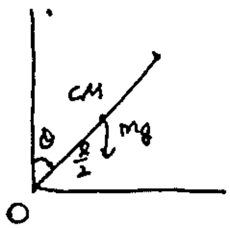
Problem Set II.



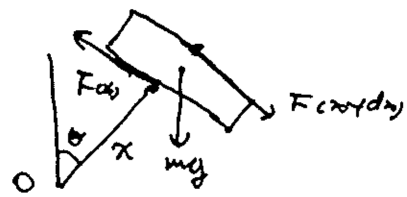
1. We consider the motion of he chimney as a whole. Regarding O as a reference point, from the torque equation, we get

$$I\ddot{\theta} = \frac{Mg\ell}{2}\sin\theta \Rightarrow \ddot{\theta} = \frac{3}{2}\frac{g}{\ell}\sin\theta \qquad (1)$$

where we used the fact that the moment of inertia of a thin bar is $I = Ml^2/3$. I will show you two methods of solving the remaining problem.

METHOD 1.

Let us consider the small (infinitesmal) piece of chimney as shown right. We directly apply D'Alembert's method to the piece. First the force balance along the perpendicular direction gives,



$$\ddot{\theta}x^2 dx - dmg\, x\sin\theta = F_\perp(x+dx) - F_\perp(x)$$

where $F_\perp$ denotes the shear. By expanding the argument of $F_\perp$ and using (1), we get

$$\frac{d}{dx}F_\perp = \frac{mg\sin\theta}{\ell}\left(\frac{3}{2}\frac{x}{\ell} - 1\right)$$

The above equation can easily be integrated to give,

$$F_\perp = \frac{3}{4}\frac{mg\sin\theta}{\ell^2}x^2 - \frac{m}{\ell}g\sin\theta\, x + C$$

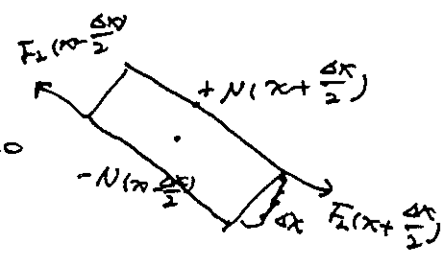
Using the boundary condition, $F_\perp=0$ at $x=l$, we determine

$$C + \frac{3}{4}mg\sin\theta - mg\sin\theta = 0$$

Consequently,

$$\therefore F_\perp = \frac{3}{4}\frac{mg\sin\theta}{\ell^2}(x^2-\ell^2) - \frac{mg}{\ell}\sin\theta\,(x-\ell) = \frac{mg\sin\theta}{4\ell^2}(3x-\ell)(x-\ell) \qquad (2)$$

Now, we consider the torque balance of the piece about the center of mass of the piece. From the figure right, we get



$$I'\ddot{\theta} - N\left(x-\frac{\Delta x}{2}\right) + N\left(x+\frac{\Delta x}{2}\right) - F_\perp\left(x+\frac{\Delta x}{2}\right)\frac{\Delta x}{2} - F_\perp\left(x-\frac{\Delta x}{2}\right)\frac{\Delta x}{2} = 0$$

where I' represents the moment of inertia of the piece. From the fact that the chimeny is thin, we can show that the first term is smaller than the second term by the factor

$$\frac{r^2}{\ell^2}$$

where r is the width of the chimney. Thus we neglect the term. Then, by expansion, we get the very simple relation.

$$\frac{d}{dx}N = F_\perp$$

We integrate the above equation using (2) and have,

$$N = \frac{Mg\sin\theta\; x(\ell-x)^2}{4\ell^2} + C'$$

where the constant can be set to 0 from the boundary condition N=0 at x=0. Thus,

$$N = \frac{Mg\sin\theta\; x(\ell-x)^2}{4\ell^2} \qquad (3)$$

By differentiating (2) and (3), we get

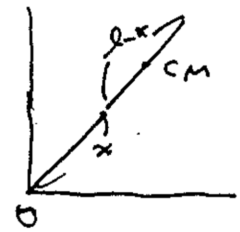
$$\frac{d}{dx}F_\perp = 0 \Rightarrow x = 2\ell/3$$

$$\frac{d}{dx}N = 0 \Rightarrow x = \ell/3$$

where each x above represents the most point of break.

