Measurement of the $B \to J/\psi K^*(892)$ Decay Amplitudes

We present a measurement of the decay amplitudes in $B \rightarrow J/\psi K^*$ (892) channels using 20.7 fb$^{-1}$ of data collected at the Y(4S) resonance with the BABAR detector at PEP-II. We measure a $P$-wave fraction $R_\perp = (16.0 \pm 3.2 \pm 1.4)$% and a longitudinal polarization fraction $(59.7 \pm 2.8 \pm 2.4)$%. The measurement of a relative phase that is neither 0 nor $\pi$, $\phi_\parallel = 2.50 \pm 0.20 \pm 0.08$ radians, favors a departure from the factorization hypothesis. Although the decay $B \rightarrow J/\psi K \pi$ proceeds mainly via $K^*$ (892), there is also evidence for $K^*_0$ (1430) and $K \pi$ $S$-wave contributions.
The decay $B^0 \rightarrow J/\psi K^{*0}$ with $K^{*0} \rightarrow K^0 \pi^0$ allows a measurement of the $CP$ violation parameter $\sin 2\beta$ that is theoretically as clean as for $B^0 \rightarrow J/\psi K_S^0$ [1]. However, due to the presence of even $L = 0, 2$ and odd $L = 1$ orbital angular momenta in the $J/\psi K^*$ system, there can be $CP$-even and $CP$-odd contributions to the decay rate. If the information contained in the decay angles is ignored, the measured time-dependent $CP$ asymmetry is reduced by the dilution factor $D_{\perp} = 1 - 2R_{\perp}$, where $R_{\perp}$ is the fraction of the $P$-wave. If the angular information is used, the $CP$ components can be separated [2].

The angular analysis also provides a test of the factorization hypothesis, the validity of which is in question for the $CP$ measurement of the $K^0$ and $K^{*0}$ amplitudes in the transversity basis [2,5] used by CLEO [6] and CDF [7]. The amplitudes $A_0$, $A_2$, and $A_3$ have $CP$ eigenvalues +1, +1, and −1, respectively, $A_0$ corresponds to longitudinal polarization, and $A_1$ is determined in the present analysis, $A_1$ and $A_2$, respectively [8]; only $J/\psi$ decays to $e^+e^-$ and $\mu^+\mu^-$ are considered. The data sample corresponds to $20.7 \times 10^6$ $B$ meson pairs.

In this Letter, we present a measurement of the decay amplitudes in the modes $B^0 \rightarrow J/\psi K^{*0}$ and $B^0 \rightarrow J/\psi K^{*+}$, where the $K^{*0}$ and $K^{*+}$ are reconstructed in the $K^0 \pi^0$, $K^{*0}\pi^0$, and $K^{*+}\pi^0$, respectively [8]; only $J/\psi$ decays to $e^+e^-$ and $\mu^+\mu^-$ are considered. The data sample corresponds to $20.7 \times 10^6$ $B$ meson pairs.

The BABAR detector is described elsewhere [9]. Charged particle track parameters are obtained from measurements in a 5-layer double-sided silicon vertex tracker and a 40-layer drift chamber located in a 1.5 T magnetic field; both devices provide $dE/dx$ information. Additional charged particle identification (PID) information is obtained from a detector of internally reflected Cherenkov (DIRC) light consisting of quartz bars that carry the light to a volume filled with water, and equipped with 10752 photomultiplier tubes. Electromagnetic showers are measured in a calorimeter (EMC) consisting of 6580 CsI(Tl) crystals. An instrumented flux return (IFR), containing multiple layers of resistive plate chambers, provides $\mu$ identification.

Electrons are identified by requiring that shower shape and energy deposition in the EMC be compatible with those expected for an electron of the measured momentum; $dE/dx$ measurements must also be compatible with the electron hypothesis. Muon candidates must penetrate at least two interaction lengths in the detector, and generate a small number of hits per layer in the IFR. If a muon candidate traverses the EMC, its energy deposition must be consistent with that of a minimum ionizing particle. Kaon candidates must survive a pion veto based on DIRC and $dE/dx$ information.

