Comparison of Muon-Pair Production to the Quark-Antiquark Annihilation Model


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New data on muon-pair production at 225 GeV/c by \( \pi^- \), \( \pi^+ \), and proton beams are analyzed with regard to the production mechanism. The observed spin alignment of the pair and the dependence of the cross section on beam-particle type are strong indications that the production is through electromagnetic quark-antiquark annihilation.

The preceding Letter\(^1\) discussed the general features of production of high-mass \( \mu \) pairs by 225-GeV/c beams of \( \pi^- \), \( \pi^+ \), and protons on C, Cu, and W targets. In this article we discuss aspects of the data in the context of a theoretical framework.

The general formalism of lepton-pair production by hadrons via a virtual intermediate photon has been discussed in the literature.\(^2\) An important special case is the description in terms of quark-antiquark annihilation, first proposed by Drell and Yan and further developed by other authors.\(^3\) The \( \mu^- \)-pair production cross section for interacting hadrons \( A \) and \( B \), neglecting \( p_T \) and including three colors is

\[
\frac{d^2 \sigma}{d M d x_F} = \frac{8 \pi \alpha^2}{9 M^5} \sum_i \sum_{x_1} \sum_{x_2} \frac{e_i^2}{x_1 + x_2} [x_i f_i^A(x_1) x_2 f_i^B(x_2) + x_i f_i^A(x_2) x_2 f_i^B(x_1)],
\]

where \( x_1, x_2 = [\pm x_T + (x_T^2 + 4M^2/s)^{1/2}] / 2 \) are the momentum fractions (Feynman \( x \)) of the annihilating quarks in the projectile and target hadrons, \( x f_i^A(x) \) is the momentum spectrum for quarks (antiquarks) of flavor \( i \) in hadron \( A \), \( x_T = 2P_{T^*}/s \) in this analysis, where \( P_{T^*} \) is the \( \mu^- \)-pair longitudinal momentum in the overall center of mass. Thus, the observed mass and Feynman-\( x \) \( x_T \) spectra for the data reflect the distributions of the annihilating quarks.

This mechanism predicts striking differences for the lepton-pair production cross section in nucleon-nucleon scattering compared to pion-nucleon scattering. In the nucleon-nucleon case the interacting particles contain no valence antiquarks and only antiquarks from the \( q\bar{q} \) sea can contribute. Since the probability density functions for the sea quarks fall steeply with \( x_i \) and \( x_2 \), the observed mass spectrum should fall rapidly with mass for fixed \( x_T \). Incident pions, on the other hand, contain a valence antiquark which has a significant probability of being found at large \( x_T \). Thus, the cross section for pion-induced pairs should fall more slowly with mass than for incident protons.

To permit comparison of \( \pi^- \)- and proton-induced cross sections at the highest possible mass values, we have used the proton results of Yoh et al.\(^4\) The cross-section ratio is shown in Fig. 1. The \( \pi^- \) to proton cross-section ratio rises to over 100 at a mass of 10 GeV/c\(^2\) in dramatic agreement with expectations.

Consider next the comparison of the \( \pi^- \)- and \( \pi^- \)-induced \( \mu^- \)-pair production cross sections. In this case, valence quarks and antiquarks from

![Graph showing the ratio of \( \pi^- \)-induced to proton-induced \( \mu^- \)-pair production cross section as a function of mass. Proton data at 225 GeV/c has been calculated from the scaling observed in 200-, 300-, and 400-GeV/c data of Ref. 4.](image)

FIG. 1. The ratio of \( \pi^- \)-induced to proton-induced \( \mu^- \)-pair cross section at \( y_{e.m.} = 0.2 \) as a function of mass. Proton data at 225 GeV/c has been calculated from the scaling observed in 200-, 300-, and 400-GeV/c data of Ref. 4.

