A Coupled Level Set/Volume-of-Fluid (CLSVOF) Method for Target Flow Simulation

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Outline

• What is CLSVOF?
• Why CLSVOF?
• How to implement CLSVOF?
What is CLSVOF?

<table>
<thead>
<tr>
<th>Volume of Fluid (VOF)</th>
<th>Level Set (LS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Volumetric phase fraction $F$</td>
<td>• Level-Set function $\Phi$</td>
</tr>
<tr>
<td>Phase 1  $F=1$</td>
<td>Phase 1  $\Phi&gt;0$</td>
</tr>
<tr>
<td>Phase 2  $F=0$</td>
<td>Phase 2  $\Phi&lt;0$</td>
</tr>
<tr>
<td>Interface  $0&lt;F&lt;1$</td>
<td>Interface  $\Phi=0$</td>
</tr>
<tr>
<td>• Transport of $F$:</td>
<td>• Transport of $F$:</td>
</tr>
<tr>
<td>$(\rho F)_t + \nabla \cdot (\rho \tilde{U} F) = 0$</td>
<td>$(\rho \phi)_t + \nabla \cdot (\rho \tilde{U} \phi) = 0$</td>
</tr>
<tr>
<td>• Mass-conservative</td>
<td>• Robust geometric information (normals and curvatures); automatic handling of topological changes (merging and pinching);</td>
</tr>
<tr>
<td>• Diffusion of the interface</td>
<td>• Not mass-conservative</td>
</tr>
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</table>

**Volume of Fluid**

**Level Set**

**CLSVOF**
Why CLSVOF?

Sussman (2000)  
Menard (2007)

Time loop complete?  
End

Initialization  
Start time loop  
Reconstruction of $F$ from $\phi$

Advection of $F$ and $\phi$

Reconstruction of $\phi$ from $F$

Reinitialisation of $\phi$

Calculation of $P$ and $u$

Mass Conservation

Why CLSVOF?

Development of the liquid jet (time step is 2.5 μm) (Menard, 2007)

<table>
<thead>
<tr>
<th>Jet characteristics</th>
<th></th>
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<tbody>
<tr>
<td><strong>Diameter, D (μm)</strong></td>
<td><strong>Velocity (m s⁻¹)</strong></td>
<td><strong>Turbulent intensity</strong></td>
<td><strong>Turbulent length scale</strong></td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>$u'/U_{\text{liq}} = 0.05$</td>
<td>0.1 D</td>
</tr>
<tr>
<td><strong>Phase</strong></td>
<td><strong>Density (kg m⁻³)</strong></td>
<td><strong>Viscosity (kg m⁻¹ s⁻¹)</strong></td>
<td><strong>Surface tension (N m⁻¹)</strong></td>
</tr>
<tr>
<td>Liquid</td>
<td>696</td>
<td>$1.2 \times 10^{-3}$</td>
<td>0.06</td>
</tr>
<tr>
<td>Gas</td>
<td>25</td>
<td>$1 \times 10^{-5}$</td>
<td></td>
</tr>
</tbody>
</table>
Why CLSVOF?

Liquid jet surface and break-up near the jet nozzle

Liquid parcels
How to implement CLSVOF?

- Couple LS with VOF within the CFD code FLUENT by implementing user defined functions (UDF)
- UDF
  - User written program that can be linked with FLUENT at run-time
  - Programmed in C and FLUENT defined macros
  - User-defined scalar (UDS) transport modeling customize FLUENT for level set equation
How to implement CLSVOF?
How to implement CLSVOF?

Initialization → Start time loop → Advection of $\phi$

Calculate $\kappa, \bar{n}$ from $\phi$

Get $u, P$, surface tension from N-S

Advection of $F$ and $\phi$

Couple LS and VOF $H(\phi) = F$

Reinitialisation of $\phi$

Time loop complete? → End

How to implement CLSVOF?

