The Best Nozzle is No Nozzle(?)

Reservoir at pressure $P$ with small aperture:

$$v_{\text{reservoir}} \approx 0, \quad v_{\text{jet}} \approx \sqrt{\frac{P}{\rho}}.$$

Jet emerges perpendicular to the plane of the aperture.

Reservoir + short nozzle:

No reservoir, just a straight tube. $v_{\text{jet}} = v_{\text{tube}}$.

Most nozzle R&D is concerned with making a jet break up quickly and uniformly (atomizing), rather than with preserving the jet.
Conservation of Energy vs. $F = dP/dt$ at a Contraction? (Borda, 1766)

Incompressible fluid $\Rightarrow V_1 A_1 = V_2 A_2$.

$$A_2 \ll A_1 \Rightarrow V_1 \ll V_2.$$ Conservation of Energy $\Rightarrow$ Bernoulli’s Law:

$$P_1 + \frac{1}{2} \rho V_1^2 = P_2 + \frac{1}{2} \rho V_2^2.$$ $V_1 \ll V_2 \Rightarrow V_2^2 \approx 2 \frac{P_1 - P_2}{\rho}.$

Argument does not depend on the area.

$F = dP/dt$:

Mass flux $= \rho VA$.

Momentum flux $= \rho V^2 A$.

Net momentum flux $= \rho (V_2^2 A_2 - V_1^2 A_1)$

$= \rho V_2 A_2 (V_2 - V_1) \approx \rho V_2^2 A_2$.

Force $\approx (P_1 - P_2) A_2$.

$$F = \frac{dP}{dt} \Rightarrow V_2^2 \approx \frac{P_1 - P_2}{\rho}.$$ Consistency $\Rightarrow$ dissipative loss of energy,

OR jet pulls away from the wall and contracts.

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Cavitation can be induced by a sharp-edged aperture.

A jet emerging from a small aperture in a reservoir contracts in area:

\[ A_{\text{jet}} = \frac{\pi}{\pi + 2} A_{\text{aperture}} = 0.62 A_{\text{aperture}} \]
\[ d_{\text{jet}} = 0.78 d_{\text{aperture}} \]

2-d potential flow (conservation of energy) \( \Rightarrow \) analytic form:

\[ x = \frac{2d}{\pi + 2} (\tanh^{-1} \cos \theta - \cos \theta), \quad y = d - \frac{2d}{\pi + 2} (1 + \sin \theta), \]
\[ \theta = \text{angle of streamline}, \quad -\frac{\pi}{2} < \theta < 0. \]

90% of contraction occurs for \( x < 0.8d \).

Good agreement between theory and experiment.