To relate the direction of the acceleration to the directions of the three spatial axes of twin A, twin B should also carry three gyroscopes with him.

where the component forms hold in the inertial frame of twin A. Taking the square root of eq. (4), and noting that $\alpha_{\rm B}$ and $d\,v_{\rm B}/d\tau_{\rm B}$ have the same sign, we find $dv_{\rm B}/(1-v_{\rm B}^2/c^2)^2 = \alpha_{\rm B}\,d\tau_{\rm B}$, and,[9]

$$\frac{v_{\rm B}}{c} = \tanh\left(\int_0^{\tau_{\rm B}} d\tau\,\frac{\alpha_{\rm B}}{c}\right), \qquad \gamma_{\rm B} = \cosh\left(\int_0^{\tau_{\rm B}} d\tau\,\frac{\alpha_{\rm B}}{c}\right), \qquad \gamma_{\rm B}\frac{v_{\rm B}}{c} = \sinh\left(\int_0^{\tau_{\rm B}} d\tau\,\frac{\alpha_{\rm B}}{c}\right), \tag{5}$$

supposing that twin B starts from rest at the origin at time $t_{\rm B} = \tau_{\rm B} = 0$. Then, from eqs. (2) and (5) we have,[10]

$$dx_{\rm B} = \gamma_{\rm B}\,v_{\rm B}\,d\tau_{\rm B}, \qquad x_{\rm B}(\tau_{\rm B}) = c\int_0^{\tau_{\rm B}} d\tau'\sinh\left(\int_0^{\tau'} d\tau\,\frac{\alpha_{\rm B}}{c}\right), \tag{6}$$

$$dt_{\rm B} = \gamma_{\rm B}\,d\tau_{\rm B}, \qquad t_{\rm B}(\tau_{\rm B}) = \int_0^{\tau_{\rm B}} d\tau'\cosh\left(\int_0^{\tau'} d\tau\,\frac{\alpha_{\rm B}}{c}\right). \tag{7}$$

In particular, twin B can compute the time $t_{\rm B}(\tau_{\rm B})$ that observers next to B (if they exist) in the inertial frame of A would find on their clocks (synchronized with A's clock) when B's clock reads $\tau_{\rm B}$. This computation, made with measurements taken only by twin B, complements that of eq. (1), which required measurements taken by a set of observers in the frame of twin A.[11]

2.2.1 Moments When $\mathbf{v}_{\rm B} = 0$

There may be moments (events) during twin B's trip when his velocity $\mathbf{v}_{\rm B}$ is zero with respect to the inertial frame of twin A. Since twin B can compute the time $t_{\rm B}(\tau_{\rm B})$ on the clock (if it existed) next to him in twin A's inertial frame, he could also infer that twin A's clock has

[9]Our eq. (5) was deduced in [229].

[10]Our eqs. (6)-(7) were deduced in sec. III of [271]. Related discussions are in sec. 97 of [63], and in [129], where $\theta \equiv \int_0^{\tau_{\rm B}} d\tau\,\alpha_{\rm B}/c$.

[11]A simple illustration of twin B's calculations is for the well-known case in which his trip is in a circle of radius r, at angular velocity ω, in the inertial frame of twin A. During this trip, twin A ages by $2\pi/\omega$, while twin B ages by only $2\pi/\gamma_B\omega$, where $\gamma_B = 1/\sqrt{1-\omega^2 r^2/c^2}$.

We suppose that twin B starts his trip at rest, next to twin A at the origin of twin A's coordinate system, and "instantaneously" accelerates to velocity $\mathbf{v}_B = \omega r\,\hat{\mathbf{x}}$, after which he experiences an acceleration of constant magnitude $\omega^2 r$ (in the inertial frame of twin A) in the x-y plane, always perpendicular to his velocity $\mathbf{v}_B$ (which remains constant in magnitude).

