Abstract

Majorana conceived that electrically neutral spin-1/2 particles might be their own antiparticles, and surmised that neutrinos could be examples thereof. If so, a "neutrino" (or "antineutrino") would actually have equal amplitudes, of strengths $1/\sqrt{2}$, to be either neutrino and antineutrino. While in general only one of the neutrino or antineutrino has significant interaction according to the $V-A$ theory of the weak interaction, charged-pion decay, $\pi \rightarrow \mu \nu$, is exceptional in that the muon is almost at rest in the frame of the pion, such that the lefthanded chirality muon has essentially equal amplitudes to have both positive and negative helicity. As the pion is spinless, the muon and neutrino must have the same helicity (opposite spin components $S_z$ along the direction of the muon momentum), such that helicity combinations $\mu^+ \nu^+$ and $\mu^- \nu^-$ exist with nearly equal amplitude if the neutrino is a Majorana state, while if the (anti)neutrino is a Dirac state only one of these is possible and parity is maximally violated in the decay. The experimental evidence (from 1957, 10 months before the formulation of the $V-A$ theory) favors maximal parity violation in charged-pion decay, which thereby excludes that the neutrino is a Majorana state. Further, the charge-pion decay $\pi \rightarrow e \nu$ would not be suppressed in case of Majorana neutrinos, and its rate would be similar to that for $\pi \rightarrow \mu \nu$, in contradiction to data available in the late 1940’s. A related argument is that muon-neutrino beams from charged-pion decay would be nearly equal mixtures of neutrino and antineutrino if these were Majorana states, whereas the beam would be essentially pure neutrino from $\pi^+ \rightarrow$ decay (and essentially pure antineutrino from $\pi^- \rightarrow$ decay) if the neutrinos are Dirac states. Then, for example, the cross sections for nominal neutrino-nucleon and antineutrino-nucleon reactions would be equal for Majorana neutrinos, rather than in the ratio 2:1 as observed for muon-neutrino charged-current interactions above the resonance region, and predicted by the assumption of Dirac neutrinos. In sum, Majorana neutrinos are excluded by the data from numerous past experiments, with the implications that searches for neutrinoless double-beta decay will continue to report null results, and that explanations for the low masses of observed neutrinos via a see-saw mechanism involving Majorana neutrinos are not viable.
The main argument of this paper, that data from past neutrino experiments have long since excluded that the neutrinos are Majorana states (without this being recognized), is very simple, as essentially given in the Abstract. In the body of the paper we elaborate the argument slightly, and review the historical context of Majorana neutrinos.

In 1937, Majorana [1] remarked that a possible variant Dirac 4-spinor states [2] of electrically neutral, spin-1/2 particles is that they are their own antiparticles, and suggested that neutrinos may be a realization of this in Nature. Pauli noted in eqs. (99)-(100) of [3] that any Dirac 4-spinor $\psi$ can be expressed as the sum of two Majorana states, $\psi_1$ and $\psi_2$, which are composed of $\psi$ and its charge-conjugate (antiparticle) state $\bar{\psi}$ [4],

$$
\psi = \frac{\psi_1 + i\psi_2}{\sqrt{2}}, \quad \psi_1 = \frac{\psi + \bar{\psi}}{\sqrt{2}}, \quad \psi_2 = \frac{\psi - \bar{\psi}}{\sqrt{2}i}, \quad \bar{\psi} = i\gamma^2 \psi^*, \quad \bar{\psi}_{1,2} = \psi_{1,2},
$$

where $\gamma^\mu = (\gamma^0, \gamma)$, $\mu = 0, 1, 2, 3$, are the Dirac $4 \times 4$ matrices [2]. For the Majorana theory to be more than the identity (1), once relations (2)-(3) are established, it must be that the Dirac state $\psi$ is replaced by either Majorana state $\psi_1$ or $\psi_2$. Here, we suppose that a Dirac neutrino (or antineutrino) state $\psi$ is instead the Majorana state $\psi_1 = (\psi + \bar{\psi})/\sqrt{2}$. A key feature is the $1/\sqrt{2}$ in the Majorana state, which factor seems to have been neglected in works like [5] that are often cited as showing how processes which could occur with Dirac neutrinos cannot distinguish these from Majorana neutrinos.

