20-T, 120-cm-I.R. Target Magnet with Layer-Wound Resistive Magnet

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This report presents designs for 20-tesla target magnets optimized by a computer program with major upgrades to its predictions of superconductor current density, magnet stresses and strains, and cost-optimization parameters. The field-and-temperature dependence of the non-copper current density in the Nb$_3$Sn in the strands of the superconducting cable is as shown in Fig. 1, generated by Eq. (1):

$$j(B, T) \approx \frac{46.630}{B} (1 - t^{1.247})^2 b^{0.437} (1 - b)^{1.727} \text{ [A/mm}^2\text{].} \tag{1}$$

The magnetic flux density $B$ is in teslas; $t$ and $b$ are, respectively, the normalized temperature $T/T_c$ and normalized magnetic flux density $B/B_c(T)$. Equation (1) resembles that of “A general scaling relation for the critical current density in Nb$_3$Sn”, by A Godecke, B ten Haken, H H J ten Kate and D C Larbalestier (2006 Supercond. Sci. Technol. 19 R100 doi: 10.1088/0953-2048/19/10/R02), but gives a much closer fit to the $j(B, T)$ data for Nb$_3$Sn of the ITER barrel magnet tabulated on page 645 of Case Studies in Superconducting Magnets, by Y. Iwasa. Fig. 2 plots the parameter $B_c(T) = 20.8 - 1.27 T - 0.0234 T^2$ needed by Eq. (1). Data points from which to generate the curve fit of Fig. 2 came from $T_c = 18.2 \text{ K}$ and the $j(B|T)$ curves of Fig. 3a&b, for which, by extrapolation, $j(B|T) = 0$ at [28.8 T, 1.8 K], [24.5 T, 4.2 K], and [16.1 T, 10 K].

Field and Temperature Dependence of Current Density of Nb$_3$Sn Strands

Fig. 1: Non-copper current density vs. field and temperature for Nb$_3$Sn strands of ITER barrel magnet.
**Temperature Dependence of Critical Field, $B_c$, of Nb$_3$Sn**

![Graph showing the temperature dependence of critical field, $B_c$.](image_url)

**Fig. 2:** Curve fit of data generated by Fig. 3a&b. $B_c(T) = 20.8 - 1.27 T - 0.0234 T^2$ teslas.

**Fig. 3a&b:** Curve fits of $j(B)\sim(B_c - B)^2$. Left: $j(B | T)$. Right: $\sqrt{j(B | T)}$. For ITER barrel conductor, extrapolation of the red curve to $j = 0$ gives $B_c = 16.1$ T at 10 K; extrapolation of the green curve gives $B_c = 24.5$ T at 4.2 K. For internal-tin conductor, $B_c = 25.1$ T at 4.2 K (blue curve) and 28.8 T at 1.8 K (black curve).
To predict the maximum stress (at the inner radius) in each solenoid, the computer program uses Eq. (5.35b) on p. 124 of *Solenoid Magnet Design* by Montgomery & Weggel or, equivalently, Eq. (3.77b) on p. 101 of *Case Studies in Superconducting Magnets*. The solenoid is of inner radius $a_1$, outer radius $a_2$, current density $j$, bore field $B_1$, and external field $B_2$, and is of isotropic material of Poisson’s ratio $\nu = 0.3$. The predicted stress is:

$$
\sigma_{\text{max}} = \frac{j(a_1^2 + 14a_1a_2 + 85a_2^2)(B_1 + B_2) + 14(a_1^2B_1 + a_2^2B_2)}{120(a_1 + a_2)}.
$$