METHOD 2.

Instead of considering the small piece of the chimney we consider two large portion of the chimney shown right. Notice that the interaction between the two portions is represented by F and N. By applying D'Alembert's method to the lower portion of the chimney we get,



$$\frac{Mx^3}{3\ell}\ddot{\theta} = \frac{Mx^2 g\sin\theta}{2\ell} + xF_\perp - N$$

where we used the fact that the moment of inertia of the lower portion is $I=1/3\ (M\ x/l)\ x^2$. We also consider the roration of the upper portion regarding its center of mass as a reference point. This consideration gives,

$$\frac{M(\ell-x)^3}{12\ell}\ddot{\theta} = \frac{(\ell-x)}{2}F_\perp + N$$

where we used the fact that the moment of inertia of the upper portion about its CM is $I= 1/12\ (M\ (l-x)/l\ )\ (l-x)^2$. By using (1), we have two equations,

$$\frac{Mg}{2}\frac{\sin\theta}{\ell^2}x^3 = \frac{Mx^2 g\sin\theta}{2\ell} + xF_\perp - N$$

$$\frac{Mg}{8}\frac{\sin\theta}{\ell^2}(\ell-x)^3 = \frac{(\ell-x)}{2}F_\perp + N$$

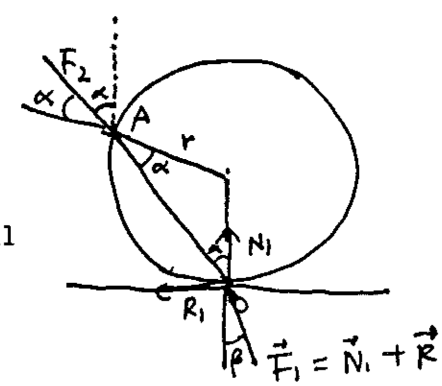
for two unknowns $F_\perp$, and N. By solving the above two equations simultaneously we get (after rather lengthy calculation!),

$$F_\perp = \frac{Mg\sin\theta}{4\ell^2}(3x-\ell)(x-\ell)$$

$$N = \frac{Mg\sin\theta}{4\ell^2}x(\ell-x)^2$$

In fact, there can be variety of other methods depending on where you put your refence point.

2. Consider the figure of right side. The direction of F_2 is along the line OA. Otherwise, if we regard O as a reference point, there would be net torque which would break the equilibrium condition. Since we considered torque already, we condier the force balance. The vertial and horizontal force balance give,



$$-F_2\cos\alpha - mg + N_1 = 0$$

$$F_2\sin\alpha - R_1 = 0$$

From the figure, we find that

$$\cot\beta = \frac{N_1}{R_1}$$

Combining these results, we find that,

$$\cot\beta = \cot\alpha + \frac{mg}{F_2\sin\alpha}$$

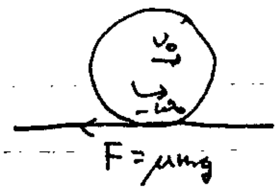
From the requirement of $R_1 < \mu_1 N_1$ at point O, we have

$$\mu_1 > \frac{R_1}{N_1} = \mu_{1,min} \quad \therefore \quad \mu_{1,min} = \frac{F_2\sin\alpha}{F_2\cos\alpha + mg}$$

and thus determine $\mu_{1,min}$. $\mu_{2,min}$ can be determined by requiring the similar condition at point A. In this case we get,

$$F_{2,tangential} \le \mu_2 F_{2,vertical} \Rightarrow F_2\sin\alpha \le \mu_2 F_2\cos\alpha \Rightarrow \mu_2 > \mu_{2,min} = \tan\alpha.$$

