Choosing Radiators for the BCD RICH Counters

These remarks follow the thinking of the notes of Tom Ypsilantis from Snowmass ’88.

**Gas Radiator**

Tom emphasizes that the ultimate limit to performance of a RICH counter is the dispersion in Čerenkov angle due to the variation of the index of refraction with photon energy (chromatic aberration). He claims that for many interesting gases

\[
\frac{\Delta \theta_C}{\theta_C} \sim 5 \times 10^{-3}.
\]

The Čerenkov angle is given by

\[
\sin^2 \theta_C = \frac{1}{\beta^2} \left( \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \right), \quad \text{so} \quad \theta_C \approx \frac{1}{\gamma_t}
\]

for large \( \gamma \). The minimum value of \( \gamma_t \) for any practical gas is 17, achieved by C\(_5\)F\(_{12}\). If we consider a RICH counter of, say, 100 cm in length, the chromatic aberration implies the Čerenkov ring has a spatial extent of

\[
\sigma_r \approx 100 \Delta \theta_C < 100 \frac{5 \times 10^{-3}}{17} \approx .03 \text{ cm}.
\]

This is likely smaller than the spatial resolution of an affordable readout of the RICH counters. Hence it is useful to display the limits of RICH-counter performance set by detector resolution (rather than by chromatic aberration as Ypsilantis does).

For a counter whose light is focused by a mirror of focal \( L \) (usually just the length of the counter itself), the radius of the Čerenkov ring of a particle of mass \( M \) is

\[
r = L \tan \theta_C \approx L \frac{1 - \frac{\gamma^2}{\gamma_t^2}}{\gamma_t} \approx \frac{L}{\gamma_t} - \frac{L\gamma_t^2}{2\gamma^2}.
\]

Noting that \( 1/\gamma = M/P \), we have

\[
r_\pi - r_K = L\gamma_t \frac{M_K^2 - M_\pi^2}{2P^2}.
\]
If the position of each photoelectron is measured to accuracy $\sigma_r$, and $N$ photoelectrons are observed, then the number of standard deviations $S$ between the Čerenkov rings for pions and Kaons of momentum $P$ is

$$S = \frac{r_\pi - r_K}{\sqrt{N\sigma_r}}.$$ 

The number of photoelectrons $N$ is given by the expression

$$N = \frac{\epsilon \alpha}{h c} \sin^2 \theta_C L dE \approx 370[\text{cm}^{-1}\text{eV}^{-1}] \frac{\epsilon L dE}{\gamma_t^2},$$

where $\epsilon$ is the efficiency for collecting the light and converting it to electrons, and $dE$ is the range of photon energies collected. With $\epsilon \sim 0.3$ and $dE \sim 1$ eV we have

$$N \approx \frac{N_0 L}{\gamma_t^2},$$

where $N_0 \sim 100/\text{cm}$. Using this last relation for $N$, the momentum at which pions and Kaons can be separated to $S$ standard deviations is

$$P[\text{GeV}/c] \approx M_K \frac{N_0^{1/4} L^{3/4}}{\sqrt{2S\sigma_r}} \approx 2(L[\text{cm}])^{3/4},$$

for $N_0 = 100/\text{cm}$, $S = 3$ standard deviations, and $\sigma_r = 1$ mm. Thus if we wish to build a RICH counter of a fixed length $L$, the same $\pi$-$K$ separation can, in principle, be obtained with any gas! It seems wise to have a large number $N$ of photoelectrons, so we should pick the gas with the minimum $\gamma_t$ such that pions produce detectable signals over the entire relevant momentum range.

However, if we wish to design the counter to provide a given number of photoelectrons, we can replace $L$ by its dependence of $N$ in the above to find

$$P \approx M_K \frac{\gamma_t^{3/2} N^{3/4}}{\sqrt{2S\sigma_r}}.$$

In this case it is advantageous to use a gas with large $\gamma_t$, but the needed length $L$ to generate the $N$ photoelectrons may become quite large.

