**Neutral Kaon Decays and CP Violation**

Along with the discovery of parity non-conservation in 1956, another milestone in weak-interaction physics is the discovery in 1964 of CP violation in $K^0$ decays. At the outset we remark that CP violation has a poorly understood role in the nature of things. It is only a small connection to the weak interaction which remains CP conserving in most instances. This contrasts with parity violation which was found to be 'maximal' - i.e., no right-handed couplings at all. Even today there is no known cause for CP violation. It is imagined that it gives us a view of some very-high-energy phenomena which is otherwise invisible in another presently accessible phenomena. Theories of the very-high-energy regime such as grand unification still have trouble explaining CP violation, in a practical sense. CP violation gives us a clue as to the apparent non-invariance of the universe. Also, it allows us to tell the difference between matter and antimatter in a way that is independent of terms like $+\text{ and }-$, or left and right (as discussed in Sec. 6 below).

Well before CP violation was discovered it was realized that the $K^0, \bar{K}^0$ system has many peculiar and interesting features. Before discussing CP violation we briefly describe strangeness oscillations and regeneration.

**References:**

1. **Strangeness Oscillations**

In 1952 Pais [P.R. 86, 663 (1952)] invented the idea of strangeness to explain the fact that $K^0$ and $\bar{K}^0$ always seemed to be produced together in reactions like $\pi^+ \rightarrow K^0 + \bar{K}^0$. In Pais' view, the $K^0$ has opposite strangeness from the $\bar{K}^0$, and so must be a distinct particle. It was clear from the start that strangeness is not conserved in the weak interaction, so that as $K^0 \rightarrow \pi^+ \pi^-$ we would also expect $\bar{K}^0 \rightarrow \pi^- \pi^+$. It is rather odd (not to say strange) that both $K^0$ and $\bar{K}^0$ but they both decay the same way, indeed Fermi objected to the whole strangeness scheme because of this. The skepticism of Fermi led C. N. Yang to [P.R. 97, 1387 (1955)] a remarkable deduction.

If $K^0 \rightarrow \pi^+ \pi^-$ and also $\bar{K}^0 \rightarrow \pi^- \pi^+$ via the weak interaction, then one can imagine the 2nd order process $K^0 \rightarrow \pi^+ \pi^- \rightarrow K^0$. Hence we have a 2-state system with a non-zero transition amplitude between two states. Then we know from quantum mechanics that the $K^0$ and $\bar{K}^0$ are not the eigenstates of the system.
Rather there will be a pair of linear combinations of the $K^0$s and $\bar{K}^0$s which actually have definite lifetimes. These are called $K_{\text{short}}^0 = K^0_s$ and $K_{\text{long}}^0 = K^0_L$.

These states will have different masses where

$$\Delta m \sim \text{amplitude} \left( K^0 \to \pi^0 \to K^0 \right) = G^2 M_{K_s} \lambda^{-1/2} \theta$$

(we pursue this in more detail in Secs. 5 and 7 below)

We note that $\Gamma_{K^0} \approx \Gamma_{K^0} \approx \langle \Delta m^2 \text{weak} | K^0 \rangle \approx \langle K^0 | \text{weak}^2 | K^0 \rangle \approx \Delta m$ so that we can anticipate $\Delta m / \Gamma_{\text{short}} \approx 1$.

By invoking CP invariance we can make a shortcut in the derivation of the expressions for $K^0_s$ and $K^0_L$ in terms of $K^0$ and $\bar{K}^0$. This was already sketched on p. 309.

With $P(K^0) = -|K^0\rangle$ and $C(K^0) = -|\bar{K}^0\rangle$, we have $CP(K^0) = +|\bar{K}^0\rangle$.

Thus if we define $|K_1^0\rangle = \frac{|K^0\rangle + |K^0\rangle}{\sqrt{2}}$ and $|K_2^0\rangle = \frac{|K^0\rangle - |K^0\rangle}{\sqrt{2}}$,

then $CP(K_1^0) = +|K_1^0\rangle$ and $CP(K_2^0) = -|K_2^0\rangle$.

We identify $K_1^0 = K^0$ so that it can decay into $\pi\pi$ which was $CP = +1$ when $\pi\pi = 0$. All this assumes CP conservation in the weak interaction--which holds for the $V-A$ theory.

Then with $K_2^0 = K^0$, it can decay to $\pi\pi$ but not $\pi\pi$, and will clearly have the longer lifetime due to the lack of phase space for the $\pi\pi$ decay.

Now things begin to get interesting. Suppose we produce a $K^0$ in a strong interaction such as $\pi^- p \rightarrow K^0 \Lambda$. Before discussing its decay we must convert the $K^0$ to the appropriate state of $K_1^0$ and $K_2^0$ as the latter are the eigenstates of the weak interaction which governs the decay. Now

$$|K^0\rangle = \frac{|K_1^0\rangle + |K_2^0\rangle}{\sqrt{2}}$$

As the $|K_1^0\rangle$ state decays away with time we can describe its wave function as

$$\psi_s(t) = \psi_s(0) e^{-i M_s t - \frac{\Gamma_s}{2} t} \quad \Gamma_s = \frac{1}{\tau_s}$$

so that the probability of still having a $K^0$ at time $t$ is

$$P(t) = P_s(0) e^{-\Gamma_s t}$$

Likewise

$$\psi_L(t) = \psi_L(0) e^{-i M_L t - \frac{\Gamma_L}{2} t}$$
Thus \( \Psi_{K^0}(t) = \frac{1}{\sqrt{2}} \left( e^{-i \frac{\pi}{2} t} + e^{i \frac{\pi}{2} t} \right) \)

So that the probability that a \( K^0 \) remains at time \( t \) is

\[ P_{K^0}(t) = \frac{1}{4} \left[ e^{-\frac{\pi t}{2}} + e^{\frac{\pi t}{2}} + \frac{\pi t}{2} \right] \]

This shows how the \( K^0 \) does not have a definite lifetime!

