Power Deposition in Graphite Targets of Various Radii

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“Thermal shock” by pulsed beams incident on solid targets will be greatest at the point of peak energy/power deposition (and greater for beams of lower duty cycle).

In large targets (beam dumps/hadron calorimeters), the longitudinal profile of energy deposition has a “shower maximum” ~ 1 pion interaction lengths into the target.

Where is the peak energy deposition in a “pencil” graphite target, of radius ~ 8 mm, as considered for a Muon Collider/Neutrino Factory?

Facts: graphite density ~ 1.8 g/cm$^3$,
\[ \text{dE/dx} = 1.5 \text{ MeV/(g/cm}^2) \],
\[ \text{pion interaction length } \sim 72 \text{ cm}, \]
\[ \text{radiation length } \sim 24 \text{ cm}. \]

The studies were done for a 4-MW beam of 6.75-GeV-kinetic-energy protons. The studies reported here were done with MARS15(2014), and FLUKA(2011).
Target at 0°

For a first study, we consider graphite targets at 0 to the magnetic axis, in 0- and 20-T uniform magnetic fields.

**B = 0 T, MARS**

The plots show the total power deposited in 1-cm-thick disks, for target of various radii.

⇒ Largest power deposition only 4 cm into a “pencil” target, but at ~ 60 cm in targets with large radius.

\[ \frac{dE}{dx} \text{ only deposits about 1870 Watts in 1 cm of graphite, for a 4-MW beam.} \]
Target at 0°, II

B = 20 T, MARS

B = 20 T, FLUKA

FLUKA indicates 5-10% more power deposition in this comparison.

The FLUKA beam is parallel, with rms radius = 2 mm, while the MARS beam is focused with spot rms radius of 2 mm and $\beta^* = 80$ cm.
The $z$-coord. of the target slice with peak power density is constant for radii > 30 cm.
The total power absorbed in the target increases from ~150 kW in a "pencil" target (length = 80 cm, radius = 8 mm) to about 1 MW (out of 4 MW) in a target of 80 cm length and 40 cm radius.
We now want to locate the coordinates of the point with peak local power deposition.

A study not shown confirmed that this point has coord. $r = 0$.

To find the $z$ coord., we plot the power deposition vs. $z$ in a cylinder with $r = 1$ mm.

The curves are essentially independent of the target radius (for $r_{\text{target}} > 8$ mm).
The z-coord. of the point with peak local power deposition is 2-3 cm into the target, independent of the target radius, as shown in the left figure below.

The peak local power deposition is about 3600 W/g for 0 magnetic field and 4-MW beam power, and about 3400 W/g for 20 T field, as shown in the right figure below.

For 60-Hz beam structure, the peak energy deposition is only about 60 J/g (and 240 J/g for 15-Hz beam structure), for 4-MW beam power.
Target at 65 mrad

Trajectory of the central proton ray for 65-mrad tilt and 20-T field.

The y-coord. of the beam at any z inside the target is essentially the same as the y-coord. of the center of the target.

But, the beam enters the target offset in x by ~ 4.3 mm from the target center.

⇒ Peak energy deposition likely offset from the target center.
Target at 65 mrad, II

The peak energy deposition is 3598 J/g (~ same as for the 0° case), and occurs for (x,y,z) = (-0.35, 2.85, -37.5) cm (2.5 cm into the target).

Power deposition in the target slice -4 mm < x < 3 mm.
Power Deposition Due to $dE/dx$

The peak power deposition of 3600 W/g occurs about 37 cm from the center of the target.

The rms radius there is $\sigma_r = 0.2 \left[ 1 + \left( \frac{37}{80} \right)^2 \right]^{1/2} \sim 0.22$ cm, for $\beta^* = 80$ cm,

$\Rightarrow$ Effective area of a Gaussian beam $= 2\pi \sigma_r^2 \sim 0.30$ cm$^2$.