In Majorana’s time the theory of the weak interaction was that due to Fermi [6, 7]. An early application of Majorana’s vision to Fermi’s theory was by Furry [8], who pointed out that Majorana neutrinos permit the phenomenon of neutrinoless double-beta decay, $A(Z) \rightarrow A'(Z + 2)e^-e^-$, of a nucleus $A(Z)$, in addition to the 2-neutrino double-beta decay $A(Z) \rightarrow A'(Z + 2)e^-e^-\nu\bar{\nu}$ studied by Goeppert-Mayer [9]. In the Fermi theory, one expects the matrix elements for the two forms of double-beta decay to be similar, but the 5-body final state of 2-neutrino double-beta decay has much smaller phase volume than does the 3-body final state of neutrinoless double-beta decay, so Furry predicted that the latter has much higher rate (shorter lifetime). Present experimental limits [10, 11] on the lifetime for neutrinoless double-beta decay are several orders of magnitude longer than that of observed 2-neutrino double-beta decays.

Following the discovery of parity violation in the weak interaction [12], the weak-interaction coupling was identified as $V - A$ [13, 14] (rather than $S + T$, scalar plus tensor, as had been believed for many years based on inaccurate experiments [15]), i.e., $(1 - \gamma^5)\gamma^\mu$, where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. At this time, the notion of right(left)-handed chirality states was also introduced [16, 17],

$$
\psi_{R,L} = \frac{1 \pm \gamma^5}{2} \psi, \quad \bar{\psi}_{R,L} = \frac{1 \mp \gamma^5}{2} \bar{\psi}.
$$

In the $V - A$ theory, the weak interaction involves lefthanded chirality spin-1/2 particles, which we write as $u_L$, and righthanded antiparticles, $v_R$. 

The chirality states for nonzero mass differ slightly from the helicity (originally “spirality” [18]) spin-1/2 states,
\[ \psi_\pm = \frac{1 \pm \gamma_0 \hat{p}(\theta, \phi) \cdot \gamma \gamma_5}{2}, \]
for which the positive(negative) helicity state has spin component parallel(antiparallel) to the direction of motion (for both particles and antiparticles). A righthanded chirality particle state is dominantly positive helicity, but has a component of order \( m/E \) to have negative helicity, etc.

Spin-1/2 particle (and antiparticle) states \( \psi \) obey the free-space Dirac equation
\[ i\gamma^\mu \partial_\mu \psi = m\psi. \]
The operator \( \gamma_0 \hat{p}(\theta, \phi) \cdot \gamma \gamma_5 \) commutes with \( \gamma^\mu \), given that \( \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2\eta^\mu\nu \) and \( \gamma^5\gamma^\mu = -\gamma^\mu\gamma^5 \), where the diagonal matrix \( \eta^\mu\nu \) has diagonal elements 1, -1, -1, -1. Hence, the helicity states (5) also obey the Dirac equation (6). In contrast, the chirality states (4) obey the coupled Dirac-like equations,
\[ i\gamma^\mu \partial_\mu \psi_{R,L} = m\psi_{L,R}, \quad i\gamma^\mu \partial_\mu \tilde{\psi}_{R,L} = m\tilde{\psi}_{L,R}, \]
as consistent with the Lagrangian for free Majorana fields [17],
\[ \mathcal{L} = \frac{1}{2} \left( \bar{\psi} i\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi \right). \]

As does the electromagnetic interaction, with its \( \gamma^\mu \) coupling to spin-1/2 charged particles, the \( V - A \) weak interaction obeys helicity conservation for (anti)particles with energy \( E \) large compared to their rest mass \( m \).

