1 Problem

Show that the trajectory of a charged particle in a magnetic field can be duplicated by that of a current-carrying wire held at rest under constant tension (by fixtures outside the field region. Deduce the current $I$ required in a wire of tension $T$ to match the trajectory of an proton of momentum $P$.

2 Solution

The equation of motion of a (relativistic) particle of charge $e$, mass $m$ and velocity $v$ in a magnetic field $B$ is (in MKSA units)

$$\frac{dP}{dt} = ev \times B. \quad (1)$$

An increment $ds$ of arc length along the particle’s trajectory can be written

$$ds = vt. \quad (2)$$

Using this to replace $v$ in eq. (1), the particle’s trajectory can be described as

$$dP = e \, ds \times B. \quad (3)$$

The momentum $P$ is along the trajectory, so we can write

$$P = Ps, \quad \text{and} \quad dP = Ps \times B, \quad (4)$$

noting that the magnetic field changes the direction, but not the magnitude, of the momentum. Hence the equation of the trajectory is

$$ds = \frac{e}{P} ds \times B. \quad (5)$$

The equation for static equilibrium of a current-carrying wire under tension $T$ in the same magnetic field is

$$\sum F = 0 = T(s + ds) - Tds + I ds \times B, \quad (6)$$

or

$$ds = -I/T ds \times B. \quad (7)$$
The trajectory of the “floating” wire can be the same as that of the charged particle when
\[
I[A] = -\frac{eT[N]}{P[\text{kg-m/s}]}.
\] (8)

We express this relation in practical units by noting that
\[
T[N] = 0.0098 \ T[\text{gm}],
\] (9)

when the tension is maintained by a weight \( T \) in grams attached to one end of the wire over a pulley, and that the momentum of the proton is
\[
P[\text{kg-m/s}] = \frac{10^6 e}{c} P[\text{MeV/c}] = \frac{e}{300} P[\text{MeV/c}].
\] (10)

Hence,
\[
I[A] = -\frac{2.94 T[\text{gm}]}{P[\text{MeV/c}]}.
\] (11)