Even more bizarre is that while the \( K^0 \) is still present, there is some probability that it has turned into a \( \bar{K}^0 \). Namely

\[ |\bar{K}^0\rangle = \frac{1}{\sqrt{2}} (K^0 + \bar{K}^0) \quad \Rightarrow \quad \Psi_{\bar{K}^0}(t) = \frac{1}{\sqrt{2}} \left( e^{-i \frac{\pi}{2} t} - e^{i \frac{\pi}{2} t} \right) \]

And so \( P_{\bar{K}^0}(t) = \frac{1}{4} \left[ e^{-\frac{\pi t}{2}} + e^{\frac{\pi t}{2}} - \frac{\pi t}{2} \right] \)

We can now answer Fermi's question and show that there is experimental meaning to this analysis. A \( K^0 \) produced in the reaction \( \pi^- p \rightarrow K^0 A \) will be moving with velocity \( v = c \) (if \( E_K \gg 1 \text{ GeV} \)). So we can place matter in the path of the kaon at various distances from the production point, thereby simulating its history in time. If a \( K^0 \) is still present we might observe reactions like

\( K^0 p \rightarrow K^+ n \)

But if a \( \bar{K}^0 \) is present we can have \( \bar{K}^0 p \rightarrow \Lambda^0 n^+ \), which is completely impossible if only \( K^0 \)'s remain. The latter reaction was indeed observed in a bubble chamber, with incident \( p^- \) beam, in 1956.

Furthermore, if one measures the number of \( \bar{K}^0 \) as a function of time, one can determine \( |\Delta M| \), by direct observation of the 'strangeness oscillation'. The first experiment of this type to perform a significant measurement of \( |\Delta M| \) was by Pitch et al. [Nuovo Cimento 22, 1660 (1961)].

2. Regeneration

Shortly after the strangeness oscillation analysis, Pais & Piccioni [P.R. 100, 1985 (1955)] noted another unusual possibility. Suppose we produce a beam of \( K^0 \) particles and then wait until the \( K^0_S \) component has died away, say after \( 10 \text{ T}_S \). We are then left with a \( K^0 \) beam of \( \frac{1}{2} \) the original intensity (\( T_L \approx 600 \text{ T}_S \); see above figure).
IF THIS BEAM STRIKES A TARGET, STRONG INTERACTIONS CAN OCCUR.

For this it is appropriate to again use a description in terms of $K^0$ and $\bar{K}^0$, rather than $K^0$ and $K^-$, as strangeness is conserved in the strong interactions. Because the $\bar{K}^0$ has the same strangeness as the $K^0$, it interacts with matter more readily than the $K^0$. So the $\bar{K}^0$ component of the beam is attenuated more than the $K^0$ component as the beam passes through matter. The wave function of the unscattered beam particles is no longer

$$|K^0\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}$$

but rather

$$\frac{f_1|K^0\rangle - f_2|K^-\rangle}{\sqrt{2}}$$

where $f_1$ and $f_2$ are the amplitudes for a $K^0$ and $\bar{K}^0$ to pass through the matter without scattering. The unscattered beam now consists of

$$\frac{f_1 + f_2}{\sqrt{2}}|K^0\rangle - \frac{f_1 - f_2}{\sqrt{2}}|K^0\rangle = \frac{f_1 + f_2}{\sqrt{2}}|K^0\rangle + \frac{f_1 - f_2}{\sqrt{2}}|\bar{K}^0\rangle$$

Once the beam has emerged from the absorbing matter, we can again observe $K^0 \rightarrow \pi^+\pi^-\pi^0$ decays. We say that a $K^0$ component of the beam has been regenerated.

![Figure 10.3. Regeneration of $K^0$](image)

GOOD [P.R. 110, 1550 (1958)] WAS THE FIRST TO REALIZE THAT THE REGENERATION PHENOMENON CAN BE USED TO MEASURE A.M.'S, $M_L - M_S$. WE DISCUSS A VERSION DUE TO FITCH.

$$K^0 \rightarrow \pi^+\pi^-\pi^0$$

A $K^0$ beam impinges on 2 thin regeneration slabs separated by a distance $d$. The measurement consists of counting the number of $K^0 \rightarrow \pi^+\pi^-\pi^0$ decays observed after the second slab, as a function of gap size $d$.

IT IS SIMPLER IN THE FOLLOWING TO THINK OF $d$ AS THE PROPER TIME NEEDED FOR A $K^0$ TO CROSS THE GAP. THEN THE AMPLITUDE FOR A $K_S$ TO EMERGE FROM THE 2ND SLAB IS

$$p_1 e^{-iM_Sd} - p_2 e^{-iM_Ld} + p_3 e^{-iM_d} + p_4 e^{-iM_d}$$

if $p_i < 1$

where $p_i$ = (complex) amplitude to regenerate a $K_S$ in slab $i$.

The exponential factors indicate that the $K_S$ component produced in the 1st slab can decay before emerging from the 2nd slab, and
That the K⁰ component can decay while crossing the gap. Then
\[ \Phi_\text{c} = \text{phase of regeneration amplitude} \]

Over the years very precise measurements of AM have been measured with this technique, which is a conceptual variation on the double slit interference of light.

**Recent results:**
- \[ |\Delta M_1| T_s = 0.477 \pm 0.03 \]
- \[ |\Delta M_1|' = 3.521 \pm 0.03 \times 10^{-6} \text{eV} \]
- \[ |\Delta M_1| / M_K = 0.7 \times 10^{-19} \]

The last ratio is certainly the smallest dimensionless quantity measured in high energy physics.

The sign of AM has been determined by another variation on the regeneration technique, by using a K⁰ beam.

![Diagram of regeneration technique](image)

Two blocks of matter are placed relatively close to a K⁰ source, so that both K⁰'s and K⁰L strike the block. The effect is based on the interference between the K⁰'s component of the beam, and the K⁰'s component regenerated from K⁰L in the two regenerators.

To determine the sign of AM we must know the phase of the strong interaction amplitude \( \Phi - \Phi \) which is responsible for the regeneration of the K⁰. Even the algebra needed to keep track of all this is somewhat complicated, but the experiments show that

\[ M_L > M_S \]

[Sjournovich et al., P.R.L. 17, 1075 (1966)]

[Melnik et al., P.R. 172, 1613 (1968)]

**Exercise:** Show that
- \( \Delta M = n \Phi \)
- \( \Phi = \sum_{i=1}^{3} (a_i + 2) \)
- \( \Phi = \sum_{i=1}^{3} (a_i + 2) \sum_{i=1}^{3} (a_i + 2) \)

Taking \( a_1 = a_2 = a \). The sign of AM can be fixed using termination of 3 interference terms.