Furthermore, the coupling factor \( G \) in Fermi’s theory of beta decay [7] is now understood to be related to the coupling (of strength \( g \)) of leptons and quarks to \( W^\pm \) vector gauge bosons (charged currents) [21] such that
\[ G = \frac{\sqrt{2}g^2}{8m_W^2} = \frac{\sqrt{2}e^2m_Z^2}{8m_W^2(m_Z^2 - m_W^2)}, \]
where \( e \) is the charge of the electron and \( Z^0 \) is the neutral vector gauge boson [22]. That is, in the Standard Model of electroweak interactions, the coupling strength in beta decay (charge-current neutrino interactions) can be determined by experiments that do not involve neutrinos.

Considering now the possibility that the light neutrinos in the Standard Model are actually Majorana states, the \( V - A \) coupling would be to the lefthanded state,
\[ \psi_{1L} = \frac{1 - \gamma^5}{2} \psi_1 = \frac{1 - \gamma^5}{2\sqrt{2}} (u + v) = \frac{u_L + v_R}{\sqrt{2}}, \]
where the Dirac 4-spinor \( v \) is the antiparticle of \( u \) [23, 24]. There would be no coupling to the righthanded state,
\[ \psi_{1R} = \frac{1 + \gamma^5}{2} \psi_1 = \frac{1 + \gamma^5}{2\sqrt{2}} (u + v) = \frac{u_R + v_L}{\sqrt{2}}, \]
which is a sterile neutrino [25], and is the antiparticle of the Majorana state \( \psi_{1L} \). That is, although a general Majorana state is its own antiparticle, the left- and righthanded Majorana states are not their own antiparticles (and do not obey the nominal Dirac equation, but rather the coupled equations \( i\gamma^\mu \partial_\mu \psi_{1L,1R} = m \psi_{1R,1L} \)).

In all neutrino experiments to date, except the PTOLEMY search for cosmic microwave background neutrinos [26], the neutrinos are ultrarelativistic \( (E_\nu \gg m_\nu) \) such that their chirality states are essentially helicity states as well, \( \nu_L \approx \nu_- \) and \( \bar{\nu}_R \approx \bar{\nu}_+ \). In the \( V - A \) theory with Dirac neutrinos, the \( \nu_L \) is paired with a righthanded-chirality charge anti lepton \( \bar{l}_R^+ \), while the \( \bar{\nu}_R \) is paired with \( l^+_L \), in a production reactions such as \( \pi^+ \rightarrow \bar{l}_R^+ \nu_L \) and \( \pi^- \rightarrow l^+_R \bar{\nu}_R \). In the decay of a spinless charged pion, the lepton and neutrino must have the same helicity. For (relativistic) neutrinos, this restricts the possibilities to \( \pi^+ \rightarrow \bar{l}_R^+ \nu_L^- \) and \( \pi^- \rightarrow l^+_R \bar{\nu}_R^+ \), where \( \bar{l}_R^+ \) is a righthanded-chirality, negative-helicity antilepton, etc. If the (anti)leptons were relativistic, as for electrons/positrons in pion decay, their right(left)-handed-chirality states would be essentially positive(negative) helicity, such that charged-pion decay electrons/positrons is suppressed. In contrast, muons from charged-pion decay have only 4-MeV kinetic energy in the frame of the pion, such that a muon chirality state is roughly an equal mixture of positive and negative helicity states, which permits decay to the “wrong helicity” muons with high probability.

For Majorana neutrinos, charged-pion decay could also proceed via \( \pi^+ \rightarrow \bar{l}_R^+ \nu_R \approx \bar{l}_{R,+}^+ \nu_{R,+} \) and \( \pi^- \rightarrow l^+_R \nu_L \approx l_{L,+}^+ \nu_{L,-} \), which violate lepton number conservation. Charged-pion decay to electrons/positrons would not be suppressed in these “right helicity” decays, and the decay rates to electrons and muons would be roughly equal, rather than in the observed ratio of \( (m_e/m_\mu)^2 \) [15].

