Space Charge in Target Region

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Space charge effects

- Consider space charge effect in target region
  - Look at linear space charge effect of pure pi+/- beam
  - Look at transverse kick from proton beam

- I'm no expert in space charge - a learning experience for me!
  - I think I got the physics right
  - ...

- Context is “Checking that we don't have to worry about this stuff”
  - I'm not expecting a problem
Quote 2d envelope evolution equation in the presence of space charge (from S.Y. Lee):

\[ x'' + K_x x - 2 \frac{K_{sc}}{(x+y)} - \frac{\varepsilon_x^2}{x^3} = 0 \]
\[ y'' + K_y y - 2 \frac{K_{sc}}{(x+y)} - \frac{\varepsilon_y^2}{y^3} = 0 \]

- \( x \) is the width in \( x \) of the beam, \( y \) is the width in \( y \) of the beam
- Prime denotes differentiation wrt \( z \)
- \( K_x, K_y \) are transverse lattice focussing strength
- \( K_{sc} \) is a constant \~\ number of muons in the beam

In solenoids, assume cylindrical symmetry

- \( K_x = K_y = \frac{B^2}{2p} \)
- \( \varepsilon_x = \varepsilon_y \)
- \( x = y \)

Such that we get

\[ x'' + K_x x - \frac{K_{sc}}{x} - \frac{\varepsilon_x^2}{x^3} = 0 \]
Linear SC in Target Region

- Evolve linear envelope equation from the target
  - Start with beam envelope radius ~ 1 cm
    - Proton spot size
  - 8 GeV beam @ 50 Hz => 6e13 protons per pulse
  - Scale to 1e5 proton MARS file from Harold
    - 5.5e4 pi+ and 5.0e4 pi-
    - => 3.3e13 pi+ per pulse and 3e13 pi-
  - Assume bunch is 30 cm long => 1e12 pi+/mm
Caveats

- But overlapping pi+ pi- beams which cancel each other's charge
  - 10% more pions => net beam has 1e11 pi+
- But significant scraping after the taper which I have ignored
- Envelope equation should not have such big beta oscillations
- Conclusion: likely no effect, may want to check in a proper space charge code

![Graph showing 1e11 pi/mm oscillation pattern]
Space Charge in Target Region

- Near to the target we have an intense proton beam moving right next to our pions
  - It is known that space charge does have an effect on the proton beam
  - What is the effect on pions?
- Assume simple, worst case scenario that proton bunch are a spherical charge with radius 1 cm and constant density inside proton bunch
- Then I think electric field goes like (in p rest frame)
  - \[ V = \frac{q}{4 \pi \varepsilon_0} r \quad \text{......} \quad r > r_{\text{min}} \]
  - \[ V = \frac{q}{4 \pi \varepsilon_0} r_{\text{min}} \quad \text{......} \quad r < r_{\text{min}} \]
- In lab frame \((A,V)\) transforms like a 4-vector to calculate the actual field
  - Jargon: retarded potential
No fields

- In the absence of fields, quite a strong effect
  - For 1e14 protons at 5 GeV, 30 cm (1 ns) long bunch, 1 cm spot size
    - Worst case, probably impractical for proton beam
  - Fire pions in momentum range 0.1 - 5 GeV
    - Do not consider pion self-charge for space charge calculation
Fields - pi+

- Use the FS2 field taper ... basically no effect
Conclusions

- This is a very basic study
  - *Maybe* see a slight effect on pion beam optics from self-charge but probably not
    - May want to check with proper space charge routines
  - No significant effect from proton beam on pion beam