**Fig. 17:** Distribution of \( N(T) \) versus \( T / T_s \) or \( D / T_s + 1 \), where \( D \) is the distance from the target to the regenerator, and \( T_s \) is the mean decay length of K⁰ [Piccioci (58) and (61)].
3. Discovery of CP Violation

In 1964 Christenson et al. [P.R.L. 13, 138 (1964)] and Abashian et al. [P.R.L. 12, 242 (1964)] observed the decay \( K^0 \rightarrow \pi^+ \pi^- \), which is forbidden by CP invariance. Christenson et al. measured

\[
\frac{\Gamma_{K^0 \rightarrow \pi^+ \pi^-}}{\Gamma_{K^0 \rightarrow \text{all}}} = 2.0 \pm 0.4 \times 10^{-3}
\]

This result agrees well with the best value now measured of \( 2.03 \pm 0.9 \times 10^{-3} \), which shows the quality of the initial measurement. Of course one must be very careful that the observed \( \pi^+ \pi^- \) are not due to the CP conservation process of regeneration of \( K^0_S \) via \( K^0_L \) interactions with matter. (Palmer: Is \( K^0_L \rightarrow K^0_S \) regeneration really \( CP \) conserving?)

The decay \( K^0_L \rightarrow \pi^+ \pi^- \) is sometimes termed the \(^1\)Vacuum regeneration\(^1\) under the supposition that the \( K^0_L \) turned into a \( K^0_S \) even in the presence of any absorber. (For this to be possible the \( K^0_L \) and \( K^0_S \) states cannot be orthogonal, as discussed in sec. 5 below). The \(^1\)Vacuum regeneration\(^1\) phenomenon allows many further variations on the already intricate subject of regeneration, which has led to many beautiful experiments. We sketch briefly the formalism used in some of these endeavors.

We start at \( t = 0 \) with a general neutral kaon state

\[ a_S |K^0_S \rangle + a_L |K^0_L \rangle \]

and observe the rate of decay to \( \pi^+ \pi^- \) as a function of time. The amplitude to decay at time \( t \) is

\[ a_S e^{-i\frac{M_{\pi \pi}}{2} t} + a_L e^{-i\frac{M_{\pi \pi}}{2} t} \]

\[ \langle \pi^+ \pi^- | \text{weak} | K^0_S \rangle + a_L e^{-i\frac{M_{\pi \pi}}{2} t} \langle \pi^+ \pi^- | \text{weak} | K^0_L \rangle \]

We introduce the CP violation parameter \( \eta_{+-} = \frac{\langle \pi^+ \pi^- | \text{weak} | K^0_S \rangle}{\langle \pi^+ \pi^- | \text{weak} | K^0_L \rangle} \)

Then decay amplitude

\[ a_S e^{-i\frac{M_{\pi \pi}}{2} t} + \eta_{+-} a_L e^{-i\frac{M_{\pi \pi}}{2} t} \]

And decay rate

\[ 19.5^2 e^{-i\frac{M_{\pi \pi}}{2} t} + 19.5^2 e^{-i\frac{M_{\pi \pi}}{2} t} \]

\[ 20 \times 9.5 |\eta_{+-}| e^{-i\frac{M_{\pi \pi}}{2} t} \]

And \( \phi = \text{phase of } (a_S, a_L) = \phi_{a_S} - \phi_{a_L} \)

For example, in the original experiment of Christenson et al. they had a \( KL \) beam \( \Rightarrow \eta_{+-} = 0 \), \( a_L = 1 \)

Then

\[ |\eta_{+-}|^2 = \frac{\Gamma_{K^0 \rightarrow \pi^+ \pi^-}}{\Gamma_{K^0 \rightarrow \text{all}}} \approx 2 \times 10^{-3} \]

\[ \Rightarrow |\eta_{+-}| \approx 2.2 \times 10^{-3} \]

But known from earlier studies.
If instead one starts with a $K^0$ beam and observes $\pi^+\pi^-$ decays before the $K^*$ component has decayed away, more information can be obtained. In this case $q_{\pi^+\pi^-} = q_L$ at $t = 0$, and

$$\text{rate} \to \pi^+\pi^- \propto e^{-\Gamma t} + 2|\eta_{+|-}| e^{-\Gamma t} \cos(\Delta m t - \phi_{+-})$$

ignoring $|\eta_{+|+}|^2$, and setting $e^{-\Gamma t} = 1$ for small $t$.

The first experiment of this type was that of Pitch et al. [PRL 13, 73 (1969)]. This demonstrated that the $\eta_{+|+}$ produced at the CP-violation decays of $K^0$ really do interfere with $K_0 - \rightarrow \eta^0\pi^0$, a non-trivial result of quantum mechanics.

The most accurate experiment of this type is that of Bewick et al. [Phys. Lett. 44B, 483 (1974); see Fig. 8] who measured

$$|\eta_{+|+}| = 2.30 \pm 0.35 \times 10^{-3}$$

$$\phi_{+-} = 45.9 \pm 1.0$$

Because $\eta_{+|+}$ is so small the interference effect is small. The interpretation of the data is very simple however.

The interference effect can be enhanced at expense of crispness of interpretation. One starts with a $K^0$ beam ($q_L = 1$) and passes it through a thin regenerator, inducing a $K^*$ component of size $q_2 = 0$, chosen $\approx \eta_{+|+}$. Then setting $t = 0$ as the beam emerges from the regenerator we find

$$\text{rate} \to \pi^+\pi^- \propto \left|\frac{1}{2} + \eta_{+|+} e^{-\Gamma t} + 2|\eta_{+|+}| e^{-\Gamma t} \cos(\Delta m t - \phi_{+-} + \phi_{\pi\pi})\right|^2$$

Details of the strong-interaction parameters $|\pi|_2$ and $\phi_{\pi\pi}$ must be known to complete the analysis. Bewick et al. [PRL 34, 1240 (1975); see Fig. 8]

Find $\phi_{+-} = 45.8 \pm 2.8^o$

by this technique.

---

Figure 10: Time distribution of $K^0 - \rightarrow \pi^+\pi^-$ events from a coherent mixture of $K^0$ and $K^0_s$ produced in pure strangeness states (77). (a) Events (histogram) and fitted distribution (dots). (b) Events corrected for detection efficiency (histogram), fitted distribution with interference term (dots), and without interference term (curve). Inset: interference term as extracted from data (dots) and fitted term (line).