Now, we consider the napkin ring moving with backspin. At first, the napking has the angular velocity and the translational velocity v_0. Since the contact point O is moving relative to ground, we should use the coefficient of the sliding friction, μ. The torque and force equation give, respectively, ($I = mr^2$, in this case)



$$m \frac{d}{dt} v = -\mu mg$$
$$mr^2 \frac{d}{dt}\omega = +\mu mgr \quad \text{(Note that this has positive sign)}$$

which can be easily solved to yield,

$$v = -\mu g t + v_0$$
$$\omega = \frac{\mu g t}{r} + \omega_0$$

As time goes on, the velocity of the contact point becomes zero and rolling without slipping occurs. In that case,

$$v = r\omega.$$

and using (1), we can find

$$-\mu g t + v_0 = \mu g t + r\omega_0 \quad \Rightarrow \quad t = \frac{v_0 + r\omega_0}{2\mu g}$$

If the translational velocity

$$v = v_0 - \mu g t$$

is negative at that time, our napkin ring will roll back. Thus we have the condition,

$$v = v_0 - \mu g \left(\frac{v_0 + r\omega_0}{2\mu g}\right) < 0 \quad \Rightarrow \quad v_0 < r\omega_0.$$

3. Since our object is rigid body we have,

$$|\vec{r}_i - \vec{r}_j| = a = \text{constant.} \qquad (1)$$

By taking variation of the above equation, we get

$$(\vec{r}_i - \vec{r}_j) \cdot (\delta\vec{r}_i - \delta\vec{r}_j) = 0$$

which means,

$$\delta\vec{r}_i = \delta\vec{r}_j + \delta k\, \hat{n} \qquad (2)$$

where $\hat{n}$ is vector which is perpendicular to $\vec{r}_i - \vec{r}_j$. For our force f_{ij}, we assume that

$$\vec{f}_{ij} = -\vec{f}_{ji}$$

and

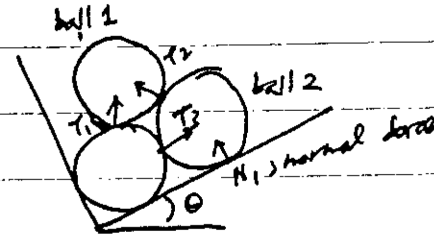
$$\vec{f}_{ij} \parallel \vec{r}_i - \vec{r}_j \qquad (3)$$

Thus,

$$\therefore \delta W_{ij} = \vec{f}_{ij} \cdot \delta r_i + \vec{f}_{ji} \cdot \delta r_j = \vec{f}_{ij} \cdot (\delta r_j + \delta k\, \hat{n}) - \vec{f}_{ij} \cdot \delta\vec{r}_j$$
$$= \vec{f}_{ij} \cdot \hat{n}\, \delta k = 0 \quad \text{(due to (3))}$$

We can interpret $\delta\vec{r}_j$ part as a translation and $\delta k\, \hat{n}$ part as a rotation. One important thing here is to note that the condition (1) is not
in (2)

$$\vec{r}_i - \vec{r}_j = \vec{a}\ ;\text{constant} \quad \Rightarrow \quad \delta\vec{r}_i = \delta\vec{r}_j.$$



4. I will present two methods for this problem.

METHOD 1. Force Balancing

The physical reason why static equilibrium is impossible is that if θ gets too small, we should have negative T_3 which is impossible. With this fact in mind, we consider the balancing of force at ball 1. This gives,

$$T_1 \sin(30-\theta) = T_2 \sin(30+\theta)$$

$$T_1 \cos(30-\theta) + T_2 \cos(30+\theta) = Mg$$

At ball 2, we have

$$N_1 \cos\theta + T_3 \sin\theta = Mg + T_2 \cos(30+\theta)$$

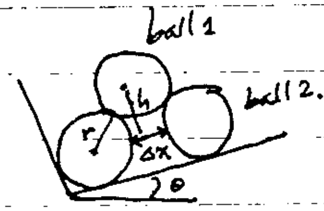
$$N_1 \sin\theta = T_3 \cos\theta + T_2 \sin(30+\theta)$$

