1. The elastic scattering reactions $\nu_\mu e \rightarrow \nu_\mu e$ and $\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$ can only take place via $Z^0$ exchange. Estimate the total cross sections for these reaction at high energies.

Recall that the piece of the gauge-covariant derivative relevant to the $Z_0$ can be written

$$i \frac{g}{\cos \theta_W} Z(I_3 - \sin^2 \theta_W Q),$$

where $g^2/\cos^2 \theta_W M_Z^2 = G/\sqrt{2}$, $I_3$ refers to weak isospin, and $Q$ is electric charge in units of $+e$. Also, the total cross section is the integral over the angular distribution, which should be examined separately for left- and right-handed electrons (in the center-of-mass frame, of course)....

2. The final-state products from $\bar{p}p$ annihilation at rest can include $K^0$ and $\bar{K}^0$ mesons which can then be used for studies of $CP$ violation. For Kaons so produced:

(a) Predict the value of the $CP$-violating asymmetry

$$A = \frac{\Gamma(K^0_{\text{phys}} \rightarrow \pi^+ e^- \bar{\nu}) - \Gamma(\bar{K}^0_{\text{phys}} \rightarrow \pi^- e^+ \nu)}{\Gamma(K^0_{\text{phys}} \rightarrow \pi^+ e^- \bar{\nu}) + \Gamma(\bar{K}^0_{\text{phys}} \rightarrow \pi^- e^+ \nu)},$$

where $K^0_{\text{phys}}$ means a neutral Kaon produced as a $K^0$. You may approximate the Kaon mass eigenstates $K_S$ and $K_L$ as

$$K_S \approx K_1 + \epsilon K_2 \quad \text{and} \quad K_L \approx K_2 + \epsilon K_1,$$

and assume $CPT$ invariance.

(b) Explain why an observation that $A \neq 0$ could be considered as direct evidence for time-reversal violation, without invoking the usual argument that $CPT$ invariance plus $CP$ violation implies $T$ violation.

3. From data on the $Z^0$ from LEP and Standard Model calculations of radiative corrections some people infer that the Higgs boson ($H^0$) has mass less than $2M_Z$. If so, the $H^0$ will be difficult (though not impossible) to detect at LHC or SSC. Here you are asked to consider the merits of producing the Higgs boson in $\mu^+\mu^-$ collisions.

When obtaining numerical values you may assume that $M_H = 120$ GeV.

(a) Estimate the total decay width $\Gamma(H \rightarrow \text{all})$ in GeV assuming $M_Z < M_H < 2M_W$, noting that the vertex factor for Higgs decay to fermions is $-im_f \sqrt{2G_F}$. Here $m_f$ is the mass of a fermion. Ignore possible effects due to virtual $W$’s and $Z$’s.

(b) Calculate the peak cross section in cm$^2$ for the reaction $\mu^+\mu^- \rightarrow H \rightarrow \text{all}$.

(c) At the hypothetical $\mu^+\mu^-$ colliding-beam facility suppose the spread in center-of-mass energy is $\Delta E/E \approx 0.1%$. Deduce the effective production cross section for the reaction of part b in view of this energy spread.
(d) Compare the result of part c to the cross section for continuum production of the dominant final state you have assumed for Higgs decay.

(e) The muon beams could be polarized. Explain what choice of polarization would suppress the ‘background’ of part d.