Fig. 4 presents the results. Note that for a radially-thin solenoid (radius ratio $\alpha \equiv a_2/a_1 \approx 1$), the peak stress is $\sigma_{\text{max}} = j_1 a_1 <B>$, where $<B> = (B_1 + B_2)/2$, the average field in the windings. For solenoids of larger aspect ratio, $\sigma_{\text{max}} > j_1 a_1 (B_1 + B_2)/2$ and is greater, even, than $j_1 [(a_1 + a_2)/2] [(B_1 + B_2)/2]$. For example, in a solenoid of radius ratio $\alpha = 1.6$ and field ratio $\beta = 0$ (appropriate for the most-upstream superconducting coil of a 20-T target magnet) the normalized stress $\sigma^* \equiv \sigma_{\text{max}} / (j_1 a_1 B_1)$ is 0.85, 31% greater than the 0.65 calculated from the mean field 0.5 $B_1$ and the mean radius $(a_1 + a_2)/2 = 1.3 a_1$. For a solenoid of radius ratio $\alpha = 2.8$ and field ratio $\beta = 0.7$ (appropriate for a pancake-wound resistive magnet of O.R. = 50 cm and I.R. = 17.5 cm) the normalized stress $\sigma^*$ is 2.85, 76% higher than the $[(1+0.7)/2] \times [(1+2.8)/2] = 1.62$ predicted from the product of the mean radius and the mean magnetic field.

**Normalized Maximum Hoop Stress** $\sigma^* \equiv \sigma_{\text{max}} / (B_1 j_1 a_1)$

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Fig. 4: Normalized maximum hoop stress $\sigma^* \equiv \sigma_{\text{max}} / (B_1 j_1 a_1)$ vs. radius ratio $\alpha \equiv \text{O.R.}/\text{I.R.}$ & field ratio $\beta \equiv B_2/B_1$. 

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Fig. 5a-d, generated by a finite-element-method program, confirms that the magnet-optimization program does indeed generate designs in which the peak strain in every coil is very nearly the \(0.4\%\) that should be acceptable for all the magnet materials: copper and stainless steel for the resistive coils, and \(\text{Nb}_3\text{Sn}\), copper stabilizer and Incoloy 908 or other conduit material for the superconducting coils.

Fig. 5a-d: Hoop strain \(\varepsilon_{\text{hoop}}\) (color & contour lines) of 20-T target magnet with layer-wound resistive coils and upstream superconducting (SC) coils of 120-cm inner radius. Total stored energy = 2.88 GJ. Resistive magnet has five nested two-layer coils of MgO-insulated hollow conductor, graded from 23.8 mm square Japanese-Hadron-Facility hollow conductor (innermost coil) to \(\sim 35\) mm square (outer two coils); supporting each coil is a stainless-steel tube or wrap of 600 MPa design stress. Field contribution = 5.3 T at 12.0 MW; field homogeneity = 3.3% peak-to-peak; \(\Delta T_{\text{max}} = 70^\circ\text{C}\) with water flow of 59 liters/sec (\(\Delta P = 40\) atm, 4 hydraulic paths per layer). The most-upstream SC coil has an outer radius of 193 cm, a length of 3.3 meters, a weight of 130 metric tons, and is 9% \(\text{Nb}_3\text{Sn}\), 51% steel, and 32% copper-helium and 8% insulation. The endmost coil shown is 1.4% superconductor, 4.5% steel, 75% copper-helium, and 19% insulation. a) Entire magnet upstream of 9 m. b) Resistive magnet. c) Upstream SC coils. d) Downstream SC coils.

Fig. 6 shows that the magnet of Fig. 5 generates an on-axis field profile that matches very closely the desired profile. Fig. 7 plots parameters of magnets optimized for minimum yearly cost of operation for a duty cycle ranging from 32% to 63% (1-3\times10^7\text{sec/yr}). The cost optimization uses parameters based on values from the NHMFL web site: 1) Power = $121$/MW-hr; 2) Fabricated copper and steel is $400$/kg (nearly twice the average for non-superconducting NHMFL magnets, to account for inflation and that the optimization ignores the mass of components such as shielding and cryostats; 3) The cost of superconducting magnets is 2 to 2\(\frac{1}{2}\) times that of non-superconducting magnets; and 4) Amortization of capital investments is at 10% per year.
Fig. 6: On-axis field profiles of superconducting magnet and layer-wound resistive magnet of 20-T target magnet optimized for $2 \times 10^7$ sec/yr of operation.

Fig. 7: Power, field allocation, mass, & yearly cost of optimized 20-T target magnets: duty cycle = $1.3 \times 10^7$ sec/yr.
On-Axis Field of 15.30-kA, 18.1-MW Target Magnet "Layer3%1e7" at 11.44 kA, 9.3 MW

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