Solving above equation for N_3 we get,

$$N_3 = \frac{Mg}{2}\left(3\sin\theta - \frac{1}{\sqrt{3}}\cos\theta\right)$$

Now, the requirement $N_2 > 0$, we have,

$$\tan\theta > \frac{1}{3\sqrt{3}} = \tan\theta_c.$$



METHOD 2. The Virtual Work.

We consider the virtual displacement of ball 2 by Δx. Since the contact should be maintained, the CM positions for ball 1 and ball 2 is shifted vertically by

ball 1; $\delta y_1 \propto 2r\cos(30-\theta)$, ball 2; $\delta y_2 \propto 4r\sin 30\cdot\sin\theta$

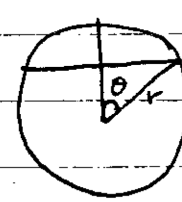
Thus the resulting change of gravitational potential energy is given by,

$$\delta W = Mg(\delta y_1 + \delta y_2) \propto (2r\cos(30-\theta) + 4r\sin 30 \sin\theta)$$

By equating above equation equation with 0, we get,

$$\tan\theta_c = \frac{1}{3\sqrt{3}}$$

This result is same as the one given by method 1. The important thing here is that we fix θ as a constant in the whole virtual displacement process. Generally method 2 is simpler than method 1.



5. We will solve this problem using the method of virtual work. From the right figure, we find the length of band is,

$$2\pi r\sin\theta$$

Thus the total mechanical energy is given by

$$E = \frac{1}{2}k(2\pi r\sin\theta - l_0)^2 + Mgr\cos\theta$$

By taking virtual change of θ, we get,

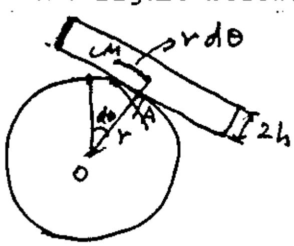
$$\delta E = k(2\pi r\sin\theta - l_0)2\pi r\cos\theta\,\delta\theta - Mgr\sin\theta\,\delta\theta = 0$$

$$\therefore\ 2\pi k \cos\theta\,(2\pi r\sin\theta - l_0) = mg\sin\theta$$

We note that this problem can be solved by direct force balancing. One caution should be mentioned if one tries to use energy conservation method. In that case, one should verify that the kinetic energy vanishes when the band is at the static equilibrium point.

6. Throughout (a) and (b), we try to calculate the change in vertical location of the center of mass. If it is lowered, the gravitational potential will be changed into kinetic energy and an instability occurs. If it is highered, we get stability, or stable equilibrium. We note that the position shown in the problem is a equilibrium point though if it is a stable one is not known.

(a) Consider the figure below.



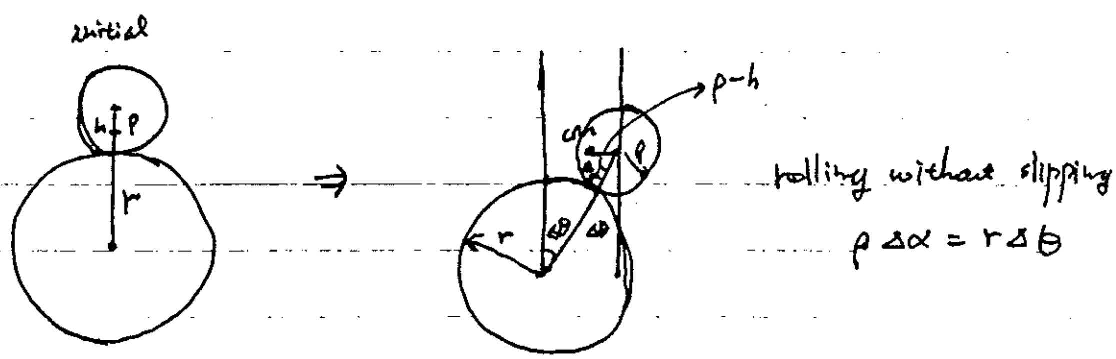
Due to the rolling of a dime, there has been change in contact point. Additionally CM has been shifted from the axis OA. The two effects are added to give,