• B.A. Nichita’s test case
  – A bubble rising in a viscous fluid due to gravity

![Level set contour (red) and volume-of-fluid contour (green) without coupling between LS and VOF (with large loss of mass).](t = 0.2s)

![Level set contour (red) and volume-of-fluid contour (green) after solving the coupling equation between LS and VOF.](t = 0.2s)
How to implement CLSVOF?

- Setup UDS for LS in FLUENT

Scalar $\phi$ Transport Equation

$$\frac{\partial \rho \phi}{\partial t} + \nabla \cdot (\rho \tilde{U} \phi) = 0$$

- Unsteady term
  $$\frac{\partial \rho \phi}{\partial t}$$
- Convection term
  $$\nabla \cdot (\rho \tilde{U} \phi)$$
- Diffusive term
  $$0$$
- Source term
  $$0$$

Additional term appear for turbulent flow such as

$$- \rho u' \phi' = \Gamma \frac{\partial \phi}{\partial x_j}$$
How to implement CLSVOF?

- Setup UDS for LS in FLUENT
  - Set number of UDS
  - Set UDS terms (Appendix A)
    - DEFINE_UDS_UNSTEADY
    - Get unsteady term for scalar equation
    - DEFINE_UDS_FLUX
    - Returns user specified flux
    - DEFINE_DIFFUSIVITY
    - Returns user diffusion coefficient ($\Gamma$)
    - DEFINE_SOURCE
  - Set UDS boundary conditions
    - Constant
    - UDF: DEFINE_PROFILE
Appendix Equations

- **Incompressible two-phase flow**

\[ \nabla \cdot U = 0 \]

\[ U_t + U \cdot \nabla U = -\frac{\nabla p}{\rho(\phi)} + \frac{1}{\rho(\phi)} \nabla \cdot (2\mu(\phi)D) - \frac{1}{\rho(\phi)} \gamma \kappa(\phi) \nabla H(\phi) + F \]

\[ \phi_t + U \cdot \nabla \phi = 0 \]

\[ F_t + \nabla \cdot (UF) = 0 \]

Density \( \rho(\Phi) \), viscosity \( \mu(\Phi) \), and curvature \( \kappa(\Phi) \) are written as,

\[ \rho(\phi) = \rho_g (1 - H(\phi)) + \rho_i H(\phi) \]

\[ \mu(\phi) = \mu_g (1 - H(\phi)) + \mu_i H(\phi) \]

\[ \kappa(\phi) = \nabla \cdot \frac{\nabla \phi}{|\nabla \phi|} \]

\( D \) is defined as the rate of deformation tensor

\[ D = (\nabla U) + (\nabla U)^T \]
Appendix Equations

• Incompressible two-phase flow

The surface tension force is

\[
\frac{1}{\rho(\phi)} \gamma \kappa(\phi) \nabla H(\phi)
\]

where \( H \) is the Heaviside function,\( \)

\[
H(\phi) = \begin{cases} 
1 & \text{if } \phi > 0 \\
0 & \text{otherwise.}
\end{cases}
\]

\( F \) will be initialized in each computational cell \( \Omega_{ij} \)

\[
F_{ij} = \frac{1}{\Delta r \Delta z} \int_{\Omega_j} H(\phi(r,z,0)) r dr dz
\]

where \( \Omega_{ij} \) is

\[
\Omega_{ij} = (r,z) \big| r_i \leq r \leq r_{i+1} \text{ and } z_j \leq z \leq z_{j+1}
\]
Appendix Equations

- Re-Initialization
  
  Reinitialize $\phi$

  $$\int_V \frac{\partial \phi}{\partial \tau} + \int_V w \cdot \nabla \phi = \int_V \text{sign } \phi_0$$

  where $w$ is the characteristic velocity pointing outward from the free surface

  $$w = \text{sign } \phi_0 \frac{\nabla \phi}{|\nabla \phi|}$$

  The sign function is

  $$\text{sign}_\epsilon(\phi_0) = 2[H_\epsilon(\phi_0) - 1/2]$$