A 4-MW beam of 6.75-GeV protons has

$N = 4 \cdot 10^6 \text{ J/s} / (6.75 \cdot 10^9 \text{ eV} \cdot 1.6 \cdot 10^{-19} \text{ J/eV}) \sim 3.7 \cdot 10^{15} \text{ p/s}.$

d$E/dx$ in graphite is 1.5 MeV/(g/cm$^2$).

The power deposition due to $dE/dx$ at 3 cm into the target is

$N \cdot dE/dx / \text{Area} = 3.7 \cdot 10^{15} / \text{s} \cdot 1.5 \cdot 10^6 \text{ eV/(g/cm}^2) \cdot 1.6 \cdot 10^{-19} \text{ J/eV} / 0.30 \text{ cm}^2$

$\sim 2950$ W/g.

This suggests that the peak power deposition (in our “pencil” target) is only about 1.2 times that due to $dE/dx$. 
Thermal Issues for Solid Targets

When beam pulse length $t$ is less than target radius $r$ divided by speed of sound $v_{\text{sound}}$, beam-induced pressure waves (thermal shock) are a major issue.

Simple model: if $U = \text{beam energy deposition in, say, Joules/g}$, then the instantaneous temperature rise $\Delta T$ is given by $\Delta T = U/C$, where $C = \text{heat capacity in Joules/g/K}$.

The temperature rise leads to a strain $\Delta r/r$ given by $\Delta r/r = \alpha \Delta T = \alpha U/C$, where $\alpha = \text{thermal expansion coefficient}$.

The strain leads to a stress $P = \text{force/area}$ given by $P = E \Delta r/r = E \alpha U/C$, where $E = \text{modulus of elasticity}$.

In many metals, the tensile strength obeys $P \approx 0.002 E$, $\alpha \approx 10^{-5}$, and $C \approx 0.3 \text{ J/g/K}$, in which case $U_{\text{max}} \approx P C / E \alpha \approx 0.002 \cdot 0.3 / 10^{-5} \approx 60 \text{ J/g}$.

Graphite @ 1400°C: $P = 42.4 \text{ Mpa}$, $E = 7.2 \text{ Gpa}$, $\alpha = 4.8 \times 10^{-5}$, $C = 1.4 \text{ J/g}$, $U_{\text{max}} \approx 1700 \text{ J/g}$.

(A nickel target at FNAL has operated with $U_{\text{max}} \approx 1500 \text{ J/g}$.)

These arguments are from *A Short Course on Targetry, KTM, NuFact03 Summer Institute*.

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How Much Beam Power Can a Solid Target Stand?

What is the maximum beam power this material can withstand without cracking, for a 6.75-GeV beam at 15 Hz with area 0.3 cm²?

Ans: MARS15 indicates that the peak energy deposition in a “pencil” target is about 1.2 times that of dE/dx, \[ \Rightarrow 1.8 \text{ MeV/(g/cm}^2\text{)} \text{ for graphite.} \]

Now, 1.5 MeV = 2.9 \cdot 10^{-13} \text{ J}, so 1500 J/g requires a proton beam intensity of \( \frac{1500 \text{ J/g}}{2.9 \cdot 10^{-13} \text{ J/cm}^2/\text{g}} \approx 5 \cdot 10^{15}/\text{cm}^2. \)

\[ \Rightarrow P_{\text{max}} \approx 15 \text{ Hz} \cdot (6.75 \cdot 10^9 \text{ eV}) \cdot (1.6 \cdot 10^{-19} \text{ J/eV}) \cdot (5 \cdot 10^{15} /\text{cm}^2) \cdot 0.3 \text{ cm}^2 \approx 2.5 \cdot 10^7 \text{ J/s} \]
\[ = 25 \text{ MW.} \]

If graphite cracks under singles pulses of > 1500 J/g, then “safe” up to 25-MW beam power @ 15 Hz and 6.75 GeV kinetic energy. (And would be “safe” up to 125 MW-beam power with a carbon-carbon target.)