Princeton University—Physics 501
Problem Set 1
Due: Sept. 16, 2004

(If you get stuck, see the hints below.)

1) A disk of radius, \(a\), has a surface charge density \(\sigma\). What is the potential and electric field a distance \(x\) from the center of the disk along its symmetry axis?

2) (J 1.4). Each of three charged spheres of radius \(a\), one conducting, one having a uniform charge density within its volume, and one having a spherically symmetric charge density that varies as \(r^n\), \(n > -3\) has a total charge \(Q\). Use Gauss’s theorem to find the electric fields inside and outside each of the spheres. Sketch the fields as a function of radius for the first two spheres, and for the third sphere with \(n = 2\) and \(n = -2\).

3) (J 1.5). The time averaged potential of a neutral hydrogen atom is given by

\[
\Phi = \frac{q}{4\pi \varepsilon_0} \frac{e^{-\alpha r}}{r} \left( 1 + \frac{\alpha r}{2} \right)
\]

where \(q\) is the magnitude of the electronic charge, and \(\alpha^{-1} = a_0/2\), \(a_0\) being the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your result physically.

4) (J 1.7) Two long, cylindrical conductors of radii \(a_1\) and \(a_2\) are parallel and separated by a distance \(d\), which is large compared with either radius. Show that the capacitance per unit length is given approximately by

\[
C \simeq \pi \varepsilon_0 \left( \ln \frac{d}{a} \right)
\]

where \(a\) is the geometrical mean of the two radii. Approximately what gauge wire (state the diameter in millimeters) would be necessary to make a two-wire transmission line with a capacitance of \(1.2 \times 10^{-11}\) F/m if the separation of the wires was 0.5 cm? 1.5 cm? 5.0 cm?
5) A parallel plate particle detector consists of a gas filled volume between two planes separated by a distance $d$ and biased such that a high electric field exists between them. When a high-energy particle traverses the gap, an ionization trail is generated. Electrons from the trail are accelerated toward the positive electrode. They rapidly gain enough energy from the field to ionize other atoms. The free electrons that are created in that process are also accelerated and create new ionizations of their own and so on: an “avalanche” quickly develops. The positive ions also move, but much more slowly. For purposes of discussion, one can assume that the plates extend infinitely.

a) Find an expression for the “signal current” $I_s$, due to one free electron or ion in terms of its charge $\pm e$ and its velocity, $v$. This is the current that is observed to flow onto the plates.

b) Explain why if one integrates the signal current with respect to time over the full duration of the pulse to obtain a “signal charge,” most of that charge is due to the motion of the ions. *Hint:* Most of the ionization will be created near the positive plate. Why?

**Hints:**

J 1.5 How does $F$ behave when $r << a$? Think of an approximate model for the hydrogen atom. Which equation relates $\Phi(\vec{x})$ and $\rho(\vec{x})$?

J 1.7 $d >> a_1, a_2$ means that the surface charge on one wire is approximately independent of the other.