ELECTROMAGNETIC INTERACTIONS

This interaction is considered to be well understood. As such, it is seldom studied for its own sake anymore, except for occasional tests of QED (Quantum Electrodynamics) at ever smaller distances. Rather, this interaction can provide probes of less understood processes.

We will use E&M to sketch some features of the Feynman diagram approach. This is a graphic way of representing terms in a perturbation expansion of the interaction.

Electromagnetic interactions involve the emission, absorption, and exchange of photons. The fewer the number of photons, the simpler the interaction (at lower order in perturbation).

One of the conceptually simplest high-energy reactions is that mentioned in Lecture 1:

\[ e^+ e^- \rightarrow \mu^+ \mu^- \]

The electron-positron pair can annihilate producing a single photon, which in turn produces a \( \mu^+ \mu^- \) pair.

The Feynman diagram:

![Feynman diagram](image)

We seldom explicitly mention the axes in a Feynman diagram, but they may be thought of as space and time as shown.

The diagram is to help us evaluate the matrix element

\[ M_{fi} = \langle \text{final} | \hat{H}_{\text{interaction}} | \text{initial} \rangle \].

As discussed in Lecture 1, the calculation is done in momentum space, consistent with the experimental situation of scattering experiments in which directions and energies are observed well, and positions known poorly.

The "Feynman Rules" allow us to read off the matrix element from the diagram. This task consists of identifying the vertex factors and the internal propagators.
A vertex in QED is the emission or absorption of a photon out
\[ a \gamma^\mu = \text{vertex factor} \]

The vertex factor is the product of \( e = \text{electric coupling} \cdot \text{charge} \) and the spin factor \( \gamma^\mu \) - a 4-vector comprised of Dirac matrices.

The spin factor \( \gamma^\mu \) is to be sandwiched between the in and out state vectors - Dirac spinors, to make up a piece of the matrix element:
\[ \langle \text{out} | \gamma^\mu | \text{in} \rangle \]

This matrix element may be thought of as describing the polarization of the photon - a 4-vector in relativistic quantum mechanics.

The propagator is a measure of the energy of the interaction, or rather its Fourier transform into momentum space.

In our example a photon is exchanged, or propagated between the initial and final states.

In Compton scattering, we say that the electron is propagated.

\[ \gamma + e \rightarrow \gamma + e \]

Recall Yukawa's idea that the field due to the exchange (propagation) of a quantum of mass \( m \) is described by the potential
\[ - \frac{e^{-\gamma m}}{\gamma} \]
\( (\hbar = c = 1) \)

The Fourier transform of this is
\[ \int e^{i\bar{\gamma} \cdot \hat{\gamma}} \frac{e^{-\gamma m}}{\gamma} d\gamma \sim \frac{1}{q^2 + m^2} \]

Where \( \hat{\gamma} = 3\text{-momentum of the propagated object.} \)

This non-relativistic argument gives a sense of the relativistic result:
PHYSICS LECTURE 3

PHOTON PROPAGATOR \( \sim \frac{1}{q^2} \)  
where \( q^2 = q \cdot q = q_0^2 - \vec{q}^2 \)
\( q_0 = 4 - \text{momentum} \)

SPIN-ZERO PROPAGATOR \( \sim \frac{1}{q_0^2 - M^2} = \frac{1}{q_0^2 - (q^2 + m^2)} \)  
EXAMPLE: PI- MESON

SPIN 1/2 PROPAGATOR \( \sim \frac{q \cdot p + m}{q^2 - m^2} \)  
EXAMPLE: ELECTRON

COMPLETE CALCULATIONS OF THE SQUARE OF THE MATRIX ELEMENT ARE "STRAIGHTFORWARD BUT LENGTHY." HERE WE INDICATE HOW THE BASIC NOTIONS OF THE FERMIAN METHOD CAN BE USED TO MAKE QUICK ESTIMATES OF CROSS-SECTIONS.

AN IMPORTANT OBSERVATION IS THAT FOR EACH PHOTON VERTEX WE GET ONE FACTOR OF THE CHARGE, \( e \), IN THE MATRIX ELEMENT.