Then, twin B's proper time interval is related by $d\tau_{\rm B} = d\,t_{\rm B}/\gamma_{\rm B}$, where $v_{\rm B} = |d\,\mathbf{x}_{\rm B}/d\,t_{\rm B}| = \omega r$ and $\gamma_{\rm B} = 1/\sqrt{(1-v_{\rm B}^2/c^2)}$. Further, during twin B's trip,

$$dt_{\rm B} = \gamma_{\rm B}\,d\tau_{\rm B}, \qquad t_{\rm B}(\tau_{\rm B}) = \int_0^{\tau_{\rm B}} d\tau'\gamma_{\rm B} = \gamma\tau_B. \tag{8}$$

Thus, (accelerated) twin B infers from his accelerometer data that clocks next to him in the inertial frame of twin A read larger than his clock by the (constant) factor γ_B throughout his circular trip. *We recall that if an observer in one inertial frame takes readings of the clocks in another inertial frame as they pass him by, those readings increase with time at a rate greater than his own clock.*

This contrasts with the result of special relativity that an inertial observer always finds a clock in motion (with respect to that observer) has a slower rate than that of (synchronized) clocks in his own frame.

this value (since for an observer at rest in twin A's frame, as is twin B when $\mathbf{v}_{\rm B} = 0$, all clocks associated with that frame are synchronized).

Thus, twin B can compute the "age" of twin A (presuming that twin A has remained at rest) at those moments when he (twin B) is at rest with respect to twin A, whether or not the two twins are in the same place.

2.3 Use of Messages Broadcast by Twin B

Meanwhile, twin A knows nothing about the "age" of twin B, except at the moment when twin B returns and twin A can read twin B's clock directly.

So, to help twin A, twin B broadcasts messages that contain his (proper) time $\tau_{\rm B}$ at which the message was sent, as well as the results of his (rapid) calculations of $\mathbf{x}_{\rm B}(\tau_{\rm B})$, $\mathbf{v}_{\rm B}(\tau_{\rm B})$ and $t_{\rm B}(\tau_{\rm B})$. Twin A receives these messages at times later than $t_{\rm B}(\tau_{\rm B})$, but when he does receive a message, twin A considers that he now knows twin B's "age" was $\tau_{\rm B}$ at twin A's time $t_{\rm B}(\tau_{\rm B})$ as calculated by twin B.

Twin B's message could be sent with an agreed carrier frequency ν_0 (from twin B's perspective). Twin A would receive the message at carrier frequency $\nu_{\rm A}$, which depends on the velocity $\mathbf{v}_{\rm B}(\tau_{\rm B})$. For example, if twin B travels on a straight line at all time, $\nu_{\rm A} = \nu_0\sqrt{((1 - v_{\rm B}/c)/(1 + v_{\rm B}/c))}$. From a measurement of $\nu_{\rm A}$ (which might be difficult if $|v_{\rm B}|/c$ is large), twin A could confirm twin B's statement of $v_{\rm B}$.

By the end of the trip, twin A has accumulated a complete record of the (proper) time $\tau_{\rm B}(t_{\rm A})$ on twin B's clock at time twin A's time $t_{\rm A}$, although he arrived at this knowledge only somewhat later than time $t_{\rm A}$, with the time delay in his knowledge decreasing to zero at the end of the trip.

2.4 Use of Messages Broadcast by Twin A

Twin A also broadcasts messages, which consist only of the time t_A when the message was sent (and possibly a confirmation that he has remained at rest). When twin B receives these messages, they do not add to the knowledge already obtained from his own accelerometer.[12]

2.5 In General, Twin B Does Not Know the "Age" of Twin A

Twin B knows the "age" of twin A only at the beginning and end of the trip, and those moments (if any) when twin B is at rest with respect to twin A (as per sec. 2.2.1 above).