\( ^{\text{1}} \)Yoh et al.,
\( ^{\text{2}} \)Drell and Yan,
\( ^{\text{3}} \)Drell and Yan,
\( ^{\text{4}} \)Yoh et al.
the interacting particles should dominate production if \( x_1 \) and \( x_2 \) are both large. In this experiment, a carbon target was used so that the target is exactly symmetric in \( u^- \) and \( d^- \) quark distributions \( (u^d = d^u, d^u = u^d) \). In the case of an incident \( \pi^- \) the valence antiquark is a \( \bar{u} \) with charge \( -\frac{2}{3} \) while for an incident \( \pi^+ \) it is a \( \bar{u} \) with charge \( \frac{1}{3} \). Since the pair production varies as the square of the quark charge, one expects \( \sigma(\pi^+ C - \mu^+ \mu^- \ldots)/\sigma(\pi^- C - \mu^- \mu^+ \ldots) \) to be \( \frac{1}{4} \). If \( x_1 \) and \( x_2 \) are not both large, sea antiquarks can contribute and the cross-section ratio approaches 1 as \( x_1 \) and \( x_2 \) approach 0. Figure 2(a) shows the measured \( \pi^+ / \pi^- \) cross-section ratio as a function of pair mass for \( x_T > 0 \). It is seen to be near unity for the \( J/\psi \) as might be expected for strong production from charge-symmetric initial states. As the pair mass increases above 3.1 GeV/c\(^2\), the ratio is consistent with a fall toward \( \frac{1}{4} \). Any deviation of the ratio from unity is indicative of an electromagnetic process and the limiting value of \( \frac{1}{4} \) is predicted by the \( q\bar{q} \) annihilation mechanism.

The solid line in Fig. 2(a) is a calculation of the \( \pi^+ / \pi^- \) cross-section ratio from the pion and nucleon structure functions together with the contribution of the observed resonances. The determination of the pion structure function is discussed in the following Letter.\(^5\) Since \( \pi^+ \) and \( \pi^- \) structure functions are the same, the approach to \( \frac{1}{4} \) is primarily dependent on the nucleon structure functions and in particular on the form of the nucleon sea-quark distributions. Using the measured sea-quark distributions of Kaplan et al.,\(^6\) we find reasonable agreement with the data although these sea-quark distributions have been determined only for \( x_2 > 0.25 \).

Figure 2(b) shows the \( \pi^+ / \pi^- \) ratio from this experiment as a function of \( M/\sqrt{s} \), together with other measurements.

Two variables, in addition to \( M \), \( x_T \), and \( p_T \), are required to specify the \( \mu^- \) pair final state. Natural choices are the polar \( (\theta^*) \) and azimuthal angles of one of the muons in the \( \mu^- \) pair rest frame. The annihilation of two massless spin-\( \frac{1}{2} \) on-shell fermions to a \( 1^- \) intermediate state would lead to a \( 1 + \cos^2 \theta^* \) distribution, where \( \theta^* \) is measured from the \( q\bar{q} \) direction in the \( \mu^- \) pair c.m. system. If the \( p_T \) of the final-state \( \mu^- \) pair is zero, the beam and target define the \( q\bar{q} \) direction. If \( p_T \) is nonzero, the \( q\bar{q} \) direction is not determined. Collins and Soper\(^7\) have suggested using the vector which bisects the angle between the beam and the reverse of the target vector as the best average estimate of the \( q\bar{q} \) direction.

The distribution of the polar (helicity) angle has been examined for mass regions below the \( J/\psi (2.0 < M < 2.7 \text{ GeV/c}^2) \), \( J/\psi (2.7 < M < 3.5 \text{ GeV/c}^2) \), and above the \( J/\psi (M > 3.5 \text{ GeV/c}) \). The distributions are shown in Fig. 3 with their best fits to the form \( 1 + \lambda \cos^2 \theta^* \). Results of the fits are given in Table I. The continuum region above

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**FIG. 2.** (a) \( R = \sigma(\pi^+ C - \mu^- \mu^+ X)/\sigma(\pi^- C - \mu^+ \mu^- X) \) vs \( M_{\mu\mu} \)

<table>
<thead>
<tr>
<th>225 GeV/c</th>
<th>40 GeV/c</th>
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<tbody>
<tr>
<td>0</td>
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</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td>4</td>
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</tr>
<tr>
<td>6</td>
<td>0.1</td>
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<tr>
<td>10</td>
<td>0.05</td>
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(b) \( R \) vs \( M/\sqrt{s} \) for data at 225 and 40 GeV/c (Cu target) for continuum pairs. The curve is the same as shown in (a) but with resonance production excluded.