$$\Delta y = (r+h)\cos\Delta\theta + r\Delta\theta\sin\Delta\theta - (r+h) \approx \frac{\Delta\theta^2}{2}(r-h)$$

where the initial position of CM is r+h. Thus, the instability condition gives,

$$\Delta y > 0 \Rightarrow \quad r > h$$

(b) As long as small rolling is concerned, the rolling of ellipsoid is equivalent to the rolling of a sphere having radius of the radius of the curvature at the initial contact point having CM h above. Thus the below figure is relavent.

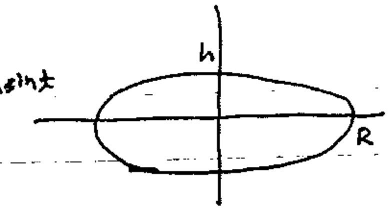


The same calculation as (a) gives, (see figure for explanations)

$$\Delta y = (r+\rho)\cos\Delta\theta - (\rho-h)\cos(\Delta\theta+\Delta\alpha) - (r+h)$$

$$= \frac{\Delta\theta^2}{2}(\rho+r)\left(-1+\frac{(\rho-h)(\rho+r)}{\rho^2}\right) \qquad \therefore\ \Delta y>0 \Rightarrow \quad \frac{1}{h} > \frac{1}{r}+\frac{1}{\rho}.$$

The radius of curvature is formally calculated as follows.



The equation of ellipsoid is given by

$$\frac{x^2}{R^2} + \frac{y^2}{h^2} = 1 \Rightarrow y = h\sqrt{1 - \frac{x^2}{R^2}} \Rightarrow x = R\cos t,\ y = h\sin t$$

and the formula

$$\kappa = \frac{1}{\rho} = \frac{[1+(y')^2]^{3/2}}{y''}$$

gives, (after a lengthy calculation)

$$\rho = \frac{R^2}{h}$$

Heuristic way of deriving this result is to expand the above equation and compare with the expansion of the equation of the circle. $(X^2 + Y^2 = \rho^2)$

$$\left.\begin{aligned} y &\approx h - \frac{h}{2R^2}x^2 \\ Y &\approx \rho - \frac{1}{2\rho}X^2 \end{aligned}\right\} \ y_{=0} \text{ value is irrelevant.} \Rightarrow \rho = \frac{R^2}{h}$$

ANOTHER METHOD: Instead of energy consideration, it is possible to consider torque. In this case, we compare the horizontal displacement of contact point and the horizontal displacement of CM. If contact point overshoots CM, we have stable equilibrium. Try this method to our problem (a) and (b)!

7. Consider the fiture in the right side. Torque equation directly gives,

$$N = -mg\sin\theta R = I\ddot{\theta} \qquad \therefore \ddot{\theta} = -\frac{MgR}{I}\sin\theta \quad (1)$$

As is well known, the simple pendulum of mass m and length l satis fies the following equation.