From the above, we see that a counter of $L = 100$ cm could perform $\pi$-$K$ separation (at the $3\sigma$ level) up to $P \approx 63$ GeV/c, which is well matched to our requirements in the Intermediate region. With $L = 400$ cm we have separation up to $P = 180$ GeV/c, which is perhaps a bit low for the Forward region. With
$L = 600$ cm, we would have separation up to $P = 240$ Gev/$c$, a better match. Hence I concur with Jorge that we should lengthen the Forward RICH to 6 m. If we desire $N = 25$ photoelectrons then we need a gas with $\gamma_t = \sqrt{N_0L/N} = 2\sqrt{L}$. Hence in the Intermediate region we need $\gamma_t = 20$, so we could use C$_5$F$_{12}$ or C$_4$F$_{10}$. In the Forward region we need $\gamma_t = 49$, and we could use argon, nitrogen, or CF$_4$. The latter has 1/3 the chromatic aberration of the first two, as well as a lower $\gamma_t$, so is probably the best choice.

**Solid Radiator**

In the Central region of the BCD the minimum pion momentum of interest is too small to be detected by a gas RICH counter. Suppose instead we use a solid (or liquid, or aerogel) radiator of thickness $T$ and index of refraction $n$, followed by a drift of distance $L$.

We consider the use of ‘proximity focusing,’ i.e., no focusing at all. Then the Čerenkov ring has radius

$$r = L \tan \theta,$$

where

$$\sin \theta = n \sin \theta_C$$

for normal incidence, according to Snell’s law. If $n \sin \theta_C > 1$ then the light will not come out of the radiator!

We consider media with index $n$ significantly larger than 1, and particles with $\gamma \gg 1$. In this regime

$$\cos \theta_C = \frac{1}{n\beta} \to \frac{1}{n},$$

$$\sin \theta_C = \sqrt{1 - \left(\frac{1}{n\beta}\right)^2} \to \frac{\sqrt{n^2 - 1}}{n},$$

$$\tan \theta_C = \sqrt{n^2\beta^2 - 1} \to \sqrt{n^2 - 1}.$$  

From this we deduce that total internal reflection will occur inside the radiator if $\sqrt{n^2 - 1} > 1$, or $n > \sqrt{2}$.

A UV-transparent solid with a low index is NaF ($n = 1.38$ at 6.5 eV, R. Arnold *et al.*, N.I.M. A273, 466 (1988). The chromatic aberration of NaF is fairly large, and Ypsilantis concludes that it can’t do $\pi$-K separation much above 3 GeV/$c$. A UV-transparent liquid is C$_6$F$_{14}$ ($n = 1.278$). Its chromatic aberration is somewhat better, and it is used by SLD and Delphi.
Again we estimate the significance of $\pi$-$K$ separation,

$$S = \frac{r_\pi - r_K}{\sigma_r \sqrt{N}},$$

supposing that spatial resolution is more important than chromatic aberration. For a proximity ‘focussed’ device, the effective spatial resolution is set by the finite width of the unfocussed Čerenkov ring.

The full radial extent of the Čerenkov ring is $T \tan \theta_C$ (not $T \tan \theta$), so the r.m.s. spread in the radii of the photoelectrons is

$$\sigma_r = \frac{T \tan \theta_C}{\sqrt{12}}.$$

The number of photoelectrons observed is

$$N = N_0 T \sin^2 \theta_C.$$

We need to expand $\tan \theta$ to first order using $\beta^2 = P^2/E^2 \approx 1 - M^2/P^2$. Some steps are

$$\tan \theta = \frac{n \sin \theta_C}{\sqrt{1 - n^2 \sin^2 \theta_C}} = \sqrt{\frac{n^2 \beta^2 - 1}{1 - \beta^2(n^2 - 1)}} \approx \sqrt{\frac{n^2 - 1}{2 - n^2}} \left(1 - \frac{M^2}{2P^2} \frac{1}{(n^2 - 1)(2 - n^2)}\right).$$

The difference in radii of the Čerenkov rings from pions and Kaons of the same momentum is

$$r_\pi - r_K \approx L \frac{M_K^2 - M_\pi^2}{2P^2} \frac{1}{\sqrt{2 - n^2}} \frac{1}{(n^2 - 1)(2 - n^2)^{3/2}}.$$ 

The number of standard deviations $S$ to which this separation is determined by the observation of $N$ photoelectrons is

$$S = \frac{r_\pi - r_K}{\sigma_r / \sqrt{N}} \approx \frac{\sqrt{12}N_0 L}{2n^2(2 - n^2)^{3/2} \sqrt{N}} \frac{M_K^2 - M_\pi^2}{P^2},$$

where we have noted that the thickness $T$ of the radiator can be written

$$T = \frac{N}{N_0 \sin^2 \theta_C} \approx \frac{N}{N_0 n^2} \frac{n^2}{n^2 - 1}.$$ 

