Finite Element Analysis of Shaped Front Endplates for the BaBar Drift Chamber

1 Introduction

This note summarizes results from a preliminary stress analysis of several shaped drift-chamber endplates using the PC-based finite-element-analysis program ALGOR.

We infer from evidence presented below that both the stresses and displacements in thin shaped endplates under uniform wire load scale as 1/thickness. A rough understanding is that the stresses are primarily tension and compression in the shaped structures (compared to primarily bending in a flat endplate). Then for a given force and maximum radius the stress varies as force/area = force/(2πrt) ∝ 1/t. (Whereas in a flat endplate the bending stresses vary as 1/t² and displacements as 1/t³.) Similar conclusions were reached in a previous study by Bénichou et al., Nucl. Instr. and Meth. 217, 163 (1983).

The material was taken as aluminum, with no downrating for holes. About 15% of the endplate volume is lost to holes, so the modulus should be downrated for at least a factor of 0.85 for aluminum. The thermoplastic carbon fiber under consideration by BaBar has a modulus about 0.7 that of aluminum, and must be downrated by another factor of 0.7 due to the holes, according to studies at McGill. The stresses are unaffected by these adjustments in the modulus, but displacements vary inversely with the modulus. Thus the displacements shown in the figures below should be scaled by

\[ \frac{5.88}{t[\text{mm}]} \text{ for aluminum and } \frac{10}{t[\text{mm}]} \text{ for carbon fiber.} \]

All endplates simulated had a 1 × 1 cm² stiffening ring at both the inner at outer radii. Further studies would be required to chose an optimal size for this important structural element.

In note TNDC-96-24 we discussed how the requirement of a factor of 10 safety against tearing of the endplate leads to a design stress limit of 24 MPa for aluminum and 16.8 MPa for thermoplastic carbon fiber. This corresponds to \(2.45 \times 10^6\) kg/m² (Al) and \(1.7 \times 10^6\) kg/m² (carbon fiber) in the units used in reporting the stresses below.
2 Toroidal-Cap Enplate

In note TNDC-96-22 we discussed a curved endplate in which the stresses could be almost purely compressive, finding a certain cubic shape for this. We have performed FEA calculations for an endplate whose section is an arc of a circle close to ‘ideal’ cubic. We call the resulting shape a toroidal-cap, shown in Fig. 1, which is also recognized as the shape of a sliced bagel. Fig. 2 shows a half section through the plate, with points plotted at the nodes of the FEA simulation.

![Illustration of the mesh used in the FEA analysis of a toroidal-cap endplate.](image)

Figures 3 and 4 show the calculated displacements and von Mises’ stress in a nominally 5-mm-thick Al endplate supported at both it inner and outer radius. Figures 5 and 6 shows results for a 2.5-mm-thick endplate. The conclusion about scaling of displacements and stress with thickness is based on comparison of the results in these four figures. We see that the scaling law is only approximate.

A 2.5-mm-thick toroidal-cap Al endplate would have the required safety factor against failure by tearing. The peak axial deflection would be only about 250 µm.
Figure 2: Radial profile of the toroidal-cap endplate. The points shown are at the nodes of the FEA simulation.

Figure 3: Radial and axial displacements of a 5-mm-thick Al toroidal-cap endplate under 3500 kg wire load.

Figure 4: Von Mises Stress in a 5-mm-thick Al toroidal-cap endplate under 3500 kg wire load.
Figure 5: Radial and axial displacements of a 2.5-mm-thick Al toroidal-cap endplate under 3500 kg wire load.

Figure 6: Von Mises Stress in a 2.5-mm-thick Al toroidal-cap endplate under 3500 kg wire load.
3 Biconical Endplate

The \textbf{BaBar} baseline design for the front endplate is a bicone, as sketched in Figs. 7 and 8. The calculated displacements and stresses for a 5-mm-thick Al bicone supported only at its outer radius are shown in Figs. 9 and 10.

![Figure 7: Illustration of the mesh used in the FEA analysis of a biconical endplate.](image)

To maintain the 10 times safety factor against tearing an Al bicone would need to be 8 mm thick and carbon fiber one would need to be 12 mm thick. The peak axial displacements would then be 1.3 mm for Al and 1.5 mm for carbon fiber.
Figure 8: Radial profile of the biconical endplate. The points shown are at the nodes of the FEA simulation.

Figure 9: Radial and axial displacements of a 5-mm-thick Al biconical endplate under 3500 kg wire load.

Figure 10: Von Mises Stress in a 5-mm-thick Al biconical endplate under 3500 kg wire load.
4 Conical Endplate

Conical endplates have been used in several large drift chambers (see Table 1 of note TNDC-96-20), and would be simpler to implement in carbon fiber than a bicone. Figures 11 and 12 show a conical endplate of the same slope as the bicone of the previous section. Figures 13 and 14 summarize the calculated displacements and stress for a cone supported only at its outer radius.

![Figure 11: Illustration of the mesh used in the FEA analysis of a conical endplate.](image)

It is noteworthy that the peak stresses in a conical plate are higher than those in a bicone of the same thickness. This shows the advantage of vaulted structures, as known to the Romans.

To maintain the 10 times safety factor against tearing an Al cone would need to be 12 mm thick and carbon fiber one would need to be 18 mm thick. The peak axial displacements would then be 1.6 mm for Al and 1.9 mm for carbon fiber.
Figure 12: Radial profile of the conical endplate. The points shown are at the nodes of the FEA simulation.

Figure 13: Radial and axial displacements of a 5-mm-thick Al conical endplate under 3500 kg wire load.

Figure 14: Von Mises Stress in a 5-mm-thick Al conical endplate under 3500 kg wire load.