For Majorana neutrinos, the charge-pion decay \( \pi \rightarrow \mu \nu \) would have roughly equal numbers of muons with positive and negative helicity, and hence would exhibit little parity violation, in contrast to experiments [27, 28] that observe “maximal” parity violation.

In case of Dirac neutrinos, \( \pi^+(\pi^-) \)-decay leads primarily to \( \nu_\mu (\bar{\nu}_\mu) \), whereas for Majorana neutrinos roughly equal numbers of \( \nu_\mu \), \( \bar{\nu}_\mu \) and \( \bar{\nu}_e \) \( (\bar{\nu}_\mu, \nu_\mu \text{ and } \nu_e) \) would be produced. The Majorana-neutrino hypothesis conflicts with evidence from experiments with neutrino beams from charged-pion decay, where in particular charged-current (anti)neutrino-nucleon scattering experiments would have equal cross sections for nominal \( \nu_\mu N \) and \( \bar{\nu}_\mu N \) initial states if the neutrinos were Majorana rather than Dirac. Rather, data from a large number of experiments, reviewed in Fig. 49.1 of [29], show that the observed ratio of these cross sections is 2:1, in agreement with models based on Dirac neutrinos.

Leptonic beta-decay, such as \( \mu^- \rightarrow e^- \nu_\nu \), can be well-calculated in the Standard \( V - A \) theory, where the reaction reads \( \mu^- \rightarrow e^- \bar{\nu}_e R \nu_\mu L \), with results that agree with experiment to within a few percent (as noted already in [13]). If the neutrinos were Majorana states (10), one might infer that the muon lifetime would be 4 times longer than if they are Dirac states, noting the the electron is relativistic in muon decay, such that only one of \( \nu \) or \( \bar{\nu} \) couples at each vertex in the decay diagram, leading to a factor of 1/2 in the decay amplitude compared to that for Dirac neutrinos. However, as pointed out in the last line of [30], in case of Majorana neutrinos there exists a second, crossed diagram, in which the “second” neutrino comes from the muon vertex rather then the electron vertex, while the “first” neutrino comes from the electron vertex. These two diagrams have equal amplitudes,
each 1/2 that of the single diagram in case of Dirac neutrinos, and hence the lifetime of the muon (and Fermi’s coupling constant $G$) is the same for either Dirac or Majorana neutrinos. For additional discussion of this issue, including possible distinguishing effects in the decay angular distribution, see [31, 32].

The neutrino was determined to have negative helicity in an extrêmement ingénieuse experiment [33] on the electron-capture process $^{63}\text{Eu}^{152} + e^- \rightarrow ^{62}\text{Sm}^{152} + \nu$, in which the neutrino energy is 840 keV and the captured electron was bound by about 100 keV. The lefthanded chirality electron had nearly equal probability to have either positive or negative helicity, such that Majorana neutrinos and antineutrinos would have roughly equal probability of being produced, and the net helicity of the final state neutrino would be near zero. The latter was excluded by about four standard deviations in the experiment, which translates into a similar exclusion of Majorana neutrinos.

Turning to nuclear-reactor experiments, in the Standard Model of beta-decay (with Dirac neutrinos) only antineutrinos are produced in reactions of the form $n \rightarrow p e^- \bar{\nu}_e$ (while only neutrinos are produced in solar fusion reactions). Hence, most reactor-based neutrino experiments are designed to observed the inverse beta-decay reaction $\bar{\nu}_e p \rightarrow e^+ n$ [36], where the neutron can be detected as a delayed coincidence after thermalization by capture on nucleus, as first proposed by Cowan and Reines [37, 38]. However, the earliest proposal to detect neutrinos produced in nuclear reactors was due to Pontecorvo [39], who supposed that neutrinos and antineutrinos were not distinct (i.e., were Majorana states), such that reactor neutrinos could lead to reactions such as $\text{Cl}^{37}(\nu, e^-)\text{A}^{37}$, which require a neutrino rather than an antineutrino.