Charged tracks are required to be in regions of polar angle for which the PID efficiency is well measured. For electrons, muons, and kaons the acceptable ranges are 0.41 to 2.41 rad, 0.3 to 2.7 rad, and 0.45 to 2.5 rad, respectively. $J/\psi$ candidates consist of a pair of identified leptons that form a good vertex. The lepton pair invariant mass must be between 3.06 and 3.14 GeV/c$^2$ for muons and 2.95 and 3.14 GeV/c$^2$ for electrons. This corresponds to a $\pm 3\sigma$ interval for muons, and accounts for the radiative tail due to bremsstrahlung for electrons. $K^0_S$ candidates consist of vertexed pairs of oppositely charged tracks with invariant mass between 489 and 507 MeV/c$^2$. In the plane perpendicular to the beam line, the $K^0_S$ flight length must be greater than 1 mm, and its direction must form an angle with the $K^0_S$ momentum vector in this plane that is less than the positive $x$ axis. The $K^\pi$ decay plane defines the $(x, y)$ plane, with $y$ such that $p_y(K) > 0$. For the $J/\psi$ rest frame, the $K^\pi$ decay plane is the $(x, y)$ plane, and the coordinate system is right handed. The transversity angles $\theta_{\perp}$ and $\phi_{\perp}$ are defined as the polar and azimuthal angles of the positive lepton from the $J/\psi$ decay; $\theta_{K^*}$ is the $K^*$ helicity angle defined as the angle between the $K$ direction and the direction opposite the $J/\psi$ in the $K^*$ rest frame. The normalized angular distribution $g(\cos \theta_{\perp}, \cos \theta_{K^*}, \phi_{\perp})$ is

$$g = \frac{1}{\Gamma} \frac{d^3\Gamma}{d\cos \theta_{\perp} d\cos \theta_{K^*} d\phi_{\perp}} = f_1 |A_0|^2 + f_2 |A_1|^2 + f_3 |A_2|^2 + f_4 \text{Im}(A_0^* A_1) + f_5 \text{Re}(A_0^* A_1) + f_6 \text{Im}(A_0^* A_2),$  

(1)

with

$$f_1 = (9/32\pi) \times 2 \cos^2 \theta_{K^*}(1 - \sin^2 \theta_{\perp} \cos^2 \phi_{\perp}),$$

$$f_2 = (9/32\pi) \times \sin^2 \theta_{K^*}(1 - \sin^2 \theta_{\perp} \sin^2 \phi_{\perp}),$$

$$f_3 = (9/32\pi) \times \sin^2 \theta_{K^*} \sin^2 \theta_{\perp},$$

$$f_4 = (9/32\pi) \times \sin^2 \theta_{K^*} \sin \theta_{\perp} \sin \phi_{\perp} \cdot \zeta,$$

$$f_5 = -(9/32\pi) \times (1/\sqrt{2}) \sin \theta_{K^*} \sin \theta_{\perp} \sin 2\phi_{\perp},$$

$$f_6 = (9/32\pi) \times (1/\sqrt{2}) \sin \theta_{K^*} \sin \theta_{\perp} \cos \phi_{\perp} \cdot \zeta.$$  

(2)

When the final state is not a $CP$ eigenstate, $\zeta$ is +1 for $B^+$ and $B^0$, and −1 for $B^-$ and $B^0$. For the $CP$ mode $K_S^0 \pi^0$, $\zeta(B^0) = -\zeta(B^-) = 1/(1 + x_d^2)$, where $x_d = \Delta m_{\psi K} / \Gamma_{\psi K} \approx 0.73$; however, since flavor is not determined in the present analysis, $\zeta$ averages to zero for this mode. We define the relative phases of the amplitudes as $\phi_{\perp} = \arg(A_1 / A_0)$ and $\phi_{\parallel} = \arg(A_0 / A_0)$.

In this Letter, we present a measurement of the decay amplitudes in the modes $B^0 \rightarrow J/\psi K^{*0}$ and $B^0 \rightarrow J/\psi K^{*+}$, where the $K^{*0}$ and $K^{*+}$ are reconstructed in the $K^0 \pi^0$, $K^{*0}\pi^0$, and $K^{*+}\pi^0$, respectively [8]; only $J/\psi$ decays to $e^+e^-$ and $\mu^+\mu^-$ are considered. The data sample corresponds to $20.7 \times 10^6$ $B$ meson pairs.
than 0.2 rad. A photon is defined as a neutral cluster of energy greater than 30 MeV in the EMC that agrees in lateral shower shape with an electromagnetic shower. A $\pi^0$ candidate consists of a pair of photons with invariant mass in the interval 106–153 MeV/c$^2$. The $J/\psi$, $K^0_S$, and $\pi^0$ are constrained to the corresponding nominal masses [10]. $K^*$ candidates must have $K\pi$ invariant mass within 100 MeV/c$^2$ of the nominal $K^*(892)$ mass [10].