The momentum at which an $S$-$\sigma$ separation of pions and Kaons can be made is then

$$P[\text{GeV}/c] \approx M_K \sqrt{\frac{\sqrt{12}N_0 L}{2Sn^2(2 - n^2)^{3/4} \sqrt{N}}} \approx \sqrt{\frac{1.5L[\text{cm}]}{n^2(2 - n^2)^{3/2}}}.$$
for a 3-σ separation, $N = 25$ photoelectrons, and $N_0 \approx 50$/cm as reported by Delphi in their liquid RICH.

For $C_6F_{14}$ with $n = 1.27$, for which the effective Čerenkov angle is $52^\circ$, and we allocate $L = 20$ cm, then we have good $\pi$-$K$ separation up to $P = 9$ GeV$/c$. This would be quite adequate for the BCD. The radiator thickness should be 1 cm to provide the 25 photoelectrons. The Čerenkov intensity has reached 80% of maximum for 250 MeV$/c$ pions, and the r.m.s. width of the Čerenkov rings would be 3 mm.

For comparison, we make a similar derivation for the effect of chromatic aberration. From $\cos \theta_C = 1/n$ we find

$$d\theta_C = \frac{dn}{n^2 \sin \theta_C} = \frac{dn}{n\sqrt{n^2 - 1}}.$$  

The index for $C_6F_{14}$ varies as

$$n = 1.2733 + 9.3 \times 10^{-3}(E[eV] - 6),$$

and the useful photon-energy interval is 1 eV, so $dn \approx 10^{-2}$ and $d\theta_C \approx 0.01$. Then $\sigma_{\theta_C} = d\theta/\sqrt{12} \approx 0.003$. This leads to an uncertainty in the radius $r = L\tan \theta$ of the Čerenkov ring given by

$$\sigma_r = \frac{L\sigma_\theta}{\cos^2 \theta} = \frac{nL\cos \theta_C \sigma_{\theta_C}}{\cos^3 \theta} = \frac{L\sigma_{\theta_C}}{(2 - n^2)^{3/2}} \approx 0.012L$$

for $C_6F_{14}$. We can use the expression for $\sigma_r$ from chromatic aberration to deduce the maximum momentum for $\pi$-$K$ separation:

$$P = M_K \sqrt{\frac{\sqrt{N}}{2S\sqrt{n^2 - 1}\sigma_{\theta_C}}} \approx 9.5 \text{ GeV}/c,$$

for $S = 3$ standard deviations. This is very comparable to the limit due to the use of proximity focussing for $L = 25$ cm. Probably we should use $L = 50$ cm, in which case we could hope for $\pi$-$K$ separation up to 9 GeV$/c$.

Comments added 8/15/90:

The above argument is for normally incident particles. When the angle of incidence $i$ is big enough, not all the Čerenkov light will come out of the radiator, some being trapped by internal reflection. This starts happening when

$$\sin(i + C) = \frac{1}{n},$$
writing $C$ for the Čerenkov angle. This can be rewritten as

$$i = \sin^{-1} \frac{1}{n} - \cos^{-1} \frac{1}{n}.$$

For NaF with $n = 1.38$ we have $i = 2.8^\circ$, and for $C_6F_{14}$ with $n = 1.27$ we have $i = 14^\circ$.

Perhaps of greater interest is the angle of incidence $i$ at which only one half of the Čerenkov cone emerges. After some geometry we find this condition is

$$i = \sin^{-1} \sqrt{2 - n^2}.$$

For NaF this implies $i = 18^\circ$ and for $C_6F_{14}$ we have $i = 38^\circ$. Note that when $i > 0$ the path length in the radiator is longer, so additional Čerenkov light is emitted, according to $1/\cos i$. This doesn’t help NaF much, but is significant for $C_6F_{14}$. The geometry is complicated, and I don’t have an simple analytic expression – but it can readily be calculated on a computer.

Similarly, the expression for the significance of $\pi/K$ separation is actually a function of the angle of incidence, and should be calculated in the future.