Classical Solution of Wave equation

\[ \nabla^2 u = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} ; \quad u : \text{displacement} \]

- One dimensional

\[ \frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \]
\[ u(x, t) = f(x - ct) + g(x + ct) \]

- Spherical Symmetry

\[ \frac{\partial^2 u}{\partial r^2} + \frac{2}{r} \frac{\partial u}{\partial r} \frac{2u}{r^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \]
\[ u(r, t) = f \frac{(r - ct)}{r} + g \frac{(r + ct)}{r} \]

- Cylindrical Symmetry

\[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \]
\[ u(r, t) = \sum_{n=1}^{\infty} c_n J_1(\varepsilon_n \xi) \cos(\varepsilon_n \theta) \]
\[ \xi = \frac{r}{R}, \theta = \frac{ct}{R} \]
\[ \sigma_T = \sigma + \delta \]
Exemple of Solutions: Free Solid, Parabolic Function

\[ v = \frac{1}{3}, \quad \xi = 0.01, \quad \Theta = 0.1, 0.2, \ldots, 10 \]

\[ r_n \text{ are the solutions of } J_0(r_n) = (1 - 2v) J_2(r_n) \]

\[
fp(\theta, \Theta, n) := \begin{cases}
\frac{1}{r_n^2} \sin(r_n \cdot \theta) & \text{if } (0 \leq \Theta) \\
\frac{1}{r_n^2 \cdot \Theta} \left[ \sin(r_n \cdot \theta) - (\sin(r_n \cdot \theta) \cdot \cos(r_n \cdot \theta) - \cos(r_n \cdot \theta) \cdot \sin(r_n \cdot \theta) \right] & \text{if } (\Theta < \theta)
\end{cases}
\]

\[
\sigma(\xi, \theta, \Theta) := \frac{4}{(1 - v)^2} \sum_{n=0}^{200} \frac{1}{\left(\frac{r_n^2 - (1 - 2v)}{(1 - v)^2}\right)} \cdot \frac{1}{\left(\frac{r_n^2}{(1 - v)^2}\right)} \cdot \frac{v}{1 - v} \cdot J_0\left(r_n^2, \frac{\xi}{r_n^2}\right) + \frac{1}{4(1 - v)} \cdot \left(1 - \xi^2\right)
\]

\[
\sigma(\xi, \theta, \Theta) := \frac{4}{(1 - v)^2} \sum_{n=0}^{200} \frac{1}{\left(\frac{r_n^2 - (1 - 2v)}{(1 - v)^2}\right)} \cdot \frac{1}{\left(\frac{r_n^2}{(1 - v)^2}\right)} \cdot \frac{1}{1 - v} \cdot J_0\left(r_n^2, \frac{\xi}{r_n^2}\right) + \frac{1}{4(1 - v)} \cdot \left(1 - 3\xi^2\right)
\]

\[
\sigma(\xi, \theta, \Theta) := \frac{4 - v}{(1 - v)^3} \sum_{n=0}^{200} \frac{1}{\left(\frac{r_n^2 - (1 - 2v)}{(1 - v)^2}\right)} \cdot \frac{1}{\left(\frac{r_n^2}{(1 - v)^2}\right)} \cdot J_0\left(r_n^2, \frac{\xi}{r_n^2}\right) \cdot fp(\theta, \Theta, n) + \frac{1}{4(1 - v)} \cdot (4 - 2v - 4\xi^2)
\]
For solids believe in Hook's Law, negative stresses allowed.

For liquids: negative pressures lead to cavitation. Wave equation fails as soon as
- \( P < P_{\text{cavitation}} \)

For instantaneous Heating at \( t = 0 \)

All \( \sigma_T (r, t = 0) = \bar{\sigma} (r) + \bar{\sigma}(r, t = 0) = \frac{E \alpha_t(r)}{1 - 2\nu} \)

independent of boundary condition. However larger stresses may occur at later times!