$B$ mesons are formed from $J/\psi$ and $K^*$ candidates. For $B \rightarrow J/\psi(K\pi^0)^*$, $\cos \theta_K$ is required to be smaller than 0.667. This reduces the cross feed (CF) from $J/\psi(K\pi^\pm)^*$ modes, where the $\pi^\pm$ is lost, and the self-cross feed (SCF) due to a wrongly reconstructed $\pi^0$. The (SC)CF is the most important background source since it tends to peak in the signal region.

The signal region is defined using two variables. The first is the difference $\Delta E = E_B^\text{exp} - E_B^\text{beam}$ between the candidate $B$ energy and the beam energy, in the $Y(4S)$ rest frame. The second is the beam-energy substituted mass $m_{\text{ES}} = (E_B^\text{exp} - p_{\text{ES}})^{1/2}$ where, in the laboratory frame, $E_B^\text{exp} = (s/2 + p_B \cdot \hat{p}_B)/E_1$ is the $B$ candidate expected energy, $p_B$ is its measured momentum, and $(E_1, \hat{p}_B)$ is the $e^+e^-$ initial-state four-momentum. $\sqrt{s}$ is the center of mass energy. For the signal region, $\Delta E$ is required to be between $-70$ and $+50$ MeV for channels involving a $\pi^0$, and within $\pm 30$ MeV otherwise. If several $B$ candidates are found in an event, the one having the smallest $|\Delta E|$ is retained. The corresponding $m_{\text{ES}}$ distributions are shown in Fig. 1.

With the signal region defined by $m_{\text{ES}} > 5.27$ GeV/c$^2$ and the above $\Delta E$ ranges, the $B$ reconstruction efficiencies are 9.9%, 23.9%, 17.2%, and 13.8% for the $K^0_S\pi^0$, $K^+\pi^-$, $K^0_L\pi^+$, and $K^+\pi^0$ modes, respectively, with corresponding total yields of 43, 547, 135, and 216 events. The CF (SCF) contamination levels, obtained from a full simulation of the BABAR detector, are 9.9(15.8)%, 1.2(2.4)%, 2.4(3.0)%, and 8.1(15.7)% of the pure signal, respectively.

The fit maximizes an unbinned likelihood that uses a probability density function (pdf) that depends on angular and $m_{\text{ES}}$ information. From the observed $m_{\text{ES}}$ value, a signal probability is computed with a Gaussian $G(m_{\text{ES}})$ to describe the signal and a phase-space background function [11] $F(m_{\text{ES}})$.

The pdf $g_{\text{obs}} = g(\hat{\omega}_i) \cdot \epsilon(\hat{\omega}_i)/(\epsilon)$ is used to describe signal events; $\hat{\omega}_i$ represents the angular variables $\cos \theta_B, \cos \theta_K, \phi_{\text{tr}}$ for event $j$, and $\epsilon(\hat{\omega}_i)$ is the efficiency at $\hat{\omega}_i$. Rewriting Eq. (1) as $g = \sum_{j=1}^{6} f_j \mathcal{A}_i$, where the $\mathcal{A}_i$ ($i = 1, \ldots, 6$) represent $|A_0|^2$, $|A_1|^2$, $|A_2|^2$, $\text{Im}(A_0^*A_1)$, Re($A_0^*A_1$), and $\text{Im}(A_0^*A_2^*)$, the mean $\langle \epsilon \rangle = \int g \epsilon \, d\hat{\omega} = \sum_{i=1}^{6} \mathcal{A}_i$, where the $\mathcal{A}_i = \int f_j \epsilon d\hat{\omega}$ are constants. The signal part of the log likelihood, $\ln L_{\text{signal}} = \sum_{j=1}^{6} \ln[g_{\text{obs}}(\hat{\omega}_j)]$, becomes $\ln L_{\text{signal}} = \sum_{j=1}^{6} \ln[g(\hat{\omega}_j)] + \sum_{j=1}^{6} \ln[\epsilon(\hat{\omega}_j)] - N_{\text{obs}} \ln(\sum_{j=1}^{6} \mathcal{A}_j)$. Since the $\epsilon(\hat{\omega}_j)$ are constants, the second term can be discarded. Only the coefficients $\mathcal{A}_i$ are required, and detailed representation of the acceptance is unnecessary [12].

The coefficients $\mathcal{A}_i$ are evaluated with a Monte Carlo simulation. Separate sets of $\mathcal{A}_i$ are used for each channel, and for $\ell = e, \mu$. The values of $\mathcal{A}_i$ ($i = 1, 2, 3$) are close to that of $\epsilon$; $\mathcal{A}_i$ is always smallest, especially in channels involving a $\pi^0$, because of the requirement on $\cos \theta_K$. The values of $\mathcal{A}_i$ ($i = 4, 5, 6$), which are related to the interference terms, are compatible with zero.