Figure 8: Time distribution of $K^0 - \rightarrow \pi^+\pi^-$ decays behind a carbon regenerator (77). (a) Detection efficiency for kaon momentum interval $3 < p_K < 6$ GeV/c. (b) Efficiency-corrected data summed over $p_K$; curves show fits without interference. (c) Data for $3 < p_K < 6$ GeV/c with fit including interference.
4. CP VIOLATION IN $K_L \rightarrow \pi^0 \pi^0$

Both $K^0$ and $\bar{K}^0$ decay to $\pi^0 \pi^0$, so we also expect evidence for CP violation in the form of the decay $K_L \rightarrow \pi^0 \pi^0$. This is much harder to demonstrate experimentally as the $\pi^0$'s decay to $2\gamma$ very quickly. Nonetheless, several experiments have succeeded in analyzing the $4\gamma$ final state. To describe the results we define the CP-violating parameter

$$\eta_{oo} \equiv \frac{\langle \pi^0 \pi^0 | \text{weak} | K_L \rangle}{\langle \pi^0 \pi^0 | \text{neutral} | K_L \rangle}$$

The best determination of $\eta_{oo}$ comes from 2 experiments which use the regeneration technique and a $K^0$ beam. They found

$$\left| \frac{\eta_{oo}}{\eta^{+-}} \right| \approx \frac{1.00 \pm 0.06}{1.03 \pm 0.07} \quad [\text{Hofler et al., Phys. Lett. B41, 141 (1972)}]$$

$$\left| \eta_{oo} \right| = 1.00 \pm 0.09 \quad [\text{Lyons et al., PRL 41, 189 (1978)}]$$

Figure 7.7. Intensity of $\pi^+\pi^-$ and $\pi^0\pi^0$ as a function of proper time $\tau$. The exponential decay is modified by a sinusoidal term shown in the lower half of the diagrams. The curve through the lower data is a best fit to $|\eta|$ and $\phi$. (From Christenson et al. 79. Reprinted with permission.)

The phase angle $\phi_{oo}$ has been measured in a vacuum regeneration type experiment, with a $K^0$ beam. Christenson et al. [PRL, 41, 1189, 1212 (1979)] observed both $\pi^+\pi^-$ and $\pi^0\pi^0$ decays in the same experiment. They found

$$\left| \frac{\eta_{oo}}{\eta^{+-}} \right| = 1.00 \pm 0.09$$

$$\phi_{oo} = 41.7 \pm 3.5^\circ$$

$$\phi_{oo} = 55.7 \pm 5.8^\circ$$

Very recent results indicate

$$\left| \frac{\eta_{oo}}{\eta^{+-}} \right| = 1.00 \pm 0.09 \quad [\text{Black et al., PRL 54, 163 (1985)}]$$

5. FURTHER PARAMETERIZATION OF CP VIOLATION

We now categorize the nature of the observed CP violation in somewhat more detail. An immediate question is whether CP violation implies CPT violation, T violation or both. We will not examine this question in these notes, but simply state that the observed CP violation is experimentally consistent with CPT invariance.

We then infer that T violation does occur, although evidence for this is indirect. For details, refer to the review of Lee and Wu mentioned on p. 326.
We now return to the question of the relation between the strong interaction eigenstates $K^0$ and $\bar{K}^0$, and the weak interaction eigenstates $K_L$ and $K_S$. We relax the assumption of CP invariance which simplified the previous discussion on p. 326.

We consider a state $\Psi = \alpha |K^0\rangle + \beta |\bar{K}^0\rangle$ produced in a strong interaction. Now we follow its subsequent history as governed by the weak interaction. The Schrödinger equation tells us

$$i \frac{\partial \Psi}{\partial t} = H \Psi$$

where the Hamiltonian $H$ is a $2 \times 2$ matrix

$$H = \begin{pmatrix}
    \langle K^0 | H | K^0 \rangle & \langle K^0 | H | K_L \rangle \\
    \langle K^0 | H | K_S \rangle & \langle K^0 | H | K_S \rangle
\end{pmatrix} \equiv \begin{pmatrix}
    H_{11} & H_{12} \\
    H_{12} & H_{22}
\end{pmatrix}$$

Because the kaons can decay, $H$ has the form $M - \frac{i}{2} \Gamma$, where

$M$ and $\Gamma$ are hermitian matrices: $H_{12} = H_{21}^*$ etc. (Wigner-Weisskopf, 1932)

$M$ is usually called the mass matrix; $\Gamma$ is the decay matrix.

We seek the eigenstates of $H$, i.e., the states $K_L$ and $K_S$ which have definite lifetimes. To be precise we assume that CP invariance holds, so that $K^0$ and $\bar{K}^0$ are indeed antiparticles. Then we have

$H_{11} = H_{22}$; otherwise $M_K + i M_B$. We then find the eigenvalues of $H$ via $|H - \lambda I| = 0 \Rightarrow \lambda = H_{11} \pm \sqrt{H_{12} H_{22}}$

If we write

$\lambda^2 \equiv i H_{12}$ and $\lambda^2 \equiv i H_{22}$

Then

$\lambda_{long} = H_{11} - i \lambda \lambda_{short} = H_{11} + i \lambda$ \lambda

\lambda

And, with some effort, $|K_2^0\rangle = \frac{\lambda |K^0\rangle - \lambda |\bar{K}^0\rangle}{\sqrt{\lambda^2 + 1\lambda^2}}$

$|K_S^0\rangle = \frac{\lambda |K^0\rangle + \lambda |\bar{K}^0\rangle}{\sqrt{\lambda^2 + 1\lambda^2}}$

It is common to express $|K_3^0\rangle$ and $|K_1^0\rangle$ in terms of the CP eigenstates

$|K_1^0\rangle = \frac{|K^0\rangle \pm |\bar{K}^0\rangle}{\sqrt{2}}$ $\Rightarrow$ $|K_3^0\rangle = |K_1^0\rangle + |K_2^0\rangle$ $|K_1^0\rangle = |K_3^0\rangle - |K_2^0\rangle$

And $|K_L^0\rangle = \frac{(p-q) |K_1^0\rangle + (p+q) |K_2^0\rangle}{\sqrt{2} \sqrt{p^2 + q^2}}$

We define $\epsilon \equiv \frac{p-q}{p+q}$

$|K_2^0\rangle = \frac{|K_0^0\rangle + \epsilon |K_2^0\rangle}{\sqrt{2} \sqrt{1+\epsilon^2}} = \frac{(1+\epsilon) |K^0\rangle - (1-\epsilon) |\bar{K}^0\rangle}{\sqrt{2} \sqrt{1+\epsilon^2}}$

$|K_3^0\rangle = \frac{|K_0^0\rangle + \epsilon |K_2^0\rangle}{\sqrt{2} \sqrt{1+\epsilon^2}} = \frac{(1+\epsilon) |K^0\rangle + (1-\epsilon) |\bar{K}^0\rangle}{\sqrt{2} \sqrt{1+\epsilon^2}}$
Clearly CP invariance $\Rightarrow$ $\epsilon = 0$. Hence $\epsilon$ is often referred to as the measure of CP violation in the $K^0_L$ and $K^0_S$ wave functions.