Twin B could take the attitude that during the trip, since his calculation of $t_{\rm B}(\tau_{\rm B})$ is the most he knows about twin A's clock at time $\tau_{\rm B}$, this could be defined as the "age" of twin A: $t_{\rm A}(\tau_{\rm B}) \equiv t_{\rm B}(\tau_{\rm B})$. This convention agrees with the direct observation of twin A's clock

[12]If twin A sent messages at the agreed carrier frequency ν_0 (now from twin A's perspective), twin B would receive them at carrier frequency $\nu_{\rm B}$, which depends on his velocity $\mathbf{v}_{\rm B}$ at the time of their receipt. For example, if twin B's traveled on a straight line at all times, $\nu_{\rm B} = \nu_0\sqrt{((1 - v_{\rm B}/c)/(1 + v_{\rm B}/c))}$. From a measurement of $\nu_{\rm B}$, twin B could confirm his computation of $v_{\rm B}$ based on his accelerometer measurements.

In addition, by comparing the time interval $d\tau_{\rm B}$ between receipt of two messages with the interval $dt_{\rm A}$ at which the messages stated they were sent, twin B could infer something about his velocity and acceleration with respect to twin A, but this is less effective than use of his accelerometer.

at the beginning and end of the trip, so there is no contradiction to use of this "age" based only on measurement by the two twins.[13]

2.5.1 Twin B's Trip Includes Frequent Stops

The discussion in sec. 2.2.1 above offers a kind of solution to the issue of distant simultaneity ("age" of a distant clock) for an accelerated observer such as twin B, provided the distant clock A is somehow known to remain in a single inertial frame at all times. Namely, the accelerated observer B brings himself to rest with respect to the distant clock in inertial frame A whenever he wants to know its "age", which is then the value, eq. (7), of the time on the clock frame of A that is next to the accelerated observer B.

In principle, such "stops" could be very brief for the accelerated observer B, such that the result of the computation (7) is little different from that made just before a "stop". Hence, the accelerated observer B could reasonably omit the (disruptive) "stops", and simply assign the "age" of the distant clock (*i.e.*, of twin A in the present example) as the value $t_B(\tau_B)$ he computes from his accelerometer data.

3 Comments

The suggestion in sec. 2.5 above is that twin B use computations based on measurements made by his accelerometer to interpret $t_B(\tau_B)$ of eq. (7) as the "age" of twin A at B's time τ_B, even if twin B is not at rest with respect to twin A at that time.[14] Other interpretations

[13]For example, consider the classic version of the "twin paradox", in which twin B quickly accelerates to velocity v along the x-axis, then travels distance D in the frame of twin A, quickly reverses velocity and returns to twin A, after elapse of total time $2D/v$ on the clock of twin A.

In this idealized scenario, the clock of twin B, according to twin A, always has rate slower than the clock of twin A by the factor $1/\gamma = \sqrt{(1 - v^2/c^2)}$. Hence, the reading on twin B's clock at the end of his trip is $\tau_B = t_A/\gamma = 2D/v\gamma$.

Using the data from his accelerometer, twin B reconstructs the history of his trip according to observers in the inertial frame of twin A, and could make the interpretation of the "age" of twin A as,

$$\text{Age}_\text{A} \text{ according to twin B} = \gamma\,\tau_B, \tag{9}$$

at his (twin B's proper) time τ_B.

Meanwhile, twin A receives messages from twin B, which explain how twin B's trip is proceeding with respect to the inertial frame of twin A. These message tell the same story that twin A could learn from a set of synchronized observers in his inertial frame, so twin A is happy with this story. Thus, he accepts that the "age" of twin B is $t_\mathbf{A}/\gamma$ at his own time $t_\mathbf{A}$, whether or not this was confirmed by other observers,

$$\text{Age}_\text{B} \text{ according to twin A} = \frac{t_A}{\gamma}\,. \tag{10}$$

[14]There exist papers that try to downplay the relevance of acceleration to the "clock paradox".

Boughn [225] considered "twins" A and B that both accelerate along a straight at the same rate with respect to an inertial frame C, but start at different places along their common line at time $t_A = t_B = t_C = 0$. The twin accelerate for the same proper time until they each end up with the same velocity with respect to frame C. They are then in the same inertial frame, say D, but according to clocks in this frame, the twins arrived in this frame at different times, and therefore have different "ages". If we note that according to observers in inertial frame D, the two twin did not start their trips at the same times/"ages", it is not surprising that their "ages" are different at the respective moments when they arrived in frame D. It is suggested that the are possible, although problematic.