Thus \( \langle e^+ e^- \rightarrow \mu^+ \mu^- \rangle \sim |M|^2 \sim e^4 \times N^2 \)  
where \( N \approx \frac{\alpha}{\sqrt{\epsilon}} \)

(For the 1-photon exchange diagram on p. 27)

REMEMBER THAT \( \alpha \) HAS DIMENSIONS OF \( \text{(LENGTH)}^2 \sim \frac{1}{\text{(ENERGY)}^2} \)

So if \( E_{\text{CM}} \gg M_\mu \) and \( M_\mu \), we expect

\[ \langle e^+ e^- \rightarrow \mu^+ \mu^- \rangle \sim \frac{\alpha^2}{E_{\text{CM}}} \]  
without any detailed calculation

With a lot of work we find \( \alpha = \frac{4\pi}{3} \frac{\alpha^2}{E_{\text{CM}}} \) 'exactly' in the 1-photon approximation

HIGHER ORDER TERMS INVOLVE MORE PHOTONS. 2ND ORDER DIAGRAMS ARE LIKE

BREMSSTRAHLUNG

Z PHOTON EXCHANGE

VERTEX CORRECTION

ELECTRON PROPAGATOR CORRECTION

PHOTON PROPAGATOR CORRECTION
OF THESE DIAGRAMS, THE VERTEX AND PROPAGATOR CORRECTIONS LEAD TO THE FAMOUS INFINITIES OF QED. COLLECTING THE INFINITE NUMBER OF SUCH HIGHER ORDER DIAGRAMS TOGETHER, THEIR NET EFFECT IS TO CHANGE THE APPARENT CHARGE AND MASS OF THE VARIOUS PARTICLES. BUT WE CAN ONLY MEASURE THE 'APPARENT' QUANTITIES, AND CANNOT DIRECTLY ACCESS THE 'BARE' CHARGE OR MASS. AS THESE CORRECTIONS ARE ALWAYS PRESENT, AND ALWAYS THE SAME SIZE, WE CAN IN EFFECT IGNORE THEM, AND DEAL ONLY WITH THE APPARENT OR CORRECTED QUANTITIES.

THIS IDEA IS CALLED RENORMALISATION. IT SEEMS QUITE REASONABLE, EXCEPT FOR THE FACT THAT THE SIZE OF THE CORRECTION IS INFINITE!

ENERGY AND MOMENTUM CONSERVATION

IN ALL FUNDAMENTAL PROCESSES OBSERVED SO FAR, ENERGY AND MOMENTUM ARE CONSERVED. AS SUCH THE ENERGY-MOMENTUM 4-VECTOR OF THE INTERNALLY PROPAGATED PARTICLE MAY APPEAR 1.1. AT FIRST.

EXAMPLE: $e^+ e^- \rightarrow \gamma \gamma^*$ AT A CONTINUOUS BEAM FACILITY

AS THE ELECTRON AND POSITRON COLLIDE HEAD ON, THEIR TOTAL MOMENTUM IS ZERO. THE THE 4-VECTOR OF THE PHOTON IS

$$ q_{\gamma} = q_{e^+} + q_{e^-} = (E_{\gamma}+E_{\gamma}, \vec{0}) = (E_{\gamma}, \vec{0}) $$

THE 'MASS' OF THE PHOTON IS

$$ m^2 = q_{\gamma} \cdot q_{\gamma} = q^2 = E_{\gamma}^2 = 0 $$

EVEN IN THE COMPTON SCATTERING DIAGRAM

THE INTERNAL ELECTRON HAS A 4-VECTOR WITH MASS $\neq 0$!

THIS STATE OF AFFAIRS CANNOT LAST, THE NEUMANNERGULL ALLOWS IT FOR A SHORT WHILE: $\Delta t \sim 1/\Delta E$

WE CALL THESE INTERNAL STATES 'VIRTUAL PARTICLES', AND SOMETIMES SAY THAT THEY ARE 'OFF THE MASS SHELL'.

