Errors & Dilutions

in Measurements of CP Violation in B-\bar{B} System

K. T. McDonald

Princeton U.

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We propose to measure a CP-violating asymmetry:

\[
A = \frac{\Gamma(B \rightarrow f) - \Gamma(\bar{B} \rightarrow \bar{f})}{\Gamma(B \rightarrow f) + \Gamma(\bar{B} \rightarrow \bar{f})}
\]

\[N = \text{Total number of } B \rightarrow f \neq \bar{B} \rightarrow \bar{f} \text{ decays}
\]

\[S = \frac{A}{\sigma_A} \text{ = Statistical significance in standard deviations}
\]

\[
S = \sqrt{N} \frac{A}{\sqrt{1 - A^2}}
\]

Extra statistical power for \(A \approx 1\)

For small to moderate \(A\), approximate:

\[S \approx \sqrt{N} A
\]

or \[N \approx \left(\frac{S}{A}\right)^2
\]

Example: \(A = 0.1\)

\[S = 3 \leq
\]

\[\Rightarrow N \times 900 \text{ events required.}
\]
In practice (at least) 4 effects 'dilute' the statistical power:

1. If \( B = B^0 \), mixing oscillations reduce the asymmetry.

2. If \( B \rightarrow f \) where \( f \) is a CP eigenstate \((\bar{f} \rightarrow \bar{f})\) must tag the particle-antiparticle character of the \( B \) by observation of the second \( B \) in the event.

3. If the second \( B \) is a \( B^0 \), its oscillations dilute the tag.

4. The signal of \( B \rightarrow f \) may be subject to a background that is CP invariant.

Effect of Mixing of the First \( B \)

Pure \( B^0 \) at \( t = 0 \):

\[
\begin{align*}
|B^0(t)\rangle &= e^{\frac{i M t}{2}} e^{\frac{\Delta M t}{2}} \\
|\bar{B}^0(t)\rangle &= i e^{-\frac{i M t}{2}} e^{-\frac{\Delta M t}{2}}
\end{align*}
\]

Etc ...

Assuming we know the particle-antiparticle character of the \( B \) at time \( t \) when it decays:

Write

\[
A(t) = \frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(B^0(t) \rightarrow \bar{f})}{\Gamma(B^0(t) \rightarrow f) + \Gamma(B^0(t) \rightarrow \bar{f})} = \sin^2 \theta \quad \sin \chi \int 2 \phi
\]

\( \chi \) = angle of unitarity triangle when \( f \) = CP eigenstate

\( \sin \chi = \frac{\Delta M}{M} = \text{mixing parameter} \); \( \sin \chi \sim 0.7 \) (measured)

\( \sin \chi \sim 5-10 \) (estimated)

CP-violating interference between mixing and decay vanishes at \( t = 0 \).
A. Small $x$ ($b^0_k$): $B^-$ decay before completing one oscillation.

- Time-resolved experiment essentially the same as time integrated:

$$A \approx \frac{\int \frac{\Gamma(B \to f)}{\Gamma(B \to f)} - \int \frac{\Gamma(B \to f)}{\Gamma(B \to f)}}{\int + \int} = \frac{k}{1 + k^2} \text{ Dilution factor } D_1$$

Small $x$ $\Rightarrow$ No asymmetry

Little point in detailed time studies.

B. Larger $x$ ($B^0_k$): Several oscillations before decay.

- Time-resolved asymmetry essentially averages over one half-cycle at a time (reversing the form of the asymmetry each half cycle).

$$A \to \frac{2}{\pi} \text{ Dilution factor } D_1$$

$\Rightarrow$ Must know which half-cycle decay occurred in.

$\Rightarrow$ Must know $N$. But $N$ cannot be measured by decays $B^0_k \to f_{CP}$. Unless there is CP violation. In particular, we don't expect CP violation in $B^0 \to J/\psi \phi$. 

C. General case: $A = D_1(x) \approx 2k$
Effect of Oscillations of the Second $B$

Observe particle-antiparticle character of the second $B$ to infer particle-antiparticle character of the first $B$ ($B \to f_{CP}$)

If second $B$ is a $B^0$, it may have oscillated to $\bar{B}^0$ before decaying.

Integrated probability that a $B^0$ at $t=0$ decays as $\bar{B}^0$ is

$$p = \frac{\lambda^2}{2(1+\lambda^2)}$$

The useful number of tags is

$$N_{(B^0 \to B^0)} - N_{(B^0 \to \bar{B}^0)} = N(1-2p) = N \frac{1}{\frac{1+\lambda^2}{\text{Dilution Factor}} D_2}$$

Second $B$ is $B^+$: $D_2 = 1$

$B_{d}^0$: $D_2 = \frac{1}{1+\lambda^2} \approx \frac{1}{3}$

$B_{s}^0$: $D_2 = \frac{1}{1+\lambda^2} \approx 0$ (useless as tag)

Proportions of second $B$:

$D^+_s : D^+_d : D^0_s = \epsilon : \epsilon : 1 - 2\epsilon$

With $\epsilon \sim \frac{1}{3}$ (to be measured!)