For finite (non instantaneous) heating:

Convolution \( \sigma(r, t, t_0) = \int_0^t \sigma(r, t - \tau) \frac{\partial T(\tau)}{\partial \tau} d\tau \)

Sound Velocity: \( c \sim \sqrt{\frac{E}{\rho}} \) for solids

\( c \sim \sqrt{\frac{1}{\kappa \rho}} \) for liquids
Material Velocity: \[ v_c \approx \alpha_L \Delta T \quad \text{for solids} \]

\[ v_c \approx 1/2 \rho \kappa = 1/2 \alpha_v \Delta T \quad \text{for liquids} \]

Example Hg:

\[ \Delta T = 200 \, K \]

\[ \alpha_v = 18.1 \times 10^{-5} \, K^{-1} \]

\[ \kappa = 0.45 \times 10^{-10} \, m^2/N \]

\[ P = 800 \, MPa \]

\[ v_c = 1.8\% \quad \text{(not yet supersonic !)} \]

\[ c = 1.3 \, km/s \]

\[ v = 23 \, m/s \]
- Get some insight into the physics.

- Identify critical areas, times.

- Use it as a "Bench Mark" to check against FE-codes which are "Black Boxes".

- Use it for scaling with R, c, t₀.
Results

(All axially constrained)

- Dynamic: \[ \tilde{P} = \frac{\alpha_v T_0}{\kappa} f(\xi, \theta, \theta_0) \]
  \[ \xi = r/R, \quad \theta = ct/R, \quad \theta_0 : \text{Burst Duration } ct_0/R \]
  \[ f : \text{depends also on initial condition, } T(\xi, 0, 0) \]
  and boundary condition

- Static: \[ \bar{P} = \frac{\alpha_v T_0}{\kappa} g(\xi) \]
  \[ g : \text{depends on } T(\xi, 0, 0) \text{ and boundary condition} \]

Total: \[ P_T = \tilde{P} + \bar{P} = \frac{\alpha_v T_0}{\kappa} \{ f(\xi, \theta, \theta_0) + g(\xi) \} \]
  radially free: \[ g(\xi) = 0 \]

- Velocity: \[ v = \alpha_v T_0 c \quad h(\xi, \theta, \theta_0) \]
  \[ c \approx 1/\sqrt{\rho \kappa} \]

For solids:

\[ \alpha_T = E \alpha_L T_0 \{ f(\xi, \theta, \theta_0) + g(\xi) \} \]
  radially free: \[ g(\xi) \neq 0 \]
At $\theta = 0$ for $\theta_0 = 0$:

$$P_T(\xi,0,0) = \frac{\alpha_v T(\xi,0,0)}{\kappa}$$

$$\sigma_T(\xi,0,0) = \frac{E\alpha_L T(\xi,0,0)}{1-2\nu}$$

Does not depend on boundary conditions!
Liquid with free boundary conditions, Step Function

$p(\xi, 0)$
$p(\xi, 0.3)$
$p(\xi, 0.6)$
$p(\xi, 0.9)$
$p(\xi, 1.2)$

$\xi$
Liquid with free boundary conditions. Step Function

\[ p(0.01, \theta) \]
\[ v(0.99, \theta) \]
Free Liquid, Step Function, Heating Time Effects
Hg:

\[ \rho = 13.5 \times 10^3 \text{ kg/m}^3 \]
\[ \kappa = 0.45 \times 10^{-10} \text{ m}^2/\text{N} \]
\[ c = 1.28 \times 10^3 \text{ m/s} \]
\[ \alpha_v = 18.1 \times 10^{-5} \text{ K}^{-1} \]
\[ c_V = 140 \text{ J/kg K} \]
\[ P_{\text{cavitation}} \approx -... \text{N/m}^2 ?? \]
\[ \sigma_{\text{el.}} = 10^6 [\Omega m]^{-1} \]
\[ T_0 = 200 \text{ K} \]
\[ \frac{\alpha_v T_0}{\kappa} = 804 \text{ MPa (8000 Bar)} \]
\[ \alpha_v T_0 = 3.6 \% \]
\[ \alpha_v T_0 \ c = 46 \text{ m/s} \]