The angular dependence of combinatorial background events, $g_{\text{BC}}$, is described by a pdf similar to that in Eq. (1) with amplitudes $B_{ij}$, $i = 0, \parallel, \perp$, and corresponding terms $\mathcal{B}_i$ ($i = 1, \ldots, 6$).

The angular distribution of the (S)CF background is amplitude dependent. We correct for the effect of this background by evaluating the modified values $\mathcal{A}_i$ of the $\mathcal{A}_i$ by

<table>
<thead>
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<th>Quantity</th>
<th>$K^0_S\pi^0$</th>
<th>$K^+\pi^-$</th>
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<th>$K^+\pi^0$</th>
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<tr>
<td>$</td>
<td>A_0</td>
<td>^2$</td>
<td>0.65 ± 0.13</td>
<td>0.60 ± 0.04</td>
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<td>$</td>
<td>A_1</td>
<td>^2$</td>
<td>0.07 ± 0.11</td>
<td>0.17 ± 0.05</td>
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<tr>
<td>$</td>
<td>A_2</td>
<td>^2$</td>
<td>0.28 ± 0.14</td>
<td>0.23 ± 0.05</td>
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<tr>
<td>$\phi_{\perp}$ (rad)</td>
<td>...</td>
<td>-0.1 ± 0.2</td>
<td>0.0 ± 0.3</td>
<td>-0.4 ± 0.4</td>
</tr>
<tr>
<td>$\phi_{\parallel}$ (rad)</td>
<td>2.1 ± 0.7</td>
<td>2.5 ± 0.3</td>
<td>2.8 ± 0.4</td>
<td>2.6 ± 0.5</td>
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TABLE II. Fitted parameter values for the combined data samples. The first uncertainty is statistical, and the second is systematic. Note that $(\phi_\perp, \phi_{||}) \rightarrow (\pi - \phi_\perp, -\phi_{||})$ is also a solution.

<table>
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<tr>
<th>Quantity</th>
<th>Value</th>
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<tr>
<td>$</td>
<td>A_0</td>
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<td>$</td>
<td>A_1</td>
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<td>A_2</td>
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<tr>
<td>$\phi_\perp$ (rad)</td>
<td>$-0.17 \pm 0.16 \pm 0.07$</td>
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including the (S)CF events, in the $m_{ES}$ signal region, in addition to the signal [12]. In contrast to the $\xi_i$, the $\tilde{\xi}_i$ depend on the amplitudes used in the simulation, but the maximum effect on the fitted amplitudes is found to be on the order of $10^{-3}$. The complete log likelihood is

$$\ln \mathcal{L} = \sum_{j=1}^{N_{\text{obs}}} \ln [x G(m_{ES}) g(\tilde{\omega}_j)]$$

$$+ (1 - x) F(m_{ES}) g_B(\tilde{\omega}_j)] - N_{\text{obs}} \ln \left( \sum_{j=1}^{N_{\text{obs}}} x A_j + (1 - x) B_j \right) - \mathcal{N},$$

where $x$ is the fraction of signal integrated over the $m_{ES}$ range 5.2–5.3 GeV/c$^2$. The normalization of $g$ and $g_B$ is relaxed in an extended likelihood approach [13], with convergence to the required condition $a^2 = |A_0|^2 + |A_1|^2 + |A_2|^2 = 1$ imposed through the additional term $\mathcal{N} = N_{\text{obs}} a^2$ while $|B_0|^2 + |B_1|^2 + |B_2|^2 = a^2$ holds by construction. The fit parameters are the mean and width of $G(m_{ES})$; the shape parameter of $F(m_{ES})$; the fraction $x$; the signal amplitudes and phases $|A_0|^2, |A_1|^2, |A_2|^2, \phi_\perp, \phi_{||}$, and the corresponding background amplitudes and phases.

The agreement among the results for the individual decay channels is shown in Table I, while the fit result for the combined sample is summarized in Table II.

A partial representation of the fit is given by the one-dimensional projections of the angular distribution in Fig. 2. As a check of the fit quality, fits were performed to Monte Carlo samples with the angular distribution and number of events observed in the data. The maximum likelihood in the data is 1.35 standard deviations below the mean obtained from the Monte Carlo fits. The probability of obtaining a lower likelihood is 8.8%.

