

Floating Wire Simulation of the Trajectory of a Charged Particle in a Magnetic Field

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1 Problem

Show that the trajectory of a charged particle in a magnetic field can be duplicated by that of a current-carrying wire held at rest under constant tension (by fixtures outside the field region). Deduce the current I required in a wire of tension T to match the trajectory of an proton of momentum P .

2 Solution

The equation of motion of a (relativistic) particle of charge e , mass m and velocity \mathbf{v} in a magnetic field \mathbf{B} is (in MKSA units)

$$\frac{d\mathbf{P}}{dt} = e\mathbf{v} \times \mathbf{B}. \quad (1)$$

An increment ds of arc length along the particle's trajectory can be written

$$ds = \mathbf{v} dt. \quad (2)$$

Using this to replace \mathbf{v} in eq. (1), the particle's trajectory can be described as

$$d\mathbf{P} = e ds \times \mathbf{B}. \quad (3)$$

The momentum \mathbf{P} is along the trajectory, so we can write

$$\mathbf{P} = P\hat{\mathbf{s}}, \quad \text{and} \quad d\mathbf{P} = P d\hat{\mathbf{s}}, \quad (4)$$

noting that the magnetic field changes the direction, but not the magnitude, of the momentum. Hence the equation of the trajectory is

$$d\hat{\mathbf{s}} = \frac{e}{P} ds \times \mathbf{B}. \quad (5)$$

The equation for static equilibrium of a current-carrying wire under tension T in the same magnetic field is

$$\sum \mathbf{F} = 0 = T(\mathbf{s} + d\hat{\mathbf{s}}) - T\hat{\mathbf{s}} + Ids \times \mathbf{B}, \quad (6)$$

or

$$d\hat{\mathbf{s}} = -\frac{I}{T} ds \times \mathbf{B}. \quad (7)$$

The trajectory of the “floating” wire can be the same as that of the charged particle when

$$I[\text{A}] = -\frac{eT[\text{N}]}{P[\text{kg}\cdot\text{m}/\text{s}]} . \quad (8)$$

We express this relation in practical units by noting that

$$T[\text{N}] = 0.0098 T[\text{gm}], \quad (9)$$

when the tension is maintained by a weight T in grams attached to one end of the wire over a pulley, and that the momentum of the proton is

$$P[\text{kg}\cdot\text{m}/\text{s}] = \frac{10^6 e}{c} P[\text{MeV}/c] = \frac{e}{300} P[\text{MeV}/c]. \quad (10)$$

Hence,

$$I[\text{A}] = -\frac{2.94 T[\text{gm}]}{P[\text{MeV}/c]} . \quad (11)$$

An application of this principle is described in C.Y. Prescott, S.U. Cheng and K.T. McDonald, *Wire Orbit Ray Tracing of Magnets Using Magnetostrictive Wire Chamber Techniques*, Nucl. Instr. and Meth. **76**, 173 (1969),

http://puhep1.princeton.edu/~mcdonald/examples/detectors/prescott_nim_76_173_69.pdf