Ta:

\[ \rho = 16.8 \times 10^3 \text{ kg/m}^3 \]
\[ E = 16 \times 10^{10} \text{ N/m}^2 \]
\[ c = 3.1 \times 10^3 \text{ m/s} \]
\[ \alpha_L = 6.5 \times 10^{-6} \text{ K}^{-1} \]
\[ c_V = 151 \text{ J/kg K (at room temperature)} \]
\[ \sigma_0 = 500...1000 \text{ MPa} \]
\[ T_0 = 200 \text{ K} \]
\[ E\alpha_L T_0 = 208 \text{ MPa} \]
Liquid with free boundary conditions. Parabolic Function

Graph 1

Graph 2
Liquid with free boundary conditions. Parabolic Function
Free Liquid, Parabolic Function, Heating Time Effects
- **Free Liquid:** \( T(\xi,0,0) = T_0(1 - \xi^2), \) *parabolic*
  all much smoother, \( \frac{dP}{d\xi} \) lower
  Similar to harmonic oscillator
  \( P \approx +1 \ldots -1: \pm 800 \text{ MPa all the time} \)
  \( \nu \approx \pm \frac{1}{3} : \pm 15 \text{ m/s (1.2\%)} \)

- **Effect of \( \theta_0 > 0 \):**
  \( P \) at \( \theta_0 = 1, \) initially \( \frac{P(\theta_0 = 1)}{P(\theta_0 = 0)} \approx 1/2 \) (400 MPa)
  later \( \pm 1 \) (± 800 MPa)
  \( \nu \) at \( \theta_0 = 1 \) ± 0.2 (9 m/s)

- **Free Solid:**
  in center \( \sigma_r = \sigma_\phi \approx +3 \ldots -2: +624 \text{ MPa} \ldots -416 \text{ MPa} \)
  \( \sigma_z = 3 \ldots -0.5: +624 \text{ MPa} \ldots -104 \text{ MPa} \)
  at rim \( \sigma_\phi \approx 0 \ldots -1.5: 0 \ldots -312 \text{ MPa} \)
  \( \sigma_z \approx 0 \ldots -0.5: 0 \ldots -104 \text{ MPa} \)
  Von Mises \( \sigma_{eq.}^2 = \sigma_r^2 + \sigma_\phi^2 + \sigma_z^2 - \sigma_r \sigma_\phi - \sigma_r \sigma_z - \sigma_\phi \sigma_z \)
  Von Mieses at center: at \( \theta = 0 \) \( \sigma_{eq.} = 0 \) (hydrostatic equilibrium)
  at \( \theta \approx 1.5 \) \( \sigma_{eq.} \approx 1.5 \)
  oscillates between 0 .... 312 MPa
Von Mises at rim: at $\theta = 0$ $\sigma_{eq.} = 0$

at $\theta \approx 1.5$ $\sigma_{eq.} \approx 1.3$

oscillates between 0….275 MPa

Looks $\rightarrow$ ok for one shot!

But for 50 Hz (4Mio / Day)???

Conclusions: For pulse duration

$$\theta_0 \leq 1, \quad t_0 = \frac{R}{C} = \frac{5 \text{mm}}{C_{Hg}} \approx 4 \mu s$$

Pressures, stresses of the same order as for $\theta_0 = 0$

For small $R$ correspondingly smaller.

Problem of cavitation in liquids (negative pressures) to be solved.

Problems of fatigue in solids in high temperature to be assessed.
Fig. 5 - Overall view of the target assembly. The aluminium container is anodised black to aid cooling via radiation. The hole in the upstream luminescent screen avoids premature aging and radiation damage of the latter.