Effective Dilution Factor:

$$D_2 = \sqrt{\epsilon} \text{ for } B^+, \text{ as have only } \epsilon \text{ as most events as for all } B's$$

$$D_2 = 0.1 + \frac{1}{1+\lambda^2} + \frac{1-2\epsilon}{1+\lambda^2} \approx \frac{5\epsilon}{3} \text{ for all } B's$$

$$\frac{D_2(\text{all } B^0s)}{D_2(\text{all } B^+)} = \frac{5\sqrt{\epsilon}}{\frac{5\epsilon}{3}} \approx 1$$

$\Rightarrow$ No advantage to tagging only with $B^\pm$. 

\[ \]
Footnote on tagging via \( B^+ \) vs. all \( B^- \)

Let \( N_0 = \# \) of reconstructed \( B^+ \to f_{\text{CP}} \) required for some measurement, not yet taking into account tagging via the second \( B \).

\[ N_1 = \# \text{ of } B^+ \to f_{\text{CP}} \text{ required so that } N_0 \text{ of them will be tagged via a second } B^+ \]

\[ N_2 = \# \text{ required if tag via any second } B^- \]

\[ E = \frac{\frac{N_2}{\text{ALL } B^-}}{\text{ALL } B^-} \leq \frac{3}{8} \]

\[ N_1 = \frac{N_0}{E (D_2)} \]

Since dilution factor \( D_2 \):

\[ \text{for } B^+ \]

\[ N_2 = \frac{N_0}{(D_2)} \cdot \frac{9}{(5E)^2} \cdot \frac{N_1}{25} = \frac{24}{25} N_1 \]

There is no statistical advantage to use of a tag on \( B^+ \) only!

[We have ignored possible differences in detection efficiency and mistagging probability for tagging via different \( B^- \)s, but expect these differences to be small.]

### (3) Mistagging of Second \( B \)

Suppose only partial reconstruction of second \( B \):

\[ b \to c \to s \]

\( \ell \text{ observe } K^+ \text{ or } K^- \)

\[ b \to c \ell \nu \]

\( \ell \text{ observe } \ell^+ \text{ or } \ell^- \)

\( P = \text{probability of assigning wrong sign} \)

Effective fraction of correct tags is \( P_{\text{right}} - P_{\text{wrong}} \)

\[ \Rightarrow D_3 = 1 - 2P \]

### (4) Non Resonant Background

The first \( B \) is fully reconstructed, but may have background in the mass plot.

\[ b = \frac{\text{background}}{\text{signal}} \]

In general, there will be no CP violation in the background events, so

\[ A \to \frac{1}{1 + b} \]

\[ \text{dilution factor } D_4 \]

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**Summary of Dilution Factors**

\[ A_{\text{obs}} = D_1 D_2 D_3 D_4 \Delta \zeta \Delta \eta \]

Need \( N \approx \left( \frac{S}{A_{\text{obs}}} \right)^2 \) Tagged Events to Measure

\[ \Delta \zeta \Delta \eta \text{ to } S \text{ standard deviations} \]

\[ N \approx \left( \frac{1}{D_1 D_2 D_3 D_4} \right)^2 \left( \frac{S}{\Delta \zeta \Delta \eta} \right)^2 \]

**Example:** For \( B_d \)

\[ D_1 = \frac{k}{1 + k^2} = \frac{1}{2} \]

\[ D_2 = \frac{S}{3} \cdot \frac{S}{3} \cdot \frac{3}{8} = \frac{5}{8} \text{ (tag on all } B') \]

\[ D_3 = 1 - 2P \approx 0.1 \text{ at hadron collider} \]

\[ D_4 = \frac{1}{1 + b} \approx 1 \text{ for clean mode (3/4 K_\text{s})} \]

\[ D_1 D_2 D_3 D_4 \approx \frac{1}{4} \]

\[ N \approx 16 \left( \frac{S}{\Delta \zeta \Delta \eta} \right)^2 \]

\[ N \text{ for } S = 3, \Delta \zeta \Delta \eta = 0.1 \text{ need } N \approx 14,400 \]

\[ N^2 \left[ \frac{1}{1 + k^2 (1 - 2P)} \right]^2 \left( \frac{S}{\Delta \zeta \Delta \eta} \right)^2 \approx 6 \left( \frac{S}{\Delta \zeta \Delta \eta} \right)^2 \]

**Tagging via Leptons and Kaons**

What fraction of \( b \to f_{CP} \) decays can be correctly tagged?

\[ b \to c \to s \text{ or } c \text{ or } k \]

\[ b \to c \to l^+ \text{ or } \mu \]

What-Side \( K, \pi, \text{ or } \mu \) occur via other stages of decay cascade.