It is useful to derive another expression for the CP violation parameter $\epsilon$:

$$
\epsilon = \frac{p - q}{p + q} = \frac{p^2 - q^2}{p^2 + 2pq + q^2} \propto \frac{p^2 - q^2}{2pq} \quad \text{noting} \quad (c c^\dagger) = pq
$$

$$
= \frac{i(\lambda_2 - \lambda_1)}{2 \iota(\lambda_2 - \lambda_s)} = \frac{M_{12} - i \frac{\Gamma_{12}}{2} - M_{12}^* + i \frac{\Gamma_{12}^*}{2}}{2 \left[ (M_{12} - M_{12}^*) - i \left( \Gamma_s - \Gamma_s^* \right) \right]}
$$

(as $\lambda_1$ and $\Gamma$ are hermitian)

$$
= \frac{\omega_M \Gamma_{12}/2 + i \omega_M \Gamma_{12}}{i \left( \Gamma_s - \Gamma_s^* \right)/2 + (M_{12} - M_{12}^*)}
$$

For the neutral kaons, $\Gamma_s \approx 600 \Gamma_L$, and $(M_{12} - M_{12}^*) \Gamma_s \approx \frac{1}{2}$

$\Rightarrow$ $\Delta M \approx \Gamma_{12}/2$, and we can write

$$
\epsilon \approx \frac{1 + i}{\Gamma_s} \left( \omega_M \Gamma_{12} - i \omega_M \Gamma_{12}/2 \right)
$$

It may also be worth noting that $\langle K^0_L | K^0_S \rangle \approx 2 \Re \epsilon$

This is, the weak eigenstates are not orthogonal, if CP violation occurs, which leads to the possibility of vacuum regeneration noted on p. 331.

Further constraint on $\epsilon$ can be found by considering the $K^0_L \rightarrow \pi \pi$ decays. We will relate $\epsilon$ to $\eta^+$ and $\eta^0$.

As noted before the $\pi \pi$ system with $J = 0$ can have either $I = 0$ or $I = 2$. The $A \Sigma \Pi \Delta$ rule suggests that transitions to the $I = 2$ $\pi \pi$ channel will be suppressed. We write

$$
q_0 \equiv \langle \pi^0, I = 0, 0 | \text{weak} | K^0 \rangle \quad \text{CPT} \quad q_0^* = \langle \pi^0, I = 0, 0 | \text{weak} | K^0 \rangle
$$

$$
q_2 \equiv \langle \pi^0, I = 2, 0 | \text{weak} | K^0 \rangle \quad q_2^* = \langle \pi^0, I = 2, 0 | \text{weak} | K^0 \rangle
$$

We have chosen the phase of $|K^0\rangle$ so that $q_0$ is real.

[We ignore a complication due to strange final-state interactions and $\eta$]

The $\eta^0$. See the review of Lee & Wu

From C-G tables

$$
|\pi^0 \eta^0\rangle = \sqrt{\frac{1}{3}} \left( |\pi^0 \eta^0, I = 2, 0\rangle - \frac{\sqrt{3}}{3} |\pi^0 \eta^0, I = 0, 0\rangle \right)
$$

$$
|\eta^+ \eta^-\rangle = \sqrt{\frac{1}{3}} \left( |\eta^+ \eta^-, I = 2, 0\rangle + \frac{\sqrt{3}}{3} |\eta^+ \eta^-, I = 0, 0\rangle \right)
$$

Thus

$$
\langle \pi^0 \eta^0 | \text{weak} | K^0 \rangle \propto \left( \frac{\Gamma_0}{2} \left( \frac{1}{3} \langle I = 2, 0 | \text{weak} | (1 + \epsilon) | K^0\rangle - (1 - \epsilon) | \eta^0 \rangle \right)
$$

$$
\approx \frac{2}{\Gamma_0} \epsilon \left( \frac{\omega_M \Gamma_{12}}{q_0} \right)
$$
Similarly

\[ <\pi^0\pi^0|\text{weak}|K^0_s> = \frac{2}{13} \alpha_0 \left( -\frac{\epsilon}{\sqrt{s}} + i \frac{\text{Im} \alpha_2}{\alpha_0} \right) \]

\[ <\pi^+\pi^-|\text{weak}|K^0_s> \approx \frac{2 \alpha_0}{\sqrt{s}} + O\left( \frac{\alpha_2}{\alpha_0} \right) \]

\[ <\pi^0\pi^0|\text{weak}|K^0_s> \approx -\frac{2 \alpha_0}{\sqrt{s}} + O\left( \frac{\alpha_2}{\alpha_0} \right) \]

Ignore these as leading to 2 no angular pieces \( \eta^* \eta \rightarrow \eta^+ \eta^- \eta^0 \).

Then

\[ \epsilon' = \frac{\sqrt{s}}{\sqrt{s}} \frac{\text{Im} \alpha_2}{\alpha_0} = \text{CP violation in } \Delta S = \frac{1}{2} \text{ amplitude} \]

From experiment \( \frac{\eta^+}{\eta^0} \rightarrow 1 \implies \left| \frac{\epsilon'}{\epsilon} \right| < 0.005 \) (1997).

It is perhaps no great surprise that \( \epsilon'/\epsilon < 0.1 \) as this is typical of \( \Delta S = \frac{1}{2} \) transitions. It is possible, of course, that \( \epsilon' = 0 \).

Thus we conclude that \( \epsilon \sim \eta^+ \sim \eta^0 \).

The results discussed on p.372 for \( \phi^+ \Rightarrow \text{phase of } \epsilon \approx 45^\circ \).

From p.338, \( \epsilon \approx \frac{1 + i}{\sqrt{5}} \left( \frac{\text{Im} M_{12} - i \text{Im} M_{12}}{\epsilon} \right) \)

To get phase of \( \epsilon \approx 45^\circ \) we must have \( \text{Im} M_{12} < \text{Im} M_{12} \).

And so \( \epsilon \approx \frac{(1 + i)}{\sqrt{5}} \frac{\text{Im} M_{12}}{\text{Re} M_{12}} \approx 2 \left( 1 + \epsilon' \right) \frac{\text{Im} M_{12}}{\text{Re} M_{12}} \)

We will compare this result with constraints from the Kobayashi-Maskawa mixing scheme in section 7.

6. CP Violation in the Decay \( K^0 \rightarrow \pi^+\pi^- \)

We compare the decays \( K_L \rightarrow \pi^+\pi^- \) and \( K_S \rightarrow \pi^+\pi^- \). If CP invariance were to hold then the decay rates for these must be equal, as \( \epsilon \pi^+\pi^- \epsilon' \approx \pi^+\pi^- \epsilon' \).

