Field Profiles of Target Magnets with $B(z>5m)=1.5$ T, 2 T, 2.5 T & 3 T

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Each of the magnets described below consists of a main coil, notched on its inner surface to improve field homogeneity, plus two sets of triplets, each ~5 m long, between $z ~ 4$ m and 14 m. The Excel spreadsheet used to design the system embodies the following goals and constraints:

1) Main-solenoid I.R. = 120 cm; current density = 18 A/mm², as typical for SC#1 (~60% steel);
2) Current density in solenoids #2 to #7 = 45 A/mm² (~10% steel); I.R. = 60 cm for coils #5-#7;
3) $B(z)$, is 15 T at $z = -0.5$ m, $B_{\text{min}}$ at 5 m, and 14.7 T ($\Delta B = 0.3$ T = 2% of 15 T) at 0 & $-1$ m;
4) Field derivative $B' \equiv dB/dz = 0$ at $z = -0.5$ m & $z = 5$ m; $B' < 0$ from $z = -0.5$ m to 5 m;
5) Goal function strongly penalizes ampere-meters of conductor usage, to reduce magnet cost;
6) Penalized gently is I.R. < 120 cm for solenoid #2 and O.R. > 100 cm for solenoids #3 & #4;
7) Penalized for $z > 5$ m is a weighted sum of the squares of $\Delta B \equiv B-B_{\text{min}}$, $B'$, & $B'' \equiv d^2B/dz^2$;

Fig. 4 embodies the constraint $B' < 0.5 \left(5 - z/100\right)$ from $z = -0.5$ m to 5 m.

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Fig. 1a&b. Target Magnet 15to1.5T5m1+3+3. Left: Conductor cross sections and field direction (arrows) & magnitude (color & contours). Inner radius of successive coils is [1.20, 1.10, 1.03, 0.83, 0.60] m. Gap between coils #1 & #2 is 2.8 m; between triplets #1 & #2 is 0.56 m = 1/3 of sum of outer radii of flanking coils. Peak ambient field is 16.0 T. Right: Field profile, $|B(z)|$ (blue); $10 \log_{10}(|B|)$ (black); $1^\text{st}$ derivative, $dB/dz$ (turquoise); and $2^\text{nd}$ derivative $d^2B/dz^2$ (green). $B(-50\text{ cm}) = 15$ T; $B(500\text{ cm}) = 1.5$ T. $B(-100\text{ cm}) = B(0) = 14.7$ T.
Fig. 2a & b. On-axis field profile, $|B(z)|$ (blue); $10 \log_{10}(|B|)$ (black); $\frac{dB}{dz}$ (turquoise); & $\frac{d^2B}{dz^2}$ (green) of target magnets with $B(-100 \text{ cm}) = B(0) = 14.7 \text{ T}$ and $B = B_{\text{min}}$ at $z = 500 \text{ cm}$. Left: $B_{\text{min}} = 1.5 \text{ T}$. Right: $B_{\text{min}} = 2.5 \text{ T}$.

Fig. 3 compares the field profiles of the target magnets in Figs. 1 & 2 and also one with $B_{\text{min}} = 3.0 \text{ T}$. Only for $B_{\text{min}} = 1.5 \text{ T}$ does the field profile extend nearly to $z = 5 \text{ m}$; the other field profiles reach their asymptotic values prematurely, at ~4.2 m, ~3.8 m and 3.5 m, respectively.

Field Profiles of Target Magnets $15\text{m}$ to $B_{\text{min}} = 1.5 \text{ T}$, $2.0 \text{ T}$, $2.5 \text{ T}$ & $3.0 \text{ T}$

Fig. 3: On-axis field profiles of target magnets as in Figs. 1 & 2, with $B_{\text{min}} = 1.5$ to $3 \text{ T}$. Upper set of curves is $B(z)$; lower set is $\Delta B / B_{\text{min}}$. 
One can avoid premature taper terminations by requiring that each field profile rapidly acquire a significant downward slope upstream of 5 m. Figure 4 introduces the constraint \(-dB/dz \geq \frac{1}{2}(5-z)\) T/m throughout the range \(2.5 < z < 5\) m.

**Fig. 3:** On-axis field profiles that avoid premature taper terminations by requiring \(-dB/dz \geq \frac{1}{2}(5-z)\) T/m for \(2.5 < z < 5\) m. Upper set of curves is \(B(z)\); lower set is \(\Delta B/B_{\text{min}}\).