$$\ddot{\theta} = -\frac{g}{\ell}\sin\theta$$

The above equation defines the center of oscillation as

$$\ell = \frac{I}{MR}$$

The point lies along the line AM. Now, if we rehang the pendulum regarding the center of oscillation as a pivot point, the corresponding moment of inertia is given by (from parallel axis theorem)

$$I' = I_{CM} + m(\ell - R)^2 = I_{cm} + mR^2 - 2m\ell R + mR^2 = I - 2\underbrace{MR\ell}_{I} + m\ell^2$$

$$= I\left(\frac{\ell}{R} - 1\right)$$

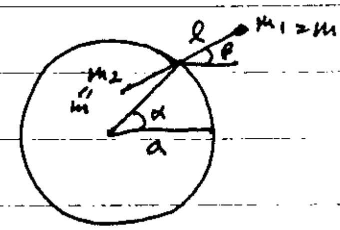
The torque equation now gives,

$$N = -mg\sin\theta(\ell - R) = I'\ddot{\theta} \quad \therefore \ddot{\theta} = -\frac{Mg(\ell-R)}{I'}\sin\theta = -\frac{MgR}{I}\sin\theta \quad (2)$$

We find that the location of the new center of oscillation is exactly the previous pivot point by comparing equation (1) and (2). Thus, the period of two pendulum is same.

8. Since there is no net external force except for the constraint force which is supplied by circular rail, the possible motion for CM is only circular motion with constant speed. If we view the whole motion from the CM, we are merely observing rod with two identical mass at the ends from the center. Since there is no external torque in this case, this motion should be rotation with constant angular speed.

Let us verify this answer using Lagrangian method.



From the right picture, we find the coordinate of one particle is given by

$$x_1 = \ell\cos\beta + a\cos\alpha\ ,\quad y_1 = \ell\sin\beta + a\sin\alpha$$

and another particle's coordinates are given by

$$x_2 = -\ell\cos\beta + a\cos\alpha\ ,\quad y_2 = -\ell\sin\beta + a\sin\alpha$$

From the above formula, the velocity is immediately found by differentiation.

$$\dot{x}_1 = -\ell\sin\beta\,\dot{\beta} - a\sin\alpha\,\dot{\alpha}\ ;\ \dot{y}_1 = \ell\cos\beta\,\dot{\beta} + a\cos\alpha\,\dot{\alpha}\ ;\ \dot{x}_2 = \ell\sin\beta\,\dot{\beta} - a\sin\alpha\,\dot{\alpha}\ ;\ \dot{y}_2 = -\ell\cos\beta\,\dot{\beta} + a\cos\alpha\,\dot{\alpha}$$

Since there is no external force, the Lagrangian is simply given by

$$L = T - V = T = \frac{1}{2}m(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m(\dot{x}_2^2 + \dot{y}_2^2)$$
$$= m\ell^2\dot{\beta}^2 + ma^2\dot{\alpha}^2$$

We can identify two generalized momenta

$$P_\alpha = \frac{\partial}{\partial\dot{\alpha}}L = 2ma^2\dot{\alpha}$$
$$P_\beta = \frac{\partial}{\partial\dot{\beta}}L = 2m\ell^2\dot{\beta}$$

where the former represents the orbital angular momenta of CM and the latter represents the angular momentum of the system with respect to CM. From the Euler-Lagrange equation, we have

$$\frac{d}{dt}P_\alpha = \frac{\partial}{\partial\alpha}L = 0 \quad\Rightarrow\quad P_\alpha = P_{\alpha 0}\ ;\ \text{constant}$$
$$\frac{d}{dt}P_\beta = \frac{\partial}{\partial\beta}L = 0 \quad\Rightarrow\quad P_\beta = P_{\beta 0}\ ;\ \text{constant.}$$

which verifies our expectation.