Fig. 6 - X-ray photo of the target ensemble after irradiation.
- To reduce $\Delta E$/Target

  Increase $f$
  Need $\Delta s = 3 \text{ cm}$
  Increase $\varnothing \sim f!!$
  Const. $\omega$

- Thermal stress (Static + Dynamic at $t = 0$)

  $$\sigma \sim \frac{E \alpha \Delta T}{1 - 2\nu}$$

  $\nu \approx 1/3$

  Tungsten $E \sim 400 \text{ GPa}$
  $\alpha \sim 4.5 \times 10^{-6} \text{ K}^{-1}$
  $\Delta T \sim 200 \text{ K}$
  $\sigma \sim 1 \text{ GPa} < \sigma_{\text{max}} \sim 1.5 \text{ Gpa}$

  $\Delta T = 625 \text{ K}$, Target will crack
- Target must be contained!
  Container Material with
  Absorption length $\lambda_C \gg \lambda_{\text{Target}}$
  $\lambda \text{ w} \sim 10 \text{ cm}$
  $\lambda \text{ Carbon} \sim 30 \text{ cm}$
  **C A R B O N**!
  Can stand $< 3000 \, ^\circ\text{C}$ in Vacuum
  Good heat conductor
  Excellent experience at CERN-$\bar{\text{P}}$-Source
- Need 200 Targets
  Target Wheel Ø 2 m
  Target Spacing $\Delta s = 3 \text{ cm}$
  \[
  \begin{array}{ll}
  \text{r.p.s.} & 0.25 \text{ t/s} \\
  \text{r.p.m.} & 15 \text{ t/min.} \\
  \text{Linear velocity} & 1.5 \text{ m/s (5.4 km/h)} \\
  \text{Centr. Force} & 0.25 \text{ g}
  \end{array}
  \]

- 1 MW in 200 Targets : 5 kW / Target
  1 MW evacuated through surface
  of $\sim 1 \text{ m}^2$ ($\pi \times \varnothing = 6.3 \times l_{\text{Target}} = 0.15 \text{ m}$)
  $100 \text{ W / cm}^2$

  into Water Cooling System

$l_{\text{Target}} = 0.15 \text{ cm}$:
longer and lighter target helpful for cooling,
but $\lambda_C \approx \lambda_{\text{Target}}$
- Average steady state temperature

\[
\begin{align*}
\Delta T & \text{ Graphite} \quad \sim 60 \text{ K} \\
\Delta T & \text{ Cu, 1 cm} \quad \sim 30 \text{ K} \\
\Delta T & \text{ Water} \quad \sim 60 \text{ K} \\
\overline{\Delta T} & \quad \sim 150 \text{ K} < \Delta T \text{ per shot}
\end{align*}
\]

- Water Cooling System

Water close to target : \quad \sim 1 \text{ cm} \\
Flow : \quad 5 \text{ l/s} \\
\Delta T_w : \quad 50 \text{ K} \\
\varnothing \text{ pipe} : \quad 5 \text{ cm} \\
v_{\text{water}} : \quad 2.5 \text{ m/s} \\
b(\text{Coriolis}) : \quad 0.8 \text{ g} \\
b_{\text{centr}} : \quad 0.25 \text{ g}

- Need well-designed heat exchanger from Cu \rightarrow Water
- Precision of Rotation

Assume a change of friction by $1\% \approx 65$ W
(expected friction of rotation $< 10 \%$, $\approx 650$ W)
Targets out of beam line by 1 mm
after 300 ms

Need control loop locked to accelerator
via angle encoder with 6000 lines / turn

+ Interlock to stop the beam
ROTATING TARGET WHEEL

WATER COOLED TARGET WHEEL

GRAPHITE

TARGET ROD

P. SIEVERS 23/05/2000
- Eddy Currents
  To move 1 cm$^3$ of Cu with v = 5 m/s
  into the solenoid takes a peak force of
  ~ 600 N!