FEYNMAN'S VIEW OF ANTI-PARTICLE'S

ANOTHER POSSIBLE DIAGRAM FOR COMPTON SCATTERING IS

INITIAL \[ \begin{array}{c}
\gamma \\
\rightarrow
\end{array} \]

FINAL \[ \begin{array}{c}
\gamma \\
\rightarrow
\end{array} \]

IN THIS CASE THE INCIDENT PHOTON CREATES AN ELECTRON-POSITRON PAIR, THEN THE POSITRON ANNIHILATES THE INCIDENT ELECTRON TO CREATE THE FINAL PHOTON.
This looks very different, but is topologically equivalent to the diagram

In which the electron radiates the final photon slightly before it absorbs the initial one.

This led to the interpretation that a positron is just an electron moving backwards in time! In the above diagrams, antiparticles have been indicated with arrows pointed opposite to their direction of motion. Then the arrowheads flow nicely around the vertices.

**Compton Scattering Cross Section**

As for $e^+e^- \rightarrow \gamma^+\gamma^-$, the Compton reaction $\gamma + e \rightarrow \gamma + e$ has 2 photon vertices. So again we are quickly led to the estimate

$$\langle \sigma \rangle \sim \frac{\alpha^2}{E_{cm}} \sim \frac{\alpha^2}{E_{cm}}$$

For low energy photons incident on an electron at rest ($E_\gamma < \epsilon M_e$) we have $E_{cm} \approx M_e$, and$$\langle \sigma \rangle \sim \frac{\alpha^2}{M_e^2}$$

This is just the Thomson cross section noted on p. 34

$$\langle \text{Thomson} \rangle = \frac{8\pi \alpha^2 (\epsilon M_e^2)}{3} = \frac{8\pi \alpha^2 (\frac{h}{M_e})^2}{3} \rightarrow \frac{8\pi \alpha^2}{3 \hbar^2} \frac{1}{M_e^2} \quad \text{if } \hbar = \text{c}\epsilon$$

Again $\alpha M_e \equiv \text{Compton wavelength of the electron.}$

Another limit is very high energy photons ($E_\gamma \gg \epsilon M_e$) incident on electrons at rest.

Then $E_{cm} \approx \left[ (K_1, 0, 0, K) + (M, 0, 0, 0) \right]^2 \approx 2MK$

So$$\langle \sigma \rangle \rightarrow \frac{\alpha^2}{2MK}$$

We may also estimate a correction due to bad behavior of the electron propagator for small angle scatterings

The matrix element $\sim \frac{1}{q^2 - M_e^2}$

Now we can write $q = p_\perp + k_\perp$ (4-vectors), noting that $p_\perp$ describes a relativistic electron; even for the initial electron at rest.
So \( q^2 - m^2 = k_F^2 + p_F^2 + 2 k_F p_F - m^2 = 2(k_{p_F} - k_{p'_F}) \rightarrow 0 \) as \( l p_{o_F} \rightarrow \infty \).

To avoid estimating an infinite cross-section, we must keep track of the small departure of \( q^2 - m^2 \) from zero.

We relate \( P_o \) to \( \vec{p} \) by

\[
P_o = \sqrt{\vec{p}^2 + m^2} \approx \left| \vec{P} \right| + \frac{m^2}{2 \left| \vec{P} \right|}
\]

so \( q^2 - m^2 \approx 2 k_0 \left( \left| \vec{P} \right| (1 - \cos \Theta) + \frac{m^2}{2 \left| \vec{P} \right|} \right) \approx \frac{k_0}{\left| \vec{P} \right|} \left( \left| \vec{P} \right|^2 \Theta + m^2 \right)
\]

Hence we might expect

\[
\sigma \approx \frac{k^2}{2 \pi k} \int \frac{d\omega \cos \Theta}{(\left| \vec{P} \right| \Theta + m^2)^2} \propto \text{square of propagator}
\]

The other factor is required to make our calculation dimensionless and easy to calculate. It goes like \( \left| \vec{P} \right|^4 \).

