If a Pig Had Wings...
Kirk T. McDonald
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544
(Feb. 11, 1999)

1 Problem

If a pig had wings, how far could it fly?
An approximate analysis will suffice.

2 Solution

If a pig had wings, these must provide lift equal to the weight of the pig:

\[ \text{lift} = m_{\text{pig}} g, \]  

(1)

where \( g \approx 10 \text{ m/s}^2 \) is the acceleration due to gravity. Now,

\[ \text{lift} = k \text{ drag}, \]  

(2)

with the lift-to-drag ratio \( k \) near 10 for a well-designed pig. Using dimensional analysis, we estimate the drag force as

\[ \text{drag} \propto \text{area}_{\text{wing}} \rho_{\text{air}} v_{\text{pig}}^2, \]  

(3)

where \( \rho_{\text{air}} \) is the mass density of air, and \( v \) is the speed of the pig in flight. Combining (1-3),

\[ \text{area}_{\text{wing}} \approx \frac{m_{\text{pig}} g}{k \rho_{\text{air}} v_{\text{pig}}^2}. \]  

(4)

As \( v_{\text{pig}} \to 0 \), \( \text{area}_{\text{wing}} \to \infty \).

So, we must consider how fast the pig can fly. The faster it flies, the more energy it must provide.

\[ \text{Power} = F v = \text{drag} v_{\text{pig}} \approx \text{area}_{\text{wing}} \rho_{\text{air}} v_{\text{pig}}^3, \]  

(5)

so

\[ \text{area}_{\text{wing}} \approx \frac{1 \text{ pigpower}}{\rho_{\text{air}} v_{\text{pig}}^3}. \]  

(6)

Equating the two expressions, (4) and (6), for the wing area, we find

\[ v_{\text{pig}} \approx \frac{k(1 \text{ pigpower})}{m_{\text{pig}} g}. \]  

(7)

Suppose \( m_{\text{pig}} = 200 \text{ kg}, k = 10, \) and 1 pigpower = 2/7.6 horsepower = 200 W. Then,

\[ v_{\text{pig}} \approx \frac{10 \cdot 200}{200 \cdot 10} = 1 \text{ m/s}. \]  

(8)
To estimate the wing area, we note that $\rho_{\text{air}} \approx 0.001 \rho_{\text{water}} = 1 \text{ kg/m}^3$. Then, from (6),

$$\text{area}_{\text{wing}} \approx \frac{200}{1 \cdot (1)^3} = 200 \text{ m}^2.$$  \hspace{1cm} (9)

The wings must, of course, be massless for the above analysis to hold.

How far can a pig fly?

Suppose the pig can dedicate his/her breakfast calories to flying. Say 25,000 calories = $4,000 \cdot 25,000 = 10^8$ J. The pig uses up this energy at the rate of 200 W, so it could fly for time $t = 10^8/200 = 5 \times 10^5$ s = 5 days. However, this seems too long for a pig to exert maximum power. I downrate the pig’s flying time to 3 hours $\approx 10,000$ s. Then, the flight distance is

$$d = v_{\text{pig}} t = 1 \cdot 10,000 = 10,000 \text{ m} = 10 \text{ km}.$$  \hspace{1cm} (10)

If the lift-to-drag ratio $k$ had been estimated as 1 rather than 10, then $v_{\text{pig}} = 0.1 \text{ m/s}$, and $d = 1 \text{ km}$.

It has been demonstrated that humans can power a large wingspan aircraft, such as Paul MacReady’s Gossamer Condor [1], at low speeds for distances of a few tens of kilometers.

3 Acknowledgment

This problem is a variant on J.A. Wheeler’s classic exam question: How far can a goose fly?

References