Optimized Target Parameters and Meson Production by IDS120h with Focused Gaussian Beam and Fixed Emittance

X. Ding, UCLA

Target Studies
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Relative normalized meson production is 0.84 of max at $\beta^*$ of 0.3 m for $\varepsilon_x = \varepsilon_y = 5 \ \mu m$.

For low $\beta^*$ (tight focus) the beam is large at the beginning and end of the interaction region, and becomes larger than the target there.
Non-Linear Fit (Growth/sigmoidal, Hill)

\[ Y = \frac{N}{1 + K_2/\beta^{-2}} \]

\[ N = 1.018 \]
\[ \sqrt{K_2} = 0.1368 \]

Linear emittance is 4.9 μm with beam radius of 0.1212 cm and \( \beta^* \) of 0.3 m.
Gaussian distribution  
(Probability density)

• In two dimensional phase space \((u,v)\):

\[
\begin{align*}
    w(u,v) &= \frac{1}{2\pi\sigma^2} \exp\left(-\frac{u^2 + v^2}{2\sigma^2}\right)
\end{align*}
\]

where \(u\)-transverse coordinate (either \(x\) or \(y\)),
\(v = \alpha u + \beta u'\)

\(\alpha, \beta\) are the Courant-Snyder parameters at the given point along the reference trajectory.

In polar coordinates \((r, \theta)\):
\[
\begin{align*}
    u &= r \cos \theta \\
    v &= r \sin \theta \\
    u' &= \frac{(v - \alpha u)}{\beta} = \frac{(r \sin \theta - \alpha u)}{\beta}
\end{align*}
\]
Distribution function method

\[ \theta = 2\pi \xi_1, \quad \theta \in [0, 2\pi] \]
\[ r = \sqrt{-2\sigma^2 \ln \xi_2}, \quad r \in [0, \infty] \]

Random number generator:

\[ \Theta = 2\pi \cdot \text{rndm}(-1) \]
\[ R = \sqrt{\text{sqrt}(-2\log(\text{rndm}(-1))) \cdot \sigma} \]
Gaussian distribution (Fraction of particles)

• The fraction of particles that have their motion contained in a circle of radius “a” (emittance $\varepsilon = \pi \frac{a^2}{\beta}$) is

$$F_{Gauss} = \int_{0}^{a} \frac{1}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} r dr = 1 - e^{-\frac{a^2}{2\sigma^2}}$$
<table>
<thead>
<tr>
<th>$k=a/\sigma$</th>
<th>$\varepsilon_{K\sigma}$</th>
<th>$F_{\text{Gauss}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\pi (\sigma)^2/\beta$</td>
<td>39.5%</td>
</tr>
<tr>
<td>2</td>
<td>$\pi (2\sigma)^2/\beta$</td>
<td>86.4%</td>
</tr>
<tr>
<td>2.5</td>
<td>$\pi (2.5\sigma)^2/\beta$ or $\sim 6\pi \sigma^2/\beta$</td>
<td>95.6%</td>
</tr>
</tbody>
</table>

Normalized emittance: $(\beta \gamma)\varepsilon_{K\sigma}$
Focused beam

• Intersection point (z=-37.5 cm):
  \( \alpha^* = 0, \beta^*, \sigma^* \)

• Launching point (z=-200 cm):
  \[ L = 200 - 37.5 = 162.5 \text{ cm} \]
  \[ \alpha = \frac{L}{\beta^*} \]
  \[ \beta = \beta^* + \frac{L^2}{\beta^*} \]
  \[ \sigma = \sigma^* \sqrt{1 + \frac{L^2}{\beta^*^2}} \]

These relations strictly true only for zero magnetic field.
Setting of simple Gaussian distribution