* Thin laminations for metallic parts with
  high electrical resistivity (Ti-alloys)

* Non-conducting materials (ceramics),
  again « Edge Technology »
TARGET WHEEL ROTATING THROUGH A SOLENOID

FRONT VIEW

SOLENOID

TARGET ROD

GRAPHITE

WATER COOLED TARGET WHEEL

P. SIEVERS 23/05/2000
Liquid Metal Target Inside Solenoid

- Injection of liquid metal target into Solenoid hampered by forces, friction, pressure:
  \[
  \frac{dB}{ds}, \; B_{(s)}, \; v_{(s)}, \; \text{target diameter, electrical conductivity of liquid.}
  \]

- Let drop « Target-lets » from above into the center of the Solenoid

- Supply « shower head » with pipes of large cross-section to keep \( v \) low

  In supply pipe: \( v_{\text{shower}} \approx \frac{v_{\text{axial injection}}}{\text{no. of shower holes}} \)

  At shower exit \( v \downarrow \approx \varnothing r_{\text{Target}} \times f \approx 0.2 - 0.5 \text{ m/s} \)

  Axial injection \( V \approx L_{\text{Target}} \times f \approx 8 - 15 \text{ m/s} \)

  \[
  \begin{align*}
  t &= 0 \quad \text{R} = 10 \text{ cm} \quad v = 0 \\
  t_1 &= 140 \text{ ms} \quad \text{R} = 3 \text{ cm} \quad v \approx 1.4 \text{ m/s} \\
  t_2 &= (140 + 20) \text{ ms} \quad \text{R} = 0 \text{ cm} \quad v \approx 1.4 \text{ m/s}
  \end{align*}
  \]
LIQUID TARGET RADIAL INJECTION INTO SOLENOID

BEAM

P. SIEVERS 23/05/2000
- Timing at shower head with 50 Hz, have to release every 20 ms a target-let. Precision ±1 ms: Target out of position by ±1.4 mm

- Electr. Polarization of target-lets?
  \[ E = v \times B \approx 1.4 \text{ m/s} \times 20 \text{ T} = 28 \text{ V/m} \]

- If target No. n destroys target No. n-1, increase distance between them, increase velocity, drop height, pressure.

- Stored cinetic energy \( E_c \) in Target (fast heating)

\[
\frac{dE_c}{dV} = \frac{(\alpha_v \Delta T)^2}{4 \kappa} \\
\frac{E_c}{E_T} \approx \frac{\alpha_v^2}{4 c_v^2 \kappa \rho} \frac{dE_T}{dm}
\]

Hg Example:
\[
\frac{dE_T}{dm} = 30 \times 10^3 \text{ J/kg}
\]

\[
\alpha_v = 18.1 \times 10^{-5} \text{ K}^{-1} \quad c_v = 140 \text{ J/kg K}
\]
\[
\kappa = 0.45 \times 10^{-10} \text{ m}^2/\text{N} \quad \rho = 13.5 \times 10^3 \text{ kg/m}^3
\]

\[
\frac{E_c}{E_T} \approx 0.7 \times 10^{-6} \times \frac{dE_T}{dm} \approx 2.1\% \quad \frac{dE_c}{dm} \approx 0.6 \text{ kJ/kg}
\]
With this accelerate Hg to
\[ v \approx 35 \text{ m/s or } 126 \text{ km/h} ! \]

or shoot it up to a height of
\[ h = 60 \text{ m} ! \]

Average kinetic power to be absorbed inside Solenoid:
\[ \bar{P}_c \approx 21 \text{kW} ! \]

Does viscosity (or magnetic field ?) prevent or reduce explosion ?
Conclusion: Wheel as such do able
Radiation resistance inside target cave is manageable (Hot Lab, Repair, maintenance).
Radioactivity confined to, but also accumulated in solid parts (except water) of wheel.
Target resistance to be verified by beam tests.

Target wheel with forward production:
Target separated from collector. Two separated problems!
The collector, pulsed at 50 Hz ?, and its efficiency may pose a limit.

Target wheel through solenoid: R+D required for target plus cooling system together with Solenoid. Superconducting Solenoid with slots? Target wheel not cheap, must make it modular to maintain it and recuperate expensive parts. Production and collection efficiency to be checked.