But we must take note of another feature of the electromagnetic interaction not yet evident: Helicity conservation.

Recall helicity = component of spin along a particle's direction of motion.

In the extreme relativistic limit the electromagnetic interaction conserves the helicity of the spin of particles at each photon vertex. We shall try to demonstrate this later.

At high energies

**Yes**

**No**

In the present case, helicity conservation tells us that the Compton cross section must vanish as \( \Theta \rightarrow 0 \) at high energies.

The configuration does not conserve electron helicity and total angular momentum at \( \Theta = 0 \), which can't happen.

We infer that the matrix element vanishes like \( \Theta \) to insure compatibility with helicity cons. of angular momentum.

\[
\sigma_{ee} \approx \frac{k^2}{mk} \int \frac{d\omega \cos \Theta}{(\left| \vec{P} \right| \Theta + m^2)^2} \approx \frac{k^2}{mk} \left( \frac{2 \Theta d\Theta}{\Theta^2 + m^2 / \left| \vec{P} \right|^2} \right)^2 \approx \frac{k^2}{mk} \frac{\left| \vec{P} \right|^2}{m^2}
\]

on \( \sigma_{ee} \approx \frac{k^2}{mk} \frac{\left| \vec{P} \right|^2}{m} \) noting \( \left| \vec{P} \right| \sim k \) [Klein-Nishina (1928)]
WITH THIS TYPE OF ARGUMENT ONE MAY ALSO ESTIMATE THE CROSS SECTION FOR PAIR PRODUCTION BY A HIGH ENERGY PHOTON IN THE VICINITY OF A HEAVY NUCLEUS (BETHE-HEITLER (1934)).

\[ E_N = K \]

\[ \sigma_{B-H} = \frac{Z^2 K^3 \lambda_n}{\hbar^2 M} \]

[See book of T.D. Lee, Chap. 8]

**Rutherford Scattering**

\[ q^2 = \text{Photon Exchange} \]

WE KNOW THAT THE TOTAL CROSS SECTION IS INFINITE — THE ELECTRIC FIELD EXTENDS TO INFINITY. WE WILL TRY TO ESTIMATE A DIFFERENTIAL CROSS SECTION.

Rutherford considered \( \frac{d\sigma}{d\Theta} \) FOR SCATTERING OF THE LIGHTER PARTICLE BY ANGLE \( \Theta \).

FOR A QUICK ESTIMATE USING OUR DIMENSIONAL ARGUMENTS IT IS SIMPLER TO CALCULATE \( \frac{d\sigma}{dq^2} \) WHERE \( q^2 = \text{Square of the exchanged photon's 4-momentum} \).

**EXCHANGED PHOTON'S 4-MOMENTUM — A RELATIVISTIC INVARIANT OF THIS PROCESS. OF COURSE, \( q^2 \) HAS DIMENSIONS \( \text{(ENERGY)}^2 \)**

\[ \text{so } \frac{d\sigma}{dq^2} \sim \frac{1}{\text{(ENERGY)}^4} \]

ANY 2 BODY PROCESS \( a + b \rightarrow c + d \) HAS ONLY 2 INDEPENDENCE RELATIVISTIC INVARIANTS OF DIMENSIONS \( \text{(ENERGY)}^2 \). THESE ARE OBTAINED BY ADDING THE KINEMATIC 4-VECTORS IN VARIOUS WAYS:

**Mandelstam Variables**

\[ S \equiv (a + b)^2 = (c + d)^2 = E_{cm}^2 \]

\[ t \equiv (a - c)^2 = (b - d)^2 = [\text{MOMENTUM TRANSFER}^2 \sim q^2 \text{ ABOVE}] \]

\[ u \equiv (a - d)^2 = (b - c)^2 \]

NOTE THAT \( S + t + u = M_a^2 + M_b^2 + M_c^2 + M_d^2 \)
The diagram approach suggests that while $S$ was the relevant variable for $e^+e^- \rightarrow \mu^+\mu^-$ (an "$S$-channel" process), $t$ is more important for $e^+p \rightarrow e^+p$ (a "$t$-channel" process).

