Ph 406 Problem Set 9
Due April 26, 1993

The spin-1 vector mesons can be taken to have quark content: \( \rho^0 = (u\bar{u} - d\bar{d})/\sqrt{2} \), \( \omega^0 = (u\bar{u} + dd)/\sqrt{2} \), \( \phi = ss \), \( J/\psi = cc \), and \( \Upsilon = bb \).

1. The decays \( V \rightarrow e^+e^- \) proceed via a single intermediate photon, where \( V \) is a vector meson. Suppose the decay rates \( \Gamma(V \rightarrow e^+e^-) \) are independent of quark mass and the meson mass, but do depend on quark charge. Predict the relative decay rates to \( e^+e^- \) for the five vector mesons.

2. In the decay \( V \rightarrow \pi^0\gamma \) the meson spin changes from 1 to 0. Hence this must be an \( M1 \) magnetic dipole transition. In the quark model the decay rate depends on the size of the relevant quark magnetic moments. Suppose the quarks have Dirac moments \( Q_q/2m_q \) where \( m_u \approx m_d \approx \frac{2}{3}m_s \). Predict the relative decay rates to \( \pi^0\gamma \) for the \( \rho^0 \), \( \omega^0 \), \( \phi \) mesons.

3. (a) The \( \psi'(3685) \) vector meson can decay to \( \chi(3415) + \gamma \). The \( \chi \) particle is believed to be a \( ^3P_0 \ c\bar{c} \) state. If so, predict the angular distribution of the \( \gamma \) relative to the direction of the electron supposing the \( \psi' \) is produced in a colliding-beam experiment \( e^+e^- \rightarrow \psi' \rightarrow \chi\gamma \). Recall that at high energies the one-photon annihilation of \( e^+e^- \) proceeds entirely via transversely polarized photons \( (S_z = \pm 1) \).

(b) The charmed meson \( D^0 \) can decay to \( K\pi \) via the Cabbibo-favored \( W \)-exchange diagram (with gluons not shown)

If this were the only possible diagram, predict the ratio of branching ratios:

\[
\frac{\Gamma(D^0 \rightarrow K^-\pi^+)}{\Gamma(D^0 \rightarrow \bar{K}^0\pi^0)}.
\]

You may assume the \( K\pi \) system is in an isospin-\( \frac{1}{2} \) state.

Draw any other Cabbibo-favored diagrams for these decays.

4. There are four spin-0 mesons that contain one bottom quark: \( B^+_u = u\bar{b} \), \( B^+_d = d\bar{b} \), \( B^+_s = s\bar{b} \), and \( B^+_c = c\bar{b} \). These decay via the weak interaction by two graphs with roughly equal strength (the ‘spectator’ model):
Here we consider only nonleptonic final states. Suppose the four final-state quarks form exactly two mesons (as happens a few percent of the time). List the two dominant two-body decays for each of the four bottom mesons.

A complication arises for the $B_c$ meson. The charm quark has a slightly shorter lifetime than the bottom quark. Hence there are two more prominent two-body decays of the $B_c$ involving $c \to W q$ rather than $b \to W q$ transitions. List these.

According to the measured values of the C-K-M matrix elements

$$\frac{V_{ub}}{V_{cb}} \approx \frac{V_{us}}{V_{ud}} \approx \frac{V_{cd}}{V_{cs}} \approx \lambda = \text{Cabbibo angle}.$$  

List the two-body nonleptonic decays of the four bottom mesons that are suppressed by one power of $\lambda$ in the matrix element (and hence by $\lambda^2 \approx 1/25$ in rate).

Note that $D^+ = c\bar{d}$, $D^0 = c\bar{u}$, and $D_s^0 = c\bar{s}$. If a meson is produced from, say, a $d\bar{d}$ state it could be a $\pi^0$, $\eta$, $\rho^0$, or $\omega^0$. Here it is sufficient to list only the $\pi^0$...

5. Both the $B_d^0 = d\bar{b}$ and $\bar{B}_d^0 = \bar{d}b$ can decay to common final states, such as $J/\psi K_S^0$ as you found in prob. 4. Hence there are transitions between $B^0$ and $\bar{B}^0$ and so the states of definite mass and lifetime are not these but

$$B_1 = \frac{B^0 + \bar{B}^0}{\sqrt{2}}, \quad \text{and} \quad B_2 = \frac{B^0 - \bar{B}^0}{\sqrt{2}}.$$  

So far this is much like the $K^0$-$\bar{K}^0$ system, ignoring the possibility of CP violation. (Can you readily show that $B_1$ and $B_2$ are the eigenstates of the $2 \times 2$ Hamiltonian, assuming the off-diagonal elements are equal, as is the case for time-reversal invariance (CP conservation)?)

In practice the lifetimes of $B_1$ and $B_2$ are essentially identical (unlike the case for $K_1$ and $K_2$), so

$$|B_1(t)| = e^{-\Gamma t/2}e^{i\Delta m_1 t}|B_1(0)|,$$

$$|B_2(t)| = e^{-\Gamma t/2}e^{i\Delta m_2 t}|B_2(0)|.$$  

Deduce the probabilities $P(t)$ and $\bar{P}(t)$ of having a $B^0$ and $\bar{B}^0$ at time $t$ in terms of the initial amplitudes $|B^0(0)|$ and $|\bar{B}^0(0)|$ and the mass difference $\Delta m \equiv m_1 - m_2$.

Suppose at $t = 0$ we have a pure $B^0$. What is the probability that it decays as a $\bar{B}^0$ rather than as a $B^0$, in terms of the ‘mixing’ parameter $x \equiv \Delta m/\Gamma$?

For the $B_d^0$, $x_d$ has been measured to be 0.7, and it is expected that for the $B_s^0$, $x_s \approx 10$. 