• INIT card in MARS.INP (MARS code)

\[\text{INIT XINI YINI ZINI DXIN DYIN DZIN WINIT}\]

\[\begin{align*}
XINI &= x0 \\
YINI &= y0 \\
ZINI &= z0
\end{align*}\]

\[\begin{align*}
DXIN &= dcx0 \\
DYIN &= dcy0 \\
DZIN &= dcz0 = \sqrt{1-dcx0^2-dcy0^2}
\end{align*}\]

(Initial starting point and direction cosines of the incident beam)
Setting with focused beam trajectories

- Modeled by the user subroutine BEG1 in m1510.f of MARS code

\[ x_v \text{ or } x_h \text{ (transverse coordinate: } u) \]

\[ x'_v \text{ or } x'_h \text{ (deflection angle: } u') \]

\[ \begin{align*}
XINI &= x_0 + x_v \\
YINI &= y_0 + x_h \\
ZINI &= z \\
DXIN &= dcx_0 + x'_v \\
DYIN &= dcy_0 + x'_h \\
DZIN &= \sqrt{1 - DXIN^2 - DYIN^2}
\end{align*} \]
Optimization of target parameters

• Fixed beam emittance \( (\varepsilon_{K\sigma}) \) to \( \pi (\sigma)^2/\beta \)

• Optimization method in each cycle
  (Vary beam radius or beam radius \( \sigma^* \), while vary the \( \beta^* \) at the same time to fix the beam emittance; Vary beam/jet crossing angle; Rotate beam and jet at the same time)

We also optimized the beam radius and target radius separately (not fixed to each other).
Effect of Solenoid Field

[Backtrack particles from $z = -37.5$ cm to $z = -200$ cm.]

(Could then do calculation of $\alpha$, $\beta$, $\sigma$ at $z = -200$ cm, but didn’t)
Effect of Solenoid Field

5/29/12
Courant-Snyder Invariant
## Optimized Target Parameters and Meson Productions at 8 GeV

(Linear emittance is fixed to be 4.9 μm )

<table>
<thead>
<tr>
<th></th>
<th>Radius (cm)</th>
<th>Beam/jet crossing angle (mrad)</th>
<th>Beam angle/Jet angle (mrad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>0.404 (target)</td>
<td>20.6</td>
<td>117/137.6</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; Run</td>
<td>0.525 (target)</td>
<td>25</td>
<td>120/145</td>
</tr>
<tr>
<td>Old 2&lt;sup&gt;nd&lt;/sup&gt; Run (vary target radius and beam radius is fixed to be 0.3 of target radius)</td>
<td>0.544 (target)</td>
<td>25.4</td>
<td>120/145.4</td>
</tr>
<tr>
<td>New 2&lt;sup&gt;nd&lt;/sup&gt; Run (vary beam radius with fixed target radius of 0.525 cm; vary target radius with fixed beam radius of 0.15 cm.)</td>
<td>Beam radius: 0.15</td>
<td>26.5</td>
<td>127/153.5</td>
</tr>
<tr>
<td></td>
<td>Target radius: 0.548</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Optimize beam radius and target radius separately

We found almost no improvement in optimized meson production if the beam radius is not fixed at 30% of target radius and optimized separately!
Optimized Meson Productions at 8 GeV

(Linear emittance is fixed to be 4.9 μm)

<table>
<thead>
<tr>
<th>Gaussian Distribution</th>
<th>Meson Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple (4.04mm/20.6mrad/117mrad)</td>
<td>32563</td>
</tr>
<tr>
<td>Focused beam (4.04mm/20.6mrad/117mrad)</td>
<td>27489</td>
</tr>
<tr>
<td></td>
<td>(-15.6% less than Simple)</td>
</tr>
<tr>
<td>Focused beam with fixed Emittance at 4.9 μm (5.44mm/25.4mrad/120mrad)</td>
<td>30025</td>
</tr>
<tr>
<td></td>
<td>(-8.9% less than Simple)</td>
</tr>
<tr>
<td></td>
<td>(8.4% more than Focused beam)</td>
</tr>
</tbody>
</table>