9. In plane polar coordiate the speed is given by

$$v^2 = \dot{r}^2 + r^2\dot{\theta}^2$$

Thus, the Lagrangian is given by

$$L = \frac{1}{2}mv^2 - V = \frac{1}{2}m(\dot{r}^2 + r^2\omega^2)$$

since there is no external force $(V=0)$. We have set $\dot{\theta} = \omega$ since the mass is rotating with the constant angular speed. From the Euler-Lagrange equation we get,

$$\frac{d}{dt}\left(\frac{\partial}{\partial\dot{r}}L\right) - \frac{\partial}{\partial r}L = 0 \quad\Rightarrow\quad \ddot{r} = \omega^2 r$$

The general solution of the above equation is,

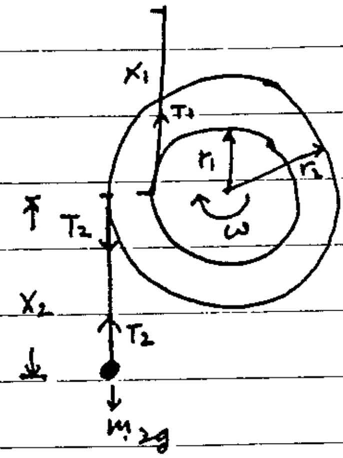
$$r = A\cosh\omega t + B\sinh\omega t$$

From the initial condition, $r = r_0$ and $\dot{r} = v_0$ at t=0, A,B are determined.

$$A = r_0\ ,\quad \omega B = v_0$$

Thus the full solution is

$$r = r_0\cosh\omega t + \frac{v_0}{\omega}\sinh\omega t.$$



10. We consider the static case of the righthand side figure. Force balancing and the torque balancing regarding CM of the double cylinder as a reference point give,

$$T_1 r_1 = T_2 r_2$$
$$T_1 = T_2 + m_1 g$$
$$T_2 = m_2 g$$

From the above equations, we get

$$(m_1 + m_2) r_1 = m_2 r_2$$

Especially if $r_1 = r_2$, the condition reduces to

$$m_1 = 0$$

From this fact, we understand that if $r_1 = r_2$, $m_1 = 0$, our acceleration calculation should give 0 result.

(b) The Newton's equation and torque equation mentioned above give, in dynamic case,

$$m_1 \ddot{x}_1 = m_1 g + T_2 - T_1$$
$$I \dot{\omega} = r_1 T_1 - r_2 T_2$$
$$m_2 (\ddot{x}_1 + \ddot{x}_2) = m_2 g - T_2$$

By solving above equation for T_1 and T_2, we get

$$T_2 = m_2 g - m_2 (\ddot{x}_1 + \ddot{x}_2), \quad T_1 = (m_1 + m_2) g - (m_1 + m_2) \ddot{x}_1 - m_2 \ddot{x}_2$$

Thus, using the additional relation (constraints),

$$\dot{x}_1 = r_1 \omega, \quad \dot{x}_2 = -r_2 \omega$$

we get,

$$\ddot{x}_1 = r_1 g \{ (m_1 + m_2) r_1 - m_2 r_2 \} / \{ I + m_1 r_1^2 + m_2 (r_2 - r_1)^2 \}$$

(c) The Lagrangian can be directly written down as,

$$L = T + V = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 (\dot{x}_1 + \dot{x}_2)^2 + \frac{1}{2} I \dot{\omega}^2 + m_1 g x_1 + m_2 (x_1 + x_2) g$$

We can rewrite above equation using the constraints mentioned above as,

$$L = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \left(1 - \frac{r_2}{r_1}\right)^2 \dot{x}_1^2 + \frac{1}{2} I \frac{1}{r_1^2} \dot{x}_1^2 + (m_1 + m_2) g x_1 - \frac{r_2}{r_1} m_2 g x_1 + \text{const.}$$

Now The Euler-Lagrange equation gives,

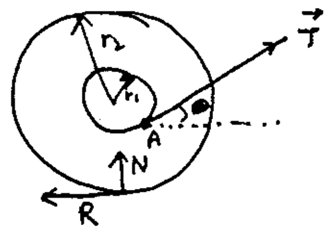
$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot{x}_1} L\right) - \frac{\partial}{\partial x_1} L = 0 \Rightarrow (I + m_1 r_1^2 + m_2 (r_2 - r_1)^2) \ddot{x}_1 = r_1 g ((m_1 + m_2) r_1 - m_2 r_2)$$

We can actually verify the special case mentioned above.