**ISAJET Study:** Tag via highest \( P_T \) \( k^+, l^+, \text{ or } \mu^+ \) in rest of event.

No secondary vertex requirement on tagging particle.

(Favored if correct tagging probability is larger than secondary vertex efficiency.)

**K's Best Options:**
Figure 4: The fraction of leptons (or Kaons) that have the wrong sign as a function of $P_T$.

Figure 5: Differential and integral tagging efficiencies of four types of tags as a function of transverse momentum. Left-hand plots: the number $N |1 - 2p|$ of useful tagged events; right-hand plots: the total efficiency of the tag as a function of the minimum-transverse-momentum requirement. The four tags are, from top to bottom, electron, muon, combined electron and Kaon, and Kaon.
Alternative Analyses of CP-Violating Asymmetries

In Sec. 6.2 of Ref. [1] we noted that our proposed method of analysis of CP violation in the neutral $B$ system would yield a null result if we integrate over time and if the $B \bar{B}$ pair was produced in a $C_{\text{odd}}$ state. As the latter condition holds for $B$'s produced at the $B(4S)$ resonance at an $e^+e^-$ collider, this analysis would be inappropriate there. A clever alternative procedure has been proposed[9] that maximizes the analyzing power at an $e^+e^-$ collider. Here we examine whether this procedure would be effective at a hadron collider.

Both $B^+$'s of a produced $B\bar{B}$ pair must be observed in a $C\bar{P}$ analysis. We label $B_0$ as the (neutral) $B$ that decays to the $C\bar{P}$ eigenstate $f$, and $B_1$ as the (charged or neutral) $\bar{B}$ that decays to a state $g \neq f$ that permits us to determine whether $B_1$ was a particle or antiparticle at the moment of its decay. We can accumulate four time distributions, where one $B$ decays at time $t$, and the other at time $t + t_f$ with $t < t_f$:

\[
I : \Gamma_{B_0 \rightarrow f(t)} \Gamma_{B_1 \rightarrow g(t)} \\
II : \Gamma_{B_0 \rightarrow f(t)} \Gamma_{B_1 \rightarrow g(t + t_f)} \\
III : \Gamma_{B_0 \rightarrow f(t)} \Gamma_{B_1 \rightarrow g(t - t_f)} \\
IV : \Gamma_{B_0 \rightarrow f(t)} \Gamma_{B_1 \rightarrow g(t + t_f)}
\]

The four distributions can be combined to form asymmetries in various ways:

\[A_1(t, t_f) \equiv \frac{I + IV - I - II}{I + IV + I + IV}.
\]

Another asymmetry is

\[A_2(t, t_f) \equiv \frac{I + III - I - IV}{I + III + I + IV},
\]

as considered in Ref. [2]. A third might be defined as

\[A_3(t, t_f) \equiv \frac{I + III - I - IV}{I + III + I + IV}.
\]

For the case that mesons 1 and 2 are of the same type the four time distributions take the form

\[
\Gamma_f(t, t_f) \propto e^{-i(t + t_f)}[1 \pm \sin 2\psi \sin z(t_0 + t)],
\]

\[
\Gamma_{II}(t, t_f) \propto e^{-i(t + t_f)}[1 \pm \sin 2\psi \sin z(t_0 + t_f)],
\]

\[
\Gamma_{III}(t, t_f) \propto e^{-i(t + t_f)}[1 \pm \sin 2\psi \sin z(t_0 + t_f)],
\]

\[
\Gamma_{IV}(t, t_f) \propto e^{-i(t + t_f)}[1 - \sin 2\psi \sin z(t_0 + t_f)],
\]

where $\psi$ is the CP-violating phase in the decay amplitude for $B_1 \rightarrow f$. $z = \Delta M / t$ is the mixing parameter for neutral $B$-meson, and the lower sign in the distributions holds for $C_{\text{odd}}$ states $|B_1\rangle |B_2\rangle - |B_1\rangle |B_2\rangle$. In the above, time is measured in units of the lifetime $1/t$.

Inserting the time distributions into the forms for the asymmetries we have

\[A_1 = \begin{cases} 0 & C_{\text{odd}} \\ \sin 2\psi \sin z(t_0 + t_f) & C_{\text{even}} \end{cases},
\]

\[A_2 = \begin{cases} \sin 2\psi \sin z(t_0 - t_f) & C_{\text{odd}} \\ 0 & C_{\text{even}} \end{cases},
\]

\[A_3 = 0.
\]

Clearly the asymmetry $A_2$ will be useful at an $e^+e^-$ collider where only $C_{\text{odd}}$ states are produced.