3.1 Twin B Does Not Start or End His Trip at Rest

Some people prefer a version of the "clock paradox" in which twin B is not required to start or end his trip at rest with respect to twin A.[15] In such variants, it is also assumed that twin B has a known, nonzero velocity with respect to twin A at the moment he passes the by latter.[16] They still agree to set their clocks to zero at this moment.

If the twins still base their considerations of each other's "age" only on quantities that they can measure themselves, the story is not essentially different from that given in sec. 2 above. However, proponents of this scenario tend to suppose that the twin B has had a constant velocity for a long time prior to passing twin A, and that both of them have arranged for a set of synchronized observers in their respective inertial frames. This leads to claims, based on information from the auxiliary observers, that each twin thinks the other is "aging" more slowly at the moment they pass each other. In turn, this leads to various perplexities of the twins, as illustrated in the following secs. 3.2-3.

3.2 Use of Twin B's Instantaneous, Comoving Inertial Frame

Once twin B has computed his velocity $\mathbf{v}_{\rm B}(\tau_{\rm B})$ and position $\mathbf{x}_{\rm B}(\tau_{\rm B})$ with respect to twin A's inertial frame, he could also compute the time $t_{\rm B}(\tau_{\rm B})$ of twin A's clock (at both $\mathbf{x}_{\rm B}$ and $\mathbf{x}_{\rm A} = 0$) according to (imagined) observers in twin B's instantaneous, comoving inertial frame (in which quantities will be labeled with a superscript *; whence, $\tau_{\rm B} = t^\star_{\rm B}$).

We again consider the classic case that twin B's outbound trip involved rapid acceleration along the x-axis to velocity v, which then stayed constant until time $t = D/v$ (in twin A's frame).[17] At time $t < D/v$, when twin B is at $x_B = vt$, his clock reads $t^\star_{\rm B} = t/\gamma = t\sqrt{(1 - v^2/c^2)}$. The clock in twin A's inertial frame that is next to twin B reads $t = \gamma\, t^\star_{\rm B}$, as twin B can compute from his accelerometer data as well.

In addition, when twin B has not yet reversed his direction, he might use information from the synchronized observers in his inertial frame. The Lorentz transformation between twin A's inertial frame and twin B's instantaneous, comoving inertial frame tells us that the time on twin A's clock is related by $t^\star_{\rm B} = \gamma\,(t_{\rm A} - v\, x_{\rm A}/c^2) = \gamma\, t_{\rm A}$, according to the observer next to twin A in the comoving inertial frame of twin B (whose synchronized clock reads

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reader learn from this that different accelerations are not required to lead to different "aging" (although there would be no different "aging" if there were no acceleration).

Gruber and Price [244] noted that if twin B underwent very large acceleration a for a very short time dt, and immediately experienced acceleration $-a$ for time $2dt$, followed by acceleration a again for time dt, he would be back at rest at his starting point after only infinitesimal time $4dt$ had elapsed, so his "aging" would be "negligible" despite the large acceleration. It was then suggested that the reader consider this to be an example of a "twin paradox" with acceleration but "no aging".

[15] This version is perhaps closer to Einstein's original discussion [4].

[16] If A and B were twins, they would no longer have the same "age" at the moment twin B passed twin A, as their prior histories would involve different accelerations.

[17] A version of this scenario may have first been given by Langevin (1911) [5].