Note that \( \pi^+\pi^- \) is not an eigenstate of CP, both \( K_L \) and \( K_S \) can decay to this.
Thus the quantity
\[
\frac{\text{Rate } (K^0 \to \pi^+\pi^-U) - \text{Rate } (K^0 \to \pi^-\pi^+U)}{\text{Rate } (K^- \to \pi^+\pi^-U) + \text{Rate } (K^+ \to \pi^-\pi^+U)}
\]
will give further evidence of CP violation if it is non-vanishing.
We give a brief analysis of this, along with another result which does not depend on CP violation.

In the quark picture of weak decay we naturally have the $\Delta S = 0 \to 0$ rule (p. 306). Thus $K^0 \to \pi^+\pi^-U$ but $K^0 \to \pi^+\pi^-U$ and $\bar{K}^0 \to \pi^+\pi^-U$ but $\bar{K}^0 \to \pi^+\pi^-U$.

Let $f^\pm = \text{Amplitude } (K^0 \to \pi^+\pi^-U)$. Then assuming CP T invariance
\[
f^- = \text{Amplitude } (\bar{K}^0 \to \pi^+\pi^-U)
\]
We again consider a neutral kaon state which has composition
\[
q \left( K_0 \right) + q \left( K_0 \right) \quad \text{at } t = 0
\]
We calculate the amplitude $A^\pm$ for this state to decay to $\pi^+\pi^-U$ at time $t$. Note that $A^\pm$ depends only on the $K_0$ component of our state at time $t$. Refer to p. 334 for $K_0, \bar{K}_0$ in terms of $K_0, \bar{K}_0$.

\[
A^+(t) = q_0 e^{ieM_0 t - \frac{\pi^+}{2}} (1 + e^{\frac{iMT}{2}})
\]
\[
A^-(t) = q_0 e^{ieM_0 t - \frac{\pi^-}{2}} (1 - e^{\frac{iMT}{2}})
\]
The decay rates $N^\pm = |A^\pm|^2$ are given by

\[
N^+(t) \sim (1 \pm 2\Re e \epsilon) \left[ q_0^2 e^{-\frac{\pi^+}{2} t} + q_0^2 e^{-\frac{\pi^+}{2} t} + q_0 q_0 \epsilon \pm 2q_0 q_0 \epsilon e^{\frac{iMt}{2}} \cos(\Delta M t + \phi) \right]
\]

\[
\phi = \arcsin \frac{q_0 q_0 \epsilon}{q_0 q_0 \epsilon}
\]

\[
\frac{N^+ - N^-}{N^+ + N^-} \sim 2q_0 q_0 \frac{\Delta M}{\Delta M^2 + q_0^2} \left( \epsilon \cos(\Delta M t + \phi) + 2\Re e \epsilon \right) \quad \text{(times a factor)}
\]

Two cases are of interest:

a. A $K_0^0$ beam $\Rightarrow q_0 = 0, q_0 = 1$ and $\epsilon = \frac{N^+ - N^-}{N^+ + N^-} = 2\Re e \epsilon$.

This allows experimental determination of the CP violation parameter $\epsilon$, independent of the $\bar{K}^0$ decay mode. The best result of this type is from Georgi et al. [Phys. Lett. B78, 483 (1978)].

They found
\[
\epsilon = 3.41 \pm 1.18 \times 10^{-3}
\]

$\Rightarrow \Re e \epsilon = 1.7 \pm 0.9 \times 10^{-3}$ (if CP $\epsilon = \Delta M > 0 \leq 0$)

This is quite consistent with the previous values of $\epsilon$, leading to a combined result $1\epsilon = 2.78 \pm 0.51 \times 10^{-3} \quad \phi \epsilon = 45^\circ$. 

\[
\text{Re e } \epsilon = 1.7 \pm 0.9 \times 10^{-3} \quad (1 \text{ if } \text{CP } \epsilon > 0 \leq 0)
\]

This is quite consistent with the previous values of $\epsilon$, leading to a combined result $1\epsilon = 2.78 \pm 0.51 \times 10^{-3} \quad \phi \epsilon = 45^\circ$.
6. A $K^0$ beam $\rightarrow q_3 < 1 = q_L$

$$S(t) = \frac{e^{-\frac{P_0 + P_1}{2}}}{2} \cos(\alpha M + \phi) + 2 Re \bar{e} e^{-\frac{P_0 + P_1}{2}}$$

This gives a very dramatic interference effect when $\epsilon < 10^{-5}$, which provides one of the best ways to measure $\Delta M$.

[Note: The same result holds if we start with a $K^0$ beam. This is not a CPT violation, as mentioned on p. 176.]

We note that this asymmetry is measured with a $K^0$ beam allows one to 'safely' distinguish the universe from the anti-universe. Namely, if a being from another universe comes to earth, do you really want to shake hands? If space is made of anti-matter, a diplomatic incident might occur. But we can ask the being to perform an experiment in their own universe before they come, to determine whether they are made of matter or anti-matter.

- Produce a beam of long-lived neutral kaons
- Measure the charge asymmetry in $K^0 \rightarrow \pi\nu\bar{\nu}$

If the lepton which is produced more frequently is made of the same kind of matter as their atoms, we don't want them to come near!

7. CP Violation and the Kobayashi–Maskawa Matrix

We have claimed that the K-M matrix and the 6 quark scheme was a place for CP violation. We now sketch how this works. This is not an explanation of CP violation, but it does relate CP violation to other features of the weak interaction of quarks.

We have seen that most features of CP violation can be related to the single parameter

$$e \sim \frac{\Delta m \Gamma_{12}/2 + i \Delta m \Gamma_{12}}{i (\Gamma_{12} - \Gamma_{12})/2 + (M_L - M_S)} = (1 + \epsilon) \frac{iM_{12}}{Re \bar{M}_{12}}$$

(PP 335, 336)

where $M_{12} = \langle \bar{K}^0|\text{weak}|K^0 \rangle$. On p. 318 we indicated how this transition might be due to a second-order weak interaction.
In the 6 quark scheme $u, c$ or $t$ quarks can be exchanged inside the diagrams.

\[
\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = (KM) \begin{pmatrix} d \\ s \\ b \end{pmatrix} = (V ud \ V us \ V ub) \begin{pmatrix} d \\ s \\ b \end{pmatrix} \]

The amplitude $M_{12} = \langle \bar{K}^0 \mid \text{weak} \mid K^0 \rangle$ then varies like

\[
\left( \frac{V_{ud} V_{us}}{q^2 - M_{u}^2} + \frac{V_{cd} V_{cs}}{q^2 - M_{c}^2} + \frac{V_{td} V_{ts}}{q^2 - M_{t}^2} \right) \]

as the square since there are 2 quark exchanges.