So \[ \frac{d\sigma}{dQ^2} = \frac{d\sigma}{dt} = \frac{\alpha^2}{t^2} = \frac{\alpha^2}{Q^4}. \]

In the laboratory frame, the electron 4-vectors relate to $q^2$:

\[ q = (E, 0, 0, 1) \rightarrow (E', E' \sin \theta, 0, E' \cos \theta) \quad (E', E' > m_e) \]

\[ q^2 = 2EE' (1 - \cos \theta) = 4EE' \sin^2 \theta/2 \]

\[ dQ^2 = 2EE' d\cos \theta \]

So \[ \frac{d\sigma}{d\cos \theta} \sim \frac{\alpha^2}{8EE' \sin^4 \theta/2}. \]

Which is very close to Rutherford's result \((E' \rightarrow E \rightarrow \frac{1}{2} m_e v_0^2)\)

**The Weak Interaction**

Fermi (1934) gave a view of the weak interaction which is a good first approximation.

This view is even more precise if we adopt the notion that the spin-$\frac{1}{2}$ quarks and leptons are the interacting particles.

The weak force was observed initially only in nuclear $\beta$-decays, and later in the decays of various mesons and hadrons. From this it was established that the range of the weak force is very short, certainly less than $10^{-13}$ cm.

Fermi supposed that the force has zero range - a contact force. But otherwise it connected spin-$\frac{1}{2}$ particles, as does electromagnetism.

Examples:

\[ \bar{\nu} \rightarrow e^+\bar{\nu}_e \]

\[ \bar{\nu}_\tau \rightarrow \mu^+\bar{\nu}_\mu \]

\[ \bar{\nu}_\tau \rightarrow \bar{\nu}_e \mu^- \bar{\nu}_e \]

Fermi wrote the interaction amplitude as the product of two vertex factors:

\[ \text{Ampli} \sim G (\bar{\nu}_\mu | \not{x} | \mu^-)(\not{e} | \not{y} | \not{\bar{\nu}}_e) \text{ etc} \]
Comparing with electromagnetism, we see that the weak coupling constant $G$ takes the place of both the $(\text{charge})^2$ and the propagator. Hence $G$ has dimensions $\frac{1}{(\text{energy})^2} = \frac{1}{(\text{mass})^2}$.

An important advance came in 1956 when Lee and Yang noted that the weak interaction violates parity conservation.

Recall that the parity transformation consists of replacing all position vectors $\vec{r}$ by $-\vec{r}$, leaving time unchanged. (This is equivalent to reflection in a mirror, followed by a rotation of 180° about an axis $\perp$ to the mirror.) Thus

velocity $\vec{v} = \frac{d\vec{r}}{dt} \rightarrow -\vec{v}$

3 momentum $\vec{p} \rightarrow -\vec{p}$

But angular momentum $\vec{L} = \vec{p} \times \vec{r} \rightarrow (-\vec{p}) \times (-\vec{r}) = \vec{L}$

A practical consequence is that in a parity-conserving interaction there can be no net angular-momentum correlation with direction of motion: terms like $\langle \vec{L} \cdot \vec{p} \rangle$ must vanish.

But such terms turn out to be large after a weak interaction: Wu et al. (1957). They are, however, complicated to measure.

In 1957 Feynman and Gell-Mann, among others, modified Fermi's theory to accommodate parity violation. They wrote, for a decay,

\[ \text{amp} \propto G (\gamma^\nu \gamma^\mu (1 - \gamma^5) |\nu^-\rangle (\nu^- | \gamma^\nu (1 - \gamma^5) | \gamma^\mu) \]

This is the so-called $V-A$ interaction which violates parity maximally.

The most prominent consequence of this concept is that neutrinos have only one spin component rather than 2 as is normal for spin-$\frac{1}{2}$ particles. We say that neutrinos are left-handed. (Anti-neutrinos are right-handed.)

Thus in $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ for $\pi$'s at rest $\bar{\nu}$

is the only possible spin orientation, as the $\pi$ has spin zero.