$t^\star_{\rm B}$). Then, twin B might suppose that twin A's "age" is,

$$\text{Age}_{\rm A} \text{ according to twin B} = \frac{t^\star_{\rm B}}{\gamma} \qquad \text{(outbound trip)}, \tag{11}$$

which is very different from eq. (9) of the scenario in sec. 2.5, $\text{Age}_{\rm A}$ according to twin B $= \gamma\,\tau_{\rm B} = \gamma\,t^\star_{\rm B}$.[18]

A famous difficulty with use of the twin B's instantaneous, comoving frame is that its velocity reverses direction at time $t = D/v$, after which the relevant Lorentz transformation is, since the origin of the comoving frame for $t^\star_{\rm B} > D/v\gamma$ was at $x_{\rm A} = 2D$ at time $t_{\rm A} = 0$,

$$t^\star_{\rm B} = \gamma\,[t_{\rm A} + v\,(x_{\rm A} - 2D)/c^2] \to \gamma\,[t_{\rm A} - (2D/v)(v^2/c^2)], \tag{12}$$

$$\text{Age}_{\rm A} \text{ according to twin B} = t_{\rm A} = \frac{t^\star_{\rm B}}{\gamma} + \frac{2D}{v}\frac{v^2}{c^2} \qquad \text{(inbound trip)}, \tag{13}$$

for twin A at $x_{\rm A} = 0$. These lead satisfactorily to $t^\star_{\rm B} = 2D/v\gamma$ and $t_{\rm A} = 2D/v$ at the end of the trip. But, during the brief time when twin B reverses his velocity, at time $t^\star_{\rm B} = D/v\gamma$, this scenario implies that the "age" of twin A, according to twin B, jumps from $D/v\gamma^2$ to $D/v\gamma^2 + (2D/v)(v^2/c^2) = 2D/v - D/v\gamma^2$.[19] This is also consistent with the "age" of twin A, according to twin B, being D/v (the average of $D/v\gamma^2$ and $2D/v - D/v\gamma^2$) at the midpoint of twin B's trip, at time $t^\star_{\rm B} = D/v\gamma$, when his velocity is momentarily equal to zero.

Discomfort with the abrupt jump in the supposed "age" of twin A according to twin B has led to consideration of many other prescriptions for computation by twin B of the "age" of the distant twin A.[20] The prescription considered in sec. 2.5 above is in some sense the "smoothest" of the alternatives.

3.2.1 Analysis Based on Accelerometer Measurements

It may be useful to illustrate the computations described in sec. 2.2 above for this scenario.

Twin B undergoes rapid, initial acceleration $\alpha_{\rm B}$ which quickly brings him to velocity $v_{\rm B} = v$ with respect to the inertial frame of twin A. Then, twin B experiences no further acceleration until time $t^\star_{\rm B} = \tau_{\rm B} = D/v\gamma$, at the end of the outbound portion of his trip.

[18]Use of eq. (11), based on observations of other observers in twin B's instantaneous inertial frame, rather than on eq. (9), based only on measurements possible by twin B, makes more sense if one supposes that twin B will not accelerate. Instead, if twin B quickly decelerated to rest with respect to twin A, he would consider twin A's "age" to be that given by eq. (9). Acceptance of eq. (11) implies acceptance that twin A would age very rapidly with respect to twin B during the brief deceleration of the latter.

[19]In a modified scenario (due to M. Fontenot), twin B accelerates from velocity v to $v' > v$ when his clock reads $t^\star_{\rm B} = D/\gamma v$, and reverses velocity only later. Then, just after this acceleration, twin B's instantaneous, comoving inertial frame has Lorentz factor $\gamma' = 1/\sqrt{(}1 - v'^2/c^2)$, and the relevant Lorentz transformation during the rest of the outbound trip is, since the origin of the (new) comoving frame was at $x_{\rm A} = D - D\,v'/v$ at time $t_{\rm A} = 0$: $t^\star_{\rm B} = \gamma'\,[t_{\rm A} - v'\,(x_{\rm A} - D + Dv'/v)/c^2] \to \gamma'\,[t_{\rm A} - D(1/v - 1/v')(v'^2/c^2)]$. According to observers in the comoving frame just after time $t^\star_{\rm B} = D/\gamma v$, twin A's clock read $t_{\rm A} = D/\gamma v[1/\gamma' + \gamma(1 - v/v')(v'^2/c^2)]$, which is less than $D/\gamma v$ for large enough v'. That is, twin A appears to become "instantaneously" younger just after the second acceleration of twin B, for large v'.