Systematic uncertainties are detailed in Table III. Limited simulation statistics (32 000 events per mode) give rise to a systematic uncertainty in the acceptance and (S)CF corrections (first row). Monte Carlo simulation has been used to estimate uncertainties due to the assumed form for the $m_{ES}$ angular distributions of the background (second row). In particular, this accounts for any possible absorption of the (S)CF background by the $F(m_{ES})$ function. The differences between simulated tracking and PID efficiencies and measurements obtained with control samples in the data lead to systematic uncertainties (third row) through their impact on acceptance corrections.

The $K\pi$ S-wave systematic uncertainty (fourth row) is obtained as follows. Although the $K\pi$ mass distribution for $B \rightarrow J/\psi K\pi$ is dominated by the $K^*(892)$ [Fig. 3(a)], a significant number of candidates are at higher mass with

![FIG. 2. The angular distributions for the channels without (top) and with (bottom) a $\pi^0$ for $m_{ES} > 5.27$ GeV/c$^2$. The data have been background subtracted and acceptance corrected. The curves correspond to the fit.](image)

![FIG. 3. (a) The background-subtracted $K\pi$ mass distribution for the $K^+\pi^-$ channel. The fit is to Breit-Wigner line shapes having nominal $K^*(892)$ and $K^*_2(1430)$ parameters [10] and a second-order polynomial (dotted line). (b) Enlargement of the 1–1.6 GeV/c$^2$ region of (a); the dashed curve denotes the sum of the Breit-Wigner contributions. (c) The background-subtracted $J/\psi$ helicity cosine distribution for events with $1.1 < m(K^+\pi^-) < 1.3$ GeV/c$^2$; the curve represents the fit of a sin$^3(\theta_{J/\psi})$ distribution to the data.](image)
a clear peak at $\sim 1.4$ GeV/$c^2$. The states in this region that couple strongly to $K\pi$ are the $K^*_0(1430)$ and the $K^*_2(1430)$ \cite{10}. Since it has width $\sim 300$ MeV/$c^2$, the $K^*_0(1430)$ alone would yield significantly more events above and below the peak than are observed. The $K^*_2(1430)$ alone describes the high mass region but, when combined with the $K^*(892)$ tail, yields too few events in the $1.1-1.3$ GeV/$c^2$ range [Fig. 3(b)]. This suggests a significant $S$-wave contribution, in which case the recoil $J/\psi K\pi$ has a helicity angle distribution $\sim \sin^2(\theta_{J/\psi})$. The observed behavior [Fig. 3(c)] agrees with this conjecture. This, together with the absence of the $S$-wave above $1.5$ GeV/$c^2$, is consistent with the mass dependence of the $S$-wave $K\pi$ scattering amplitude \cite{14}. If the $K\pi$ $S$-wave in $B$ decay behaves like this, a coherent $S$-wave amplitude should also be present in the $K^*(892)$ region; $S$-$P$ interference should occur, which, if ignored, can affect the $P$-wave amplitudes extracted from the data.

The effect of the $S$-wave in the $K^*(892)$ region has been estimated by including a scalar term in the total amplitude. This yields a more complicated angular distribution $g_S$, with ten $f_l$ functions. A fit of $g_S$ to the data in the $1.1-1.3$ GeV/$c^2$ region yields an $S$-wave fraction of $(62 \pm 9)\%$, in agreement with the failure of a $P$- and $D$-wave fit to describe the mass spectrum. By repeating the analysis using $g_S$, we find the $S$-wave contribution in the $K^*(892)$ region to be $(1.2 \pm 0.7)\%$. The differences in the $P$-wave results with and without $S$-wave are taken as estimates of systematic uncertainty (Table III, fourth row) since, with the present statistics, the presence of the $S$-wave in the $K^*(892)$ region cannot be confirmed.

Table IV compares our results to those of CLEO \cite{6} and CDF \cite{7}. They are consistent, but the present measurement is significantly more precise. Longitudinal polarization is seen to dominate and the $P$-wave intensity is small. If $\sin 2\beta$ were measured in the $B \to J/\psi K^0_S \pi^0$ channel from the decay-time information only, the value of the dilution from the present measurement, $D_\perp = 0.68 \pm 0.07$, would contribute a $10\%$ uncertainty.

Finally, we find that $|\phi_\parallel|$ differs significantly from $\pi$. This agrees with the CDF measurement, and indicates a departure from the factorization of the hadronic currents.

In addition, there is evidence that $S$- and $D$-wave amplitude contributions are necessary for a description of the $K\pi$ mass spectrum from $B \to J/\psi K\pi$ decay.

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*Also with: Università di Perugia, Perugia, Italy.
† Also with Università della Basilicata, Potenza, Italy.

[7] *Throughout the paper, the use of charge conjugate modes is implied wherever relevant.