→ cf. Notice that in (c), $Q_{frictional} = 0$, $Q_{tension} = T(\cos\theta - \frac{r_1}{r_2})$ not $Q_{fric} = -T\frac{r_1}{r_2}$, $Q_{tension} = T\cos\theta$.

11. (a) From the figure right, we have three conditions for static equilibrium. That is, two force equations

$$-mg + T\sin\theta + N = 0\ ,\quad T\cos\theta = F$$



and one torque equation regarding CM as a reference point.

$$Tr_1 = Fr_2.$$

From the above equation one finds that

$$\cos\theta = \frac{r_1}{r_2}$$

Additionally from the relation, $R \le \mu N$, , we get

$$\mu \ge \mu_{min} = \frac{R}{N} = \frac{T\cos\theta}{mg - T\sin\theta}$$

(b) In dynamic case, the force equation and torque equation is given by

$$Fr_2 - Tr_1 = I\ddot{\alpha} = \frac{I}{r_2}\ddot{x}$$

$$T\cos\theta - F = m\ddot{x}$$

where the condition of rolling without slipping implies

$$r_2\dot{\alpha} = \dot{x}$$

By solving above equations, we get

$$\ddot{x} = \frac{T(\cos\theta - r_1/r_2)}{m + I/r_2^2}. \qquad (1)$$

If F=0, we should have,

$$\ddot{x} = \frac{T}{m}\cos\theta$$

By equating this with (1), we get

$$\frac{T}{m}\cos\theta = \frac{T(\cos\theta - r_1/r_2)}{m + I/r_2^2} \Rightarrow \cos\theta = -\frac{mr_1r_2}{I}$$

(c) The construction of the kinetic part of laglangian is straightforwardly given by

$$T = \frac{1}{2}I\dot{\alpha}^2 + \frac{1}{2}m\dot{x}^2 = \frac{1}{2}I\left(\frac{\dot{x}}{r_2}\right)^2 + \frac{1}{2}m\dot{x}^2.$$

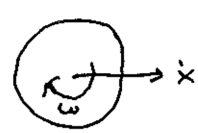
The generalized force corresponding to tension is given by,

$$^{*}\ Q_{tension} = \vec{T}\cdot\frac{\partial\vec{r}}{\partial x} = T\cos\theta - T\frac{r_1}{r_2}$$

where using the same method, the generalized force for friction is calculated to be,

$$Q_{friction} = \cancel{\quad}\ 0$$

Thus, the full equation becomes



$$\frac{d}{dt}\left(\frac{\partial}{\partial\dot{x}}T\right) = Q_{tension} + Q_{friction}$$

$$\left(\frac{I}{r_2^2} + m\right)\ddot{x} = T\left(\cos\theta - \frac{r_1}{r_2}\right) \quad \text{: same as (1)}$$

If $\cos\theta > r_1/r_2$, i.e., θ is small, grandmother can receive the spool.

* $\vec{r}$ should be point A. The velocity of the point from Lab frame is

$$\vec{v} = r_1\omega\frac{-\vec{T}}{T} + v\hat{x} = \dot{x}\hat{x} - \frac{r_1}{r_2}\dot{x}\left(\frac{\vec{T}}{T}\right)$$

$$\therefore Q_{tension} = \vec{T}\cdot\frac{\partial\vec{r}}{\partial x} = \vec{T}\cdot\frac{\partial\dot{\vec{r}}}{\partial\dot{x}} = \vec{T}\cdot\hat{x} - \frac{r_1}{r_2}\frac{1}{T}\vec{T}\cdot\vec{T} = T\left(\cos\theta - \frac{r_1}{r_2}\right)$$

In case of friction, the contact point momentarily stops.

$$\therefore Q_{friction} = \vec{F}\cdot\frac{\partial\vec{r}}{\partial x} = 0$$