In the $V-A$ theory, any high energy $(E \gamma \mu)$ spin-$\frac{1}{2}$ particle can have a significant weak interaction only if its spin orientation is left-handed.

Note that in our picture of $\pi^\rightarrow \mu^- \bar{\nu}_\mu$, the $\mu^-$ is right-handed due to angular-momentum conservation. We infer that this decay is actually somewhat suppressed, and would be almost forbidden if $M_{\pi} \gg M_{\mu}$ (since $E_{\mu} \sim M_{\mu}$).
Compare the decay $\overline{\tau} \rightarrow e^- \overline{\nu}_e$. In this case $M_{\overline{\tau}} \ll M_e$ but $E_\overline{\tau} \approx M_{\overline{\tau}}/2$ and indeed this decay is very rare.

In contrast consider the decay $\mu^- \rightarrow e^- \overline{\nu}_e \nu_\mu$.

The decay configuration shown at right has all particles left handed, all anti-particles right handed, and so is not suppressed.

From these pictures we can extract some dimensional estimates for the decay rate:

$$\Gamma = \left| \frac{1}{T} \right| \sim \left| \text{Ampl} \right|^2 \quad \text{so} \quad \Gamma \sim G^2 \cdot \text{factor}$$

Now $G$ has dimensions $\left(\text{mass}^{-1}\right)^2$, while $\Gamma$ has dimensions mass.

Thus the factor must have dimensions of $(\text{mass})^5$.

For $\tau \rightarrow \mu^- \nu_\mu$:

$$\Gamma \sim G^2 M_\tau^5 M_{\mu}$$

For $\tau \rightarrow e^- \nu_e$:

$$\Gamma \sim G^2 M_\tau^5 M_e$$

For $\mu^- \rightarrow e^- \nu_e \nu_\mu$

$$\Gamma \sim G^2 M_{\mu}^5$$

We saw that $\tau \rightarrow e^- \nu_e$ is suppressed to the extent that $M_e$ is small. An educated guess is that $\text{Ampl} \sim M_e$ to provide this suppression.

$$\Gamma_{\tau \rightarrow e^- \nu_e} \sim G^2 M_\tau^3 M_e^2$$

$$\Gamma_{\tau \rightarrow \mu^- \nu_\mu} \sim G^2 M_\tau^3 M_{\mu}^2$$

(See Table III, Appendix E)

There is no suppression in $\mu^- \rightarrow e^- \nu_e$, which then suggests

$$\Gamma_{\mu^- \rightarrow e^- \nu_e} \sim G^2 M_{\mu}^5$$

A complete calculation yields

$$\Gamma_{\mu^- \rightarrow e^- \nu_e} = \frac{G^2 M_{\mu}^5}{192 \pi^3}$$

Comparison of this result with experimental data yields the value

$$G \approx 10^{-5} \frac{1}{M_{\text{proton}}}$$

A memorable fact

Exercise: Estimate the lifetime of the neutron: $\nu \rightarrow p^- \overline{\nu}_e$

Experiment: $\tau = 925 \pm 11$ sec
HIGH ENERGY NEUTRINO SCATTERING

IF WE CAN PRODUCE A BEAM OF ANTI-NEUTRINOS, THEN WE CAN INDUCE THE 'INVERSE $\beta$-DECAY' REACTION

$$\bar{\nu}_e + p \rightarrow n + e^+$$

THE CROSS SECTION FOR THIS GOES LIKE $g^2 N_0$ FACTOR

NOW $N_0 \sim \frac{1}{(\text{ENERGY})^2}$, $g^2 \sim \frac{1}{(\text{ENERGY})^4} \Rightarrow$ FACTOR $N_0 (\text{ENERGY})^2$

AT HIGH ENERGIES THE ONLY RELEVANT ENERGY IS $E_{\text{CM}}$, AND AS BEFORE $E_{\text{CM}} \sim 2 M_p E_Y$ FOR A PARTON AT REST IN THE LAB.