[20]This jump was noted by Lange (1927), p. 24 of [23], as having bothered Bergson [20] (1922), a famous opponent of the theory of relativity.

For times $\tau_{\rm B}$ during the outbound portion after the brief initial acceleration, we have from eq. (5) that,

$$\int_0^{\tau_{\rm B}} d\tau \, \frac{\alpha_{\rm B}}{c} = \tanh^{-1}\left(\frac{v}{c}\right). \tag{14}$$

From eqs. (6)-(7) we have,

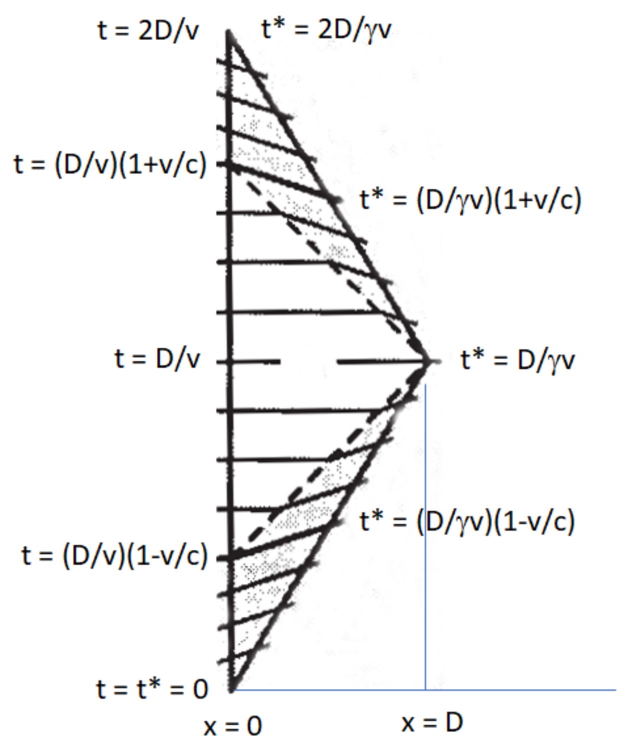
$$x_{\rm B} = c\int_0^{\tau_{\rm B}} d\tau' \sinh\left(\int_0^{\tau'} d\tau \, \frac{\alpha_{\rm B}}{c}\right) = c\int_0^{\tau_{\rm B}} d\tau' \sinh\left[\tanh^{-1}\left(\frac{v}{c}\right)\right] = c\int_0^{\tau_{\rm B}} d\tau' \gamma\frac{v}{c} = \gamma\, v\, \tau_{\rm B}, \tag{15}$$

$$t_{\rm B} = \int_0^{\tau_{\rm B}} d\tau' \cosh\left(\int_0^{\tau'} d\tau \, \frac{\alpha_{\rm B}}{c}\right) = \int_0^{\tau_{\rm B}} d\tau' \cosh\left[\tanh^{-1}\left(\frac{v}{c}\right)\right] = \int_0^{\tau_{\rm B}} d\tau' \gamma = \gamma\, \tau_{\rm B}. \tag{16}$$

At the end of the outbound trip, when $\tau_{\rm B} = D/v\gamma$, the computations (15)-(16) indicate that $x_{\rm B} = D$ and $t_{\rm B} = D/v$, as expected.

If we continue these calculations for the brief additional time, starting at $\tau_{\rm B} = D/v\gamma$, when twin B's velocity is reduced from v to zero with respect to twin A, the computations of $x_{\rm B}$ and $t_{\rm B}$ are essentially unchanged. Then, according to the discussion in sec. 2.2.1 above, twin B knows that the "age" of twin A is D/v at this moment (when twin B is a rest with respect to twin A), which result was noted above by a different argument.

3.3 Use of Marzke-Wheeler Coordinates



As an example of an alternative prescription, we consider use of the so-called Marzke-Wheeler coordinates [153].[21] Of course, any consideration of spatial coordinates goes beyond what twins A and B could measure by themselves.

