The spin-1 vector mesons can be taken to have quark content: $\rho^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$, $\omega^0 = (u\bar{u} + d\bar{d})/\sqrt{2}$, $\phi = s\bar{s}$, $J/\psi = c\bar{c}$, and $\Upsilon = b\bar{b}$.

1. The decays $V \to e^+e^-$ proceed via a single intermediate photon, where $V$ is a vector meson. Suppose the decay rates $\Gamma(V \to e^+e^-)$ are independent of quark mass and the meson mass, but do depend on quark charge. Predict the relative decay rates to $e^+e^-$ for the five vector mesons.

2. In the decay $V \to \pi^0\gamma$ the meson spin changes from 1 to 0. Hence this must be an $M1$ magnetic dipole transition. In the quark model the decay rate depends on the size of the relevant quark magnetic moments. Suppose the quarks have Dirac moments $Q_q/2m_q$ where $m_u \approx m_d \approx 2/3m_s$. Predict the relative decay rates to $\pi^0\gamma$ for the $\rho^0$, $\omega^0$, and $\phi$ mesons.

3. There are four spin-0 mesons that contain one bottom quark: $B_u^+ = u\bar{b}$, $B_d^0 = d\bar{b}$, $B_s^0 = s\bar{b}$, and $B_c^+ = c\bar{b}$. These decay via the weak interaction by two graphs with roughly equal strength (the ‘spectator’ model):

Here we consider only nonleptonic final states. Suppose the four final-state quarks form exactly two mesons (as happens a few percent of the time). List the two dominant two-body decays for each of the four bottom mesons.

A complication arises for the $B_c$ meson. The charm quark has a slightly shorter lifetime than the bottom quark. Hence there are two more prominent two-body decays of the $B_c$ involving $c \to Wq$ rather than $b \to Wq$ transitions. List these.

According to the measured values of the C-K-M matrix elements

$$\frac{V_{ub}}{V_{cb}} \approx \frac{V_{us}}{V_{ud}} \approx \frac{V_{cd}}{V_{cs}} \approx \lambda = \text{Cabbibo angle.}$$

List the two-body nonleptonic decays of the four bottom mesons that are suppressed by one power of $\lambda$ in the matrix element (and hence by $\lambda^2 \approx 1/25$ in rate).

Note that $D^+ = c\bar{d}$, $D^0 = c\bar{u}$, and $D_s^0 = c\bar{s}$. If a meson is produced from, say, a $d\bar{d}$ state it could be a $\pi^0$, $\eta$, $\rho^0$, or $\omega^0$. Here it is sufficient to list only the $\pi^0$...
4. Both the $B^0_d = d\bar{b}$ and $\bar{B}^0_d = \bar{d}b$ can decay to common final states, such as $J/\psi K^0_S$ as you found in prob. 3. Hence there are transitions between $B^0$ and $\bar{B}^0$ and so the states of definite mass and lifetime are not these but

$$B_1 = \frac{B^0 + \bar{B}^0}{\sqrt{2}}, \quad \text{and} \quad B_2 = \frac{B^0 - \bar{B}^0}{\sqrt{2}}.$$  

So far this is much like the $K^0 - \bar{K}^0$ system, ignoring the possibility of $CP$ violation. (Can you readily show that $B_1$ and $B_2$ are the eigenstates of the $2 \times 2$ Hamiltonian, assuming the off-diagonal elements are equal, as is the case for time-reversal invariance ($CP$ conservation)?)

In practice the lifetimes of $B_1$ and $B_2$ are essentially identical (unlike the case for $K_1$ and $K_2$), so

$$|B_1(t)\rangle = e^{-\Gamma t/2}e^{im_1 t}|B_1(0)\rangle,$$
$$|B_2(t)\rangle = e^{-\Gamma t/2}e^{im_2 t}|B_2(0)\rangle.$$  

Deduce the probabilities $P(t)$ and $\bar{P}(t)$ of having a $B^0$ and $\bar{B}^0$ at time $t$ in terms of the initial amplitudes $|B^0(0)\rangle$ and $|\bar{B}^0(0)\rangle$ and the mass difference $\Delta m \equiv m_1 - m_2$.

Suppose at $t = 0$ we have a pure $B^0$. What is the probability that it decays as a $\bar{B}^0$ rather than as a $B^0$, in terms of the ‘mixing’ parameter $x \equiv \Delta m/\Gamma$?

For the $B^0_d$, $x_d$ has been measured to be 0.7, and it is expected that for the $B^0_s$, $x_s \approx 10$.

5. The process $\gamma e \rightarrow W \nu$ probes the triple-gauge-boson $\gamma WW$ vertex (and hence the magnetic moment of the $W$). Draw the Feynman diagram for this process, and a second possible diagram that does not involve a $\gamma WW$ vertex. Show that the second graph can be eliminated by a choice of the photon polarization.

What are the diagram(s) for the related process $\gamma e \rightarrow Z e$? Can this ‘background’ process be eliminated by use of certain photon and electron polarizations?