SO $N_0 \sim g^2 M_p E_Y$

NUMERICALLY, FOR $E_Y = 1 \text{ GeV} \sim M_p \quad (M_p = 938 \text{ MeV/c}^2)$

$$g \approx \frac{10^{-10}}{M_p^2} \cdot M_p^2 \approx 10^{-10} \approx \frac{1}{15} \text{ (Fermi)}^2 \approx 10^{-38} \text{ cm}^2$$

THIS ESTIMATE IS NOT TOO FAR FROM EXPERIMENTAL FACT.

NOTE HOWEVER THAT $N_0 \sim E_Y$, SO AS $E_Y$ RISES THE CROSS SECTION GETS INFINITELY BIG. THIS SEEMS CONTRARY TO THE IDEA OF A CONTACT INTERACTION, AND INDICATES A FUNDAMENTAL "WEAKNESS" OF THE FERMI-/FEYNMAN- GELL-MANN THEORY, FIRST POINTED OUT BY HEISENBERG IN 1936.

YUKAWA ALREADY SAW A WAY OUT. WE COULD MAKE THE WEAK INTERACTION APPEAR MORE LIKE THE ELECTROMAGNETIC INTERACTION, WHILE REMAINING SHORT RANGE, IF A HEAVY QUANTUM WERE EXCHANGED

$$\mu \rightarrow \nu \rightarrow \nu$$

THE HEAVY PROPAGATOR IS $\frac{1}{q^2 - M_{\text{HEAVY}}^2} \sim \frac{1}{M_{\text{HEAVY}}^2}$ FOR 'LOW ENERGY'

REACTIONS SUCH AS MUON DECAY. THEN WE EXPECT THERE IS ALSO A DIMENSIONLESS WEAK COUPLING CONSTANT $g^2/\lambda$ SUCH THAT

AMPLI $\sim G \sim \frac{g^2}{\lambda} \frac{1}{M_{\text{HEAVY}}^2}$ (LOW ENERGY LIMIT)
Unification of the Weak and Electromagnetic Interactions

The idea of Weinberg and Salam (1967) is that \( q = 2 \) that the basic coupling strength of the electromagnetic (E) weak force is the same, although the propagators and vertex spin factors have some differences. (Actually, since the \( \tau \) has no electric charge, but does interact weakly, the introduced a new charge \( \gamma \), for notations, numerically \( q' = q \).)

Then \( G = 10^{-5} = \frac{e^2}{4\pi} \frac{1}{m_w^2} = \frac{1}{137} \frac{1}{m_w^2} \), so \( m_{heavy} \approx 30 \text{ MeV} \).

Actually Weinberg & Salam predicted a amplification: there should be both charged and neutral heavy quanta, related by an additional parameter, the 'Weinberg angle \( \theta_w \) (which relates the two charges) \( q \) and \( q' \).

\[
M_W^2 = \frac{376 \text{ GeV}}{\sin \theta_w} \quad M_{Z^0} = \frac{M_W^2}{\cos \theta_w}
\]

The \( W^\pm \) are the charged quanta which carry the weak force of Fermi's theory. The \( Z^0 \) is a heavy photon, which carries a new form of the weak force, the 'neutral current.' This last effect was first observed in 1973.

During the '70s the Weinberg angle was measured in various ways to be \( \theta_w \approx 45 \cdot 5 \Rightarrow M_W \approx 80 \text{ GeV} \), \( M_{Z^0} \approx 90 \text{ GeV} \).

This is almost exactly the mass found by Broggia et al. in 1982-83 when the \( W^\pm \) and \( Z^0 \) were first observed directly. They took advantage of the divergent nature of the propagator; \( \frac{1}{q^2 - M_{Z^0}^2} \).

C.M. energy of a process like \( u + \bar{d} \rightarrow \mu^+ \nu \) reaches \( M_W \) the cross-section gets very large (- too not actually infinite as the \( W \) has a short lifetime \( \Rightarrow \) finite spread of masses \& 'resonance' behavior of the propagator as indicated in lecture 1.)