**FIG. 3.** Helicity angular distributions in three different mass intervals. The \( M > 3.5 \text{ GeV/c}^2 \) interval is also shown divided in two \( p_T \) intervals. The Collins–Soper angle \( (\theta^*) \) is defined in the text.
and below the $J/\psi$ shows strong evidence of the expected spin alignment. The $J/\psi$ data are consistent with a flat angular distribution. The mass dependence of these distributions reflects a clear change in the underlying production mechanism for the $J/\psi$ compared with the continuum. Similar results for the continuum are also obtained using either the $s$, $t$, or $u$-channel reference directions for the helicity angle.

A number of attempts have been made to explain the broad $p_T$ spectra for the $\mu^+\mu^-$ pair in terms of quantum-chromodynamic corrections to the Drell-Yan mechanism. The corrections are expected to be most significant at higher $p_T$ and may modify the helicity angular distribution. Figure 3 and Table I give the helicity angular distribution for transverse momenta above and below 1 GeV/c. Evidence for spin alignment and a value of $\lambda$ consistent with 1.0 are seen in both $p_T$ intervals.

Equation (1) implies the scaling result that $M^2 d\sigma/dM$ is a function only of $M^2/s$. It should be noted that this is not an especially unique feature of the annihilation mechanism since such a result also follows from dimensional arguments and has been found to apply also to the production of some narrow resonances. Nevertheless, it is a condition which should be satisfied by the data.

Figure 4(a) shows our $\pi^-N\rightarrow\mu^+\mu^-X$ data compared with measurements at lower energies. This scaling prediction is not well satisfied by currently available $\pi^-$ data. However, some caution should be exercised in interpreting this result. The mass region covered by the low-energy experiments is below the $J/\psi$ where nonscaling resonance or continuum production may contribute. In this lower-mass region both $\langle p_T \rangle$ and the $A$ dependence are observed to vary with mass, while they do not for the mass region of this experiment. A clear lack of scaling or an experimental discrepancy is observed in the comparison with data of Ref. 13.

As a consistency check Fig. 4(b) shows our proton-induced cross sections in comparison

<table>
<thead>
<tr>
<th>Data Interval</th>
<th>$p_T$ (GeV/c)</th>
<th>Fit results</th>
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</thead>
<tbody>
<tr>
<td>$2.0\sim2.6$</td>
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<td>Flat</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\chi^2$/DF</td>
</tr>
<tr>
<td>$J/\psi$</td>
<td>All</td>
<td>79.7/9</td>
</tr>
<tr>
<td>$&gt;3.5$</td>
<td>All</td>
<td>44.6/9</td>
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<td>$&gt;3.5$</td>
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<tr>
<td>$&gt;3.5$</td>
<td>$&gt;1.0$</td>
<td>36.4/9</td>
</tr>
</tbody>
</table>

FIG. 4. $M^2 d\sigma/dM$ vs $M^2/s$, the scaling form of the cross section for (a) $\pi^-N\rightarrow\mu^+\mu^-X$ and (b) $pN\rightarrow\mu^+\mu^-X$. Data from Refs. 4 and 15 are converted from the measured $d^2\sigma/dmdy_{\mu\mu}$ to $d\sigma/d\cos\theta$ by using the anti-quark functions reported by the authors of Ref. 6 together with valence-quark distributions from deep-inelastic lepton scattering. The resulting $\chi^2$ distributions agree well with the observed spectrum in this experiment.
Determination of the Pion Structure Function from Muon-Pair Production


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Data on muon-pair production by pions are used to determine the momentum distribution for valence quarks in the pion. The shape of a nucleon structure function is also obtained and is compared with a calculation based on existing data.

In the two preceding Letters, we have presented features of high-mass muon-pair production and compared the data with predictions from a quark-antiquark annihilation model. In this Letter, the data are used within the framework of the model to obtain the momentum spectrum of valence quarks in the charged pion.

The general form of the Drell-Yan cross section in terms of the quark distribution functions is given in Ref. 1. There are a number of simplifications in its application to this experiment. For a pion it follows from charge conjugation and isospin invariance that the quark distribution function is the same for both valence quarks. Fur-