These coordinates, as related to the "clock paradox", have been discussed in [250, 256]. For the idealized example of the "clock paradox" with accelerations only at the beginning, midpoint and end of twin B's trip, the Marzke-Wheeler times t for twin A and $t^\star$ for twin B are illustrated in the figure above (adapted from Fig. 4c of [250]), which is a kind of "Minkowski diagram" of twin A's coordinates, also showing lines of constant $t^\star_{\rm B}$. The world lines of two light signals between twins A and B are shown with dashes.

In this convention, twin B considers that twin A "aged" slowly at the beginning and end of B's trip, while twin A "aged" rapidly (but not "instantaneously" from twin B's perspective) during the central portion of the trip. Still, it would seem somewhat arbitrary to twin B that twin A's rate of "aging" made discontinuous jumps at his times $t^\star_{\rm B} = (D/\gamma v)(1 \pm v/c)$, especially as at time $t^\star_{\rm B} = (D/\gamma v)(1 - v/c)$ twin B might not have yet decided to reverse his velocity at time $t^\star_{\rm B} = D/\gamma v$.

3.4 Minguzzi's Concept of "Differential Aging"

Minguzzi [271, 284] extended the computations that twin B could make based only on his own measurements, our sec. 2.2 above, with a calculation of a quantity he called the "age differential", $\Delta = \tau_{\rm X} - \tau_{\rm B}$. In this, $\tau_{\rm X}$ is the reading (proper time) on the clock of an auxiliary observer X who also left twin A at time $\tau_{\rm A} = \tau_{\rm B} = \tau_{\rm X} = 0$ and then traveled directly, with constant velocity $v_{\rm X}$ (*i.e.*, on a geodesic), arriving at time $\tau_{\rm X}$ at the location of (accelerated) twin B at his proper time $\tau_{\rm B}$.[22] A possible appeal of the concept of the "age differential" is that at the end of the trip its value is just the final "age" difference of the twins A and B (since twin A also serves as the final auxiliary observer).[23]

One could define the "age" of twin A, according to twin B, to be $\tau_{\rm B} + \Delta = \tau_{\rm X} =$ the "age" of the auxiliary observer, although this is inconsistent with the comment at the end of footnote 22, and Minguzzi did not advocate it.[24]

[21]These coordinates are a generalization of earlier work by Milne, Whitrow, and Page [33, 36, 37, 38].

[22]If this scenario were to be implemented with actual observers, different such observers (called "imaginary twins" by Minguzzi) would be required for each time $\tau_{\rm B}$. Hence, direct measurement of Minguzzi's "age differential" is not practical, but it is calculable by twin B based only on measurements by his accelerometer.

[23]For the scenario in which twin B accelerates (rapidly) only at the beginning, midpoint and end of his trip, the "age differential" is zero on the outbound trip. During the return trip, at time $\tau_{\rm B} > D/\gamma v$, the time at the location of twin B (and the auxiliary observer X) in the frame of twin A is $t_{\rm B} = t_{\rm X} = \gamma\tau_{\rm B}$, so twin B, and the auxiliary observer X, are at distance $x_{\rm B} = x_{\rm X} = 2D - v\,t_{\rm X}$. The auxiliary observer traveled this distance in time $t_{\rm X}$ with velocity $v_{\rm X} = x_{\rm X}/t_{\rm X} < v$. The auxiliary observer's clock reads $\tau_{\rm X} = t_{\rm X}/\gamma_{\rm X} = t_{\rm B}/\gamma_{\rm X} = t_{\rm B}\sqrt{(1 - v_{\rm X}^2/c^2)} < t_{\rm B}$. The "age differential" now is $\Delta = \tau_{\rm X} - \tau_{\rm B} = t_{\rm B}/\gamma_{\rm X} - \tau_{\rm B}$.

At the end of the trip, $\tau_{\rm A} = t_{\rm B} = \tau_{\rm X}$, $\gamma_{\rm X} = 1$ and $\Delta_{\rm final} = \tau_{\rm A} - \tau_{\rm B}$, in agreement with the usual analysis.