Now that the \( W \) mass is known we may calculate the characteristic range of the weak interaction:

\[
\gamma \sim \frac{1}{M_W} \sim \frac{1}{100 \text{ MeV}} \sim 2 \times 10^{-16} \text{ cm}\]

Hence Fermi's idea of a contact interaction is quite good for nuclear processes with scale \( 10^{-13} \text{ cm} \).

The unification of the weak and electromagnetic interactions is a first large step towards fulfilling Einstein's vision of the unity of all forces in nature.
Grand Unification

Following the success of the Weinberg-Salam model, Georgi & Glashow (1974) suggested a unification of the strong, weak and electromagnetic interactions. This might occur at extremely high energies when the exchange of a super-heavy object could provide direct coupling of leptons to quarks.

This theory has 2 accessible predictions:

- $\theta_{\text{Weinberg}} \approx \frac{\sqrt{2}}{2}$, which is nearly true

- Protons can decay, with a mean lifetime of $\sim 10^{32}$ years

\[ P \rightarrow e^+ \pi^0 \]

Thus far there is no clear evidence for proton decay, with a limit of about $10^{32}$ year lifetime. But watch the newspapers for updates.

We can give a very approximate sense of the grand unification argument.

We mentioned in Lecture 2 that the strong force is mediated by the gluons obeys 'asymptotic freedom'. Roughly, if one includes higher-order gluon exchange (multiple-gluon effects), the effect is to weaken the apparent strength of the 1 gluon process. The higher the energy, the weaker the interaction.

If $q$ = characteristic energy of the process, the strength then depends on $q$:

\[ \frac{1}{\alpha_s} \approx 10 + \ln \left( \frac{q^2}{10^{6} \text{GeV}^2} \right) \quad \text{A very slow change} \]

Likewise, even the electromagnetic interaction is subject to an apparent increase of strength at very high energies:

\[ \frac{1}{\alpha} \approx 137 - \ln \left( \frac{q^2}{10^{6} \text{GeV}^2} \right) \]

We say that at high energies we probe deeper into the charge distribution and see more of the 'bare' charge, which is larger than the apparent charge at low energies.

If we plot the strong and electromagnetic coupling constants as a function of energy, they eventually cross.
To have $\sqrt{\lambda} (q^2) = \sqrt{\lambda} (q^4)$ each must change by a 65 units from their values at $q = 10^{-6}$ GeV. This suggests that at $q = 10^{15}$ GeV the interactions have equal strengths: $\lambda = \lambda_\gamma = \lambda_\pi = \lambda_\nu$.

Perhaps the eventual equality of the interaction strengths is actually a manifestation of their basic unity. This in turn suggests the existence of the K particle, sketched on p. 29, which might have mass $\approx 10^{15}$ GeV.

Then the proton decay rate

$$P = \frac{M}{m} = \frac{10^{14} \text{ MeV}}{10^{-8} \text{ MeV}} = 10^{22} \text{ sec}^{-1}$$

**Dimensional Argument:**

$$M = \frac{1}{P} = \text{ Lifetime} = 10^{22} \text{ sec} \approx 10^{33} \text{ years} \approx 1 \text{ sec} \times 10^{-21} \text{ sec}.$$ 

**Gravity**

The gravitational force $F = G\frac{mm'}{r^2}$ has a coupling $G$ constant with dimensions $G = \frac{10^{-38}}{m^2}$ (1.8 Newtons).

(1.8 Newtons)

(2.8 Newtons)

**Constant converted to our units:** In a unified view, following the above pattern, we might expect

$$G \approx \frac{\frac{g^2}{4\pi}}{M^2}$$

where $\frac{g^2}{4\pi} \approx 1$ is the true dimensionless coupling constant.

Then $M \approx 10^{19} M_p = \text{Planck Mass}$

(After Planck who first noted the special interest of $M \approx \frac{\hbar c}{G}$ in 1905!)

It is fervently believed by all that when we reach energies of $10^{19} M_p$ in laboratory reactions of fundamental particles that we will at last understand how to unify all 4 fundamental interactions. The recent 10-dimensional string theories may give insight as to how this will happen....