\[
\approx \left( \frac{s_1 c_1 c_3}{q^2 - M_{u}^2} + \frac{-s_1 c_2 c_s c_3 + s_1 s_2 c_2 s_3}{q^2 - M_{c}^2} \right) \left( \frac{s_1 c_2 c_2 c_3}{q^2 - M_{t}^2} + \frac{-s_1 c_2 c_2 s_2 c_3}{q^2 - M_{t}^2} \right)
\]

\[
\approx \left( \frac{s_1 c_1 c_3}{q^2 - M_{u}^2} - \frac{c_2^2}{q^2 - M_{c}^2} - \frac{s_2^2}{q^2 - M_{t}^2} \right) \left( \frac{s_1 s_2 c_2 s_3}{(q^2 - M_{c}^2)(q^2 - M_{t}^2)} \right)^2
\]

Then $\epsilon = (1 + i) \frac{M_{12}}{M_{12}} \approx (1 + i) \frac{(s_1 c_1 c_3)}{(s_1 c_1 c_3)^2} \approx s_2 s_3 \Delta M \Delta \phi$

Putting $\Delta M = 1$. From the analysis of $b \rightarrow u$ and $b \rightarrow c$ decays (P325) we conclude $s_2 \approx 0.08$ and $s_3 \approx 0.04$.

Hence $\epsilon \approx (1 + i) 2.8 \times 10^{-3} \Delta M \Delta \phi$

At first sight the agreement is rather impressive. $s \approx 90^o$ will do.

Apparently however the detailed numerical agreement is not so good if the calculation is done more precisely. Ellis et al. [Nuc. Phys. B199, 213 (1976)], or Gianfrate et al. [PRL. 50, 1415 (1983)]

Claim $\epsilon \approx \frac{1}{10} s_2 s_3 \Delta M \Delta \phi \left[ 1 + s_2 \frac{M_{t}^2}{M_{c}^2} + \ln \left( \frac{M_{t}^2}{M_{c}^2} \right) \right]$

Then it is claimed that the small forward values of $s_2$ and $s_3$ require the mass of the top quark to be $M_t > 750$ GeV!
The K-M analysis puts no special restriction on $\epsilon'$, the CP violating parameter for $K^0 \rightarrow (\pi^0, \eta)$. In CP conserving decays we note that

$$\frac{a_{\text{CP}}}{a_{\text{CP}}} \approx 0.05 \ (p \text{bar})$$

Already it is known that $\frac{\epsilon'}{\epsilon} < 0.1$. Gilman & Naveau [Phys. Lett. B133, 443 (1983)] argue that $\epsilon'/\epsilon$ depends on $M_t$ in such a way that

$$M_{\pi \pi 0} \approx \epsilon'/\epsilon \approx 0.01$$

People at Princeton are presently involved in an experiment to measure $\epsilon'/\epsilon$ to accuracy 0.001. Perhaps their eventual result can be interpreted as indicating the top quark mass.

8. CP violation in $D^0, \bar{D}^0$ and $T^0, \bar{T}^0$ meson systems

In the 6 quark model there are several more meson pairs which might show CP violation:

$D^0 - \bar{D}^0 = c \bar{u} \not\equiv \bar{c} u$

$B^0 - \bar{B}^0 = b \bar{d} \not\equiv \bar{b} d$

$K^0 - \bar{K}^0 = s \bar{s} \not\equiv \bar{s} s$

$T^0 - \bar{T}^0 = t \bar{c} \not\equiv \bar{t} c$

However it will not be easy to demonstrate the CP violation in these systems even if it exists.

Reviewing the situation for $K^0$ decay we note several features conspire to make CP violation measurement possible:

- $T_L > T_S$ so $K^0, \bar{K}^0$ are experimentally quite distinct
- $T_L$ long enough to observe rate, vs $T$

- All decays of $K^0_S$ are to CP eigenstates, hence interference between $K^0_S \rightarrow CP$ states $\not\equiv K^0_L \rightarrow CP$ state will be fairly strong
- CP violation parameter $\epsilon$ is not too small, $\epsilon \approx 2 \times 10^{-3}$

On the first 3 counts the heavy quark systems are not favored.
For example, a decay of $D^0$ to a CP eigenstate is

$$D^0 \rightarrow \pi^+ \pi^- \quad \text{or} \quad D^0 \rightarrow K^+ K^-$$

Either way, amplifies an um ... c

So rate $\sim \Lambda^\frac{5}{2} \Theta c$ and these decays are suppressed.

In fact $D^0 \rightarrow K^+ K^- = 2.0 \times 10^{-3}$ has been measured.

Then we must expect $\Phi_{D^0} \approx \Phi_{D^0}$ as both can go to almost all possible final states with equal rates.

A look at the $K_{-} M$ matrix (p339) shows similar difficulty for the $D^0$ mesons. Thus a particular decay will be like

$$b \bar{u} \rightarrow b \bar{u} + u \bar{d} + c \bar{s} \quad \text{while} \quad b \bar{u} \rightarrow d \bar{u} + u \bar{d} + (s_{1/2})$$

The best hope seems to be for the $B^0_s$, as $b \bar{s} \rightarrow c \bar{s} + \bar{c} s \rightarrow F^+ F^-$ is a dominant branching decay mode. The $B^0_s$ has not been observed yet, but if it proves to have a lifetime $> 10^{-10}$ sec, as the $B^0$ apparently does, experiments may be possible.

We can make simplified estimates of the $CP$ violation parameter $\epsilon$ as in Section 7.

$$\epsilon \sim \frac{\text{Quark Mix}}{\text{Mix}}$$

$$D^0 \quad c \quad \text{disab} \quad \epsilon \sim \frac{\text{Quark Mix}}{\text{Mix}} \sim s_{1/2} S_{1/2} c_2 0 \quad \frac{s_{1/2} S_{1/2} c_2}{s_{1/2} c_2} \quad \text{small}$$

But

$$B^0_s \quad b \quad \text{disab} \quad \epsilon \sim \frac{\text{Quark Mix}}{\text{Mix}} \sim s_{1/2} S_{1/2} c_2 0 \quad \frac{s_{1/2} S_{1/2} c_2}{s_{1/2} c_2}$$

Direct reconstruction of $B^0_s \rightarrow F^+ F^-$ decays may prove difficult.

A better experimental signal will probably be in the charge asymmetry in $B^0_s \rightarrow \pi^\pm \pi^\mp$.

For $B^0_s - \bar{B}^0_s$ pairs produced in $e^+ e^-$ collisions, one would look for 2 leptons, one from each mesons, and measure $N^+ - N^-$ ...