At the midpoint of the trip, when $\tau_{\rm B} = D/2v\gamma$, twin B comes momentarily to rest with respect to twin A, and computes that twin A's clock reads $D/2v$ (as discussed at the end of sec. 3.1.1 above). The difference in "age" between twins A and B at this moment is $(1 - 1/\gamma)D/2v$, while the "age differential" Δ (at this moment) is zero. Thus, Minguzzi's "age differential" Δ cannot, in general, be the difference in "ages" of twins A and B.

[24]The language of Minguzzi's papers may lead some readers to infer this was his intent.

It remains a difficulty for many that special relativity, a classical theory, does not have a unique answer to the apparently simple question as to what twin B thinks is the "age" of twin A during the trip.[25] Instead, an interpretation is required, which affects the answer.

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The Principles of Mathematical Physics, Congr. Arts Sci. 604 (1905), p. 611,
http://physics.princeton.edu/~mcdonald/examples/GR/poincare_cas_604_05.pdf
On p. 621, Poincaré speculated: *Perhaps, we should construct a whole new mechanics, of which we only succeed in catching a glimpse, where inertia increasing with the velocity, the velocity of light would become an impassable limit.*

[4] A. Einstein, *Zur Elektrodynamik bewegter Körper*, Ann. d. Phys. **17**, 891 (1905),
http://physics.princeton.edu/~mcdonald/examples/EM/einstein_ap_17_891_05.pdf
http://physics.princeton.edu/~mcdonald/examples/EM/einstein_ap_17_891_05_english.pdf
On pp. 10-11 of the English translation, Einstein wrote: *From this there ensues the following peculiar consequence. If at the points A and B of K there are stationary clocks which, viewed in the stationary system, are synchronous; and if the clock at A is moved with the velocity v along the line AB to B, then on its arrival at B the two clocks no longer synchronize, but the clock moved from A to B lags behind the other which has remained at B by $tv^2/2c^2$ (up to magnitudes of fourth and higher order), t being the time occupied in the journey from A to B.*
It is at once apparent that this result still holds good if the clock moves from A to B in any polygonal line, and also when the points A and B coincide.
If we assume that the result proved for a polygonal line is also valid for a continuously curved line, we arrive at this result: If one of two synchronous clocks at A is moved in a closed curve with constant velocity until it returns to A, the journey lasting t seconds, then by the clock which has remained at rest the travelled clock on its arrival at A will be $tv^2/2c^2$ seconds slow. Thence we conclude that a balance-clock (not dependent on gravity) *at the equator must go more slowly, by a very small amount, than a precisely similar clock situated at one of the poles under otherwise identical conditions.*

[5] P. Langevin, *L'Évolution de l'espace et du temps*, Scientia **10**, 31 (1911),
http://physics.princeton.edu/~mcdonald/examples/GR/langevin_scientia_10_31_11.pdf
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[25]Twin A, of course, knows his own age. The fact that twin B does not know this age does not imply that twin A has no age.

No observer in the classical Universe can know everything about this Universe.

[6] A. Einstein, *Die Relativitäts-Theorie*, Viertel. Naturf. Gesell. Zürich **56**, 1 (1911),
http://physics.princeton.edu/~mcdonald/examples/GR/einstein_vngz_56_1_11.pdf
Translated from p. 12: *If we placed a living organism in a box, and made it carry through the same to-and-fro motions as the clock formerly did, then one could arrange that this organism, after any arbitrary lengthy flight, could be returned to its original spot in a scarcely altered condition, while corresponding organisms which had remained in their original positions had already long since given way to new generations. For the moving organism the lengthy time of the journey was a mere instant, provided the motion took place with approximately the velocity of light.*

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On p. 187 of the English translation: *The life-processes of mankind may well be compared to a clock. Suppose we have two twin-brothers who take leave from one another at a world-point A, and suppose one remains at home (that is, permanently at rest in an allowable reference-space), whilst the other sets out on voyages, during which he moves with velocities (relative to "home") that approximate to that of light. When the wanderer returns home in later years he will appear appreciably younger than the one who stayed at home.*

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