As we have noted elsewhere, in $B_d$ decays where $x_d \approx 0.7$ there are about nine lifetimes per oscillation, and so a time-resolved analysis is actually little different than a time-integrated one. Hence it is relevant to consider the time-integrated forms of the asymmetries.

Because of the time ordering in the definition of the distributions $I$-$IV$, the form of the integrals is

\[
\int_0^\infty dt \int_0^\infty dt_f \Gamma_f(t, t_f),
\]

etc. On evaluating these integrals for the case that meson $B_1$ is of the same type as $B_2$, we find

\[A_1 = \begin{cases} 0 & C_{\text{odd}} \\ 2z \sin 2\psi / (1 + x^2) & C_{\text{even}} \end{cases},
\]

while

\[A_2 = \begin{cases} z \sin 2\psi / (1 + x^2) & C_{\text{odd}} \\ 0 & C_{\text{even}} \end{cases}.
\]

At a hadron collider the $B\bar{B}$ pairs are produced in $C_{\text{even}}$ and $C_{\text{odd}}$ states with equal probability, so the question arises as to which asymmetry is to be preferred to attain maximum sensitivity to the CP-violating factor $sin 2\psi$. Note that the nonzero cases of the asymmetries are affected by the dilution due to mixing in different ways:

\[A_1(C_{\text{even}}) = \frac{2}{1 + x^2} A_3(C_{\text{odd}}).
\]

For the case of $B_d \bar{B}_d$ production where $x_d \approx 0.7$, the factor $2 / (1 + x^2) \approx 4 / 3$, so asymmetry $A_1$ is slightly to be preferred over $A_3$.

However, at a hadron collider a $B_d$ meson can be produced along with any of a $\bar{B}_d$, $\bar{B}_s$, or $B_s$ Table 1 lists the coefficients $K$ of $sin 2\psi$ for the various possibilities of $B\bar{B}$ production for the two asymmetries. On weighting by the relative production rates we estimate that $A_1$ is about 1.5 times as large as $A_2$ at a hadron collider, so clearly should be used.
Table 1: The coefficient $K$ in time-integrated $CP$-violating asymmetries of the form $A = K \sin 2\phi$ for various possibilities for $B_d \bar{B}_d$ production at a hadron collider. The weighted coefficient is obtained supposing $\pi_d = 0.7, \pi_u \gg \pi_d$, and that $B_d, B_u$, and $B_s$ mesons are produced along with a $B_d$ in the proportion $0.375 : 0.375 : 0.25$. We have assumed that the lifetimes of all three flavors of $B$ mesons are the same.

<table>
<thead>
<tr>
<th>Asymmetry</th>
<th>$B_d \bar{B}_d$</th>
<th>$B_u \bar{B}_u$</th>
<th>$B_s \bar{B}_s$</th>
<th>Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$\frac{\pi_d}{1+\pi_d}$</td>
<td>$\frac{\pi_u}{1+\pi_u}$</td>
<td>$\frac{\pi_s}{1+\pi_s}$</td>
<td>$0.25$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$\frac{\pi_u}{1+\pi_u}$</td>
<td>$\frac{\pi_u}{1+\pi_u}$</td>
<td>$\frac{\pi_s}{1+\pi_s}$</td>
<td>$0.16$</td>
</tr>
</tbody>
</table>

In Ref. [1] we considered the asymmetry

$$A(t_4, t_6) = \frac{\Gamma_{B_d \rightarrow \ell \ell}(t_4) \Gamma_{B_{u} \rightarrow \ell \ell}(t_6)}{\Gamma_{B_d \rightarrow \ell \ell}(t_4) + \Gamma_{B_{u} \rightarrow \ell \ell}(t_6)} = \sin 2\phi \sin (\tau_4 t_4 \pm \epsilon_4 t_4),$$

where there was no restriction on $t_4$ and $t_6$, and the minus sign holds for $C(\text{odd})$ states. This asymmetry is not quite the same as $A_1$ or $A_2$, but the time integrated version of this is identical to the time integrated version of $A_1$. That is, in the time integrated version of $A_1$, we effectively lose sight of the time ordering of $t_4$ and $t_4$.

For a final comparison, the coefficient $K$ that holds for use of asymmetry $A_2$ at an $e^+e^-$ collider is 0.5. This means that the average dilution due to mixing at an $e^+e^-$ collider is one half of that at a hadron collider. Equivalently, we will need four times as many tagged reconstructed $B_d \bar{B}$ decays at a hadron collider as at an $e^+e^-$ collider to achieve the same sensitivity to $\sin 2\phi$. Stated yet another way, the smallest value of $\sin 2\phi$ that can be resolved to three standard deviations with $N$ events at a hadron collider is $12/\sqrt{N}$, while at an $e^+e^-$ collider this would be $6/\sqrt{N}$.

1 References