[UPDATE: CP Violation in the B-hadron System Turned Out to Be Large ...]
A Footnote on $D^0\bar{D}^0$ Mixing

A less dramatic possibility than CP violation in the $D^0\bar{D}^0$ (or $B^+\bar{B}^0$...) system is charm (or beauty) oscillations. This is the analogue of strangenessless oscillations discussed on p. 326. Because the $D^0$ lifetime is so short, actual patterns of oscillation are very difficult to demonstrate.

But we can consider experiments which integrate over the entire lifetime of the $D^0\bar{D}^0$ mesons. For example, a prominent semi-leptonic decay of the $D^0$ is

$$ D^0 \rightarrow K^- \mu^+ \nu $$

while $\bar{D}^0 \rightarrow K^+ \mu^- \bar{\nu}$

If we start with a pure $D^0$ state, we can ask, what fraction of the decay muons are $\mu^+$, due to $D^0\bar{D}^0$ mixing?

Comparing with the expressions for $1^0_0$ and $1^0_-$ on top of p. 328, we infer:

$$ \frac{\# \text{ of } \mu^+}{\# \text{ of } \mu^-} = \frac{\int_0^{\infty} P_{D^0}(t) d\lambda}{\int_0^{\infty} P_{\bar{D}^0}(t) d\lambda} = \frac{\Delta m^2 + \Delta \Gamma^2}{2 \Gamma_{\text{ave}}^2 + \Delta m^2 + \Delta \Gamma^2} $$

where $\Delta m = M_{D^0_L} - M_{D^0_S}$; $\Delta \Gamma = \Gamma_{D^0_S} - \Gamma_{D^0_L}$; $\Gamma_{\text{ave}} = \Gamma_{D^0_S} + \Gamma_{D^0_L}$

In an actual experiment, we are likely to have produced a $D^0\bar{D}^0$ pair rather than only a $D^0$ (why?). This complicates matters, but allows a measurement of

$$ \frac{\#(\mu^+\mu^-)}{\#(\mu^+\mu^-)} $$

Here, like even muons occur if one of the $D^0$ or $\bar{D}^0$ has 'mixed' to its antiparticle. An analysis of the above ratio is given by Kingsley, Phys. Lett. 63B, 329 (1976). He claims the ratio should be

$$ \frac{\Delta m^2 + \Delta \Gamma^2}{2 \Gamma_{\text{ave}}^2 + \Delta m^2 + \Delta \Gamma^2} $$

Also:

$$ \frac{\Delta \Gamma}{\Gamma_{\text{ave}}} $$

The best (two errors too good) limit on this ratio is $< 0.006$.

Since $\Gamma_{\text{ave}} \approx 4 \times 10^{13}\text{ s}^{-1}$, we infer $M_{D^0_L} - M_{D^0_S} < 10^{-3} \text{eV}$

(Compare with $M_{K^0_L} - M_{K^0_S} = 3 \times 10^{-6} \text{eV}$). Similarly $\frac{\Delta \Gamma}{\Gamma_{\text{ave}}} < 0.2$.

Needs better experiments!
3/12/98

\[ K^0 \rightarrow \pi^0 \nu \bar{\nu} \]

**Penguin Diagram**

\[ (1 - \frac{A^2}{A^2}) \frac{1}{A^2}(1 - \frac{A^2}{A^2}) \frac{1}{A^2} \]

**\( A(\pi^0 \rightarrow \pi^0 \nu \bar{\nu}) \)**

\[ V_{ts}^* V_{td} \]  

**\( A(\bar{K}^0 \rightarrow \pi^0 \nu \bar{\nu}) \)**

\[ V_{ts}^* V_{td} \]  

**\( K_1 = \frac{K_0 \pm \bar{K}_0}{2} \)  \quad K_2 = \frac{K_0 - \bar{K}_0}{2} \)**

**\( A(K_1 \rightarrow \pi^0 \nu \bar{\nu}) = -\frac{\lambda^2}{\lambda^2} (1 - p) \sqrt{2} \)**

**\( A(K_2 \rightarrow \pi^0 \nu \bar{\nu}) = \frac{\lambda^2}{\lambda^2} \eta \sqrt{2} \)**

**\( K_L = K_2 + \epsilon K_1 \)**

**\( A(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \frac{\lambda^2}{\lambda^2} \eta \sqrt{2} (i \eta - \epsilon (1 - p)) \)**

\(|\eta| \ll |1 - p| \)

**So \( A(K_L \rightarrow \pi^0 \nu \bar{\nu}) \) is dominated by \( A(K_2 \rightarrow \pi^0 \nu \bar{\nu}) \)**

which is called **Direct CP Violation**

The smaller piece, \( \epsilon (1 - p) \), is called **Indirect CP Violation**, in

**CP Violation in the Mixing**, since it depends on \( K_L \neq K_{2} \) due to

**CP Violations**

\[ \beta(\pi^0 \nu \bar{\nu}) = C(\nu^0 \pi^0) = C(\bar{\nu} \bar{\pi}^0) \]

\[ C(\nu^0) = +1 \quad \beta(\bar{\nu} \bar{\pi}^0) = -1 \quad C(\bar{\nu} \bar{\pi}^0) = -1 \]

\[ \text{S\&Y} \quad \text{get fame if } \bar{\nu} \bar{\pi}^0 \rightarrow \nu \pi^0 \text{ vizual} \]

\[ \bar{\nu} \bar{\pi}^0 \text{ comes from } Z \Rightarrow S = 0 \text{ and } \beta = +1 \]

\[ S \text{ must have } S_{\nu \bar{\pi}} = 1 \]

\[ \text{Don't need to know } \text{LW} \]

\[ J_{L^0} = 0, \quad J_{P^0} = 0, \quad S_{\nu \bar{\pi}} = 1 \Rightarrow L_{\nu \bar{\pi}}(\bar{\nu} \bar{\pi}^0) = 1 \]

\[ \Rightarrow \text{CP}(\pi^0 \nu \bar{\nu}) = (1)(1)(-1)(-1)(-1)(-1)(-1)(-1) = +1 \]

**What is different if \( K^0 \rightarrow \pi^0 \mu^+ \nu \)**

Cannot conclude that \( S_{\nu \mu} = 1 \) only, though. This is a good approx

For \( \mu^+ \nu \)**

\[ \text{Since the } \nu \text{'s are neutrinistic} \]

**CP(\pi^0 \mu^+) Most of +1**

**CP(\pi^0 \mu^-) Some +1, some -1**
For $\theta$, the $Y$ diagram may dominate, but shouldn't change the argument about $V_{Ud}$. 