Pontecorvo’s suggestion was implemented by Davis [40, 41] in a 4-m$^3$ detector close to the Brookhaven Lab nuclear reactor, but only 6 m underground, with a result consistent with the rate of interactions expected from cosmic-ray electron neutrinos. Nonetheless, this result has been considered as evidence against Majorana neutrinos [42]. Davis’ reactor-experiment search for Majorana neutrinos has never been repeated, although he went on to make important measurements of solar neutrinos (not antineutrinos) via the chlorine reaction in a 400-m$^3$ detector deep underground [43].

Solar-neutrino experiments other than Davis’ have searched for a inverse beta-decay events, $\bar{\nu}_e p \rightarrow e^+ n$, producing limits [44, 45, 46] that the solar-antineutrino flux is small compared to that of solar neutrinos, which appears to exclude that solar neutrinos are Majorana states, although a quantitative assessment of the exclusion remains to be made.

Following the success of Cowan and Reines’ reactor-antineutrino experiment [47, 48], and the experimental evidence that the electron and muon are associated with different neutrinos [49], Pontecorvo suggested [25] the possibility of oscillations between the two (or more) neutrino types. Initial experiments used a single detector at a single distance from a nuclear reactor, and compared the rate of detected antineutrino interactions (inverse beta-decay) with expectations based on the neutrino flux as inferred from the reactor power. The first experiment of this type to obtain a reasonably significant result was [50] which reported (1981) a ratio $0.955 \pm 0.035$ (stat.) $\pm 0.110$ (syst.) of the observed to expected rates of inverse beta-decay reactions. It is now known that for a detector at the optimal distance from the reactor the observed rate would be 0.91 of that expected in the Standard Model assuming no oscillations [51]. A recent result [52], taking into account the existence of neutrino oscillations, found a ratio $0.946 \pm 0.022$ of the observed rate of inverse beta-decay reactions.
reactions compared to that of a particular model of the beta-decays in the nuclear reactor complex.

If the neutrinos produced by the nuclear reactor were lefthanded Majorana states (10), for a given flux of such neutrinos the rate of inverse beta-decay $\bar{\nu}_R p \rightarrow e^+ n$ would be only $1/2$ that from Dirac neutrinos, as lefthanded Majorana neutrinos have only a 50% probability of being a righthanded antineutrino. Hence, reactor experiments that study inverse beta-decay have excluded that the (low-mass) neutrinos produced in nuclear beta decays are Majorana states, by 5 standard deviations in 1981 [50] and more recently by 20 standard deviations [52].

With light Majorana neutrinos excluded by past experiments, it is not surprising that searches for neutrinoless double-beta decay have produced null results [53]. However, it is still possible that neutrinoless double-beta decay occurs via exchange of a heavy neutral particle (with mass $> m_Z/2$) associated with lepton-number violation, which particle would effectively be a Majorana state [54]. Of course, in this case the rate for neutrinoless double-beta decay would be extremely low compared to that for mediation by a light Majorana neutrino.

Despite lack of experimental evidence for Majorana neutrinos, enthusiasm for their existence remains high in view of the “see-saw” mechanism [55, 56, 57, 58] that provides a possible explanation for low-mass Majorana neutrinos together with partners of mass at the grand-unification scale. A conclusion of this paper is that one must look further for explanations of the low mass of observed neutrinos.

Quasiparticles labeled Majorana fermions have been reported in condensed-matter experiments [59, 60]. These nonpropagating “Majorana zero modes” have only one spin state, with a participating electron that is shared between two surfaces of the sample. Such states have been described [61] as “spin-zero half-fermions,” that have only electromagnetic interactions, and are rather different entities than the weakly interacting Majorana-neutrino chirality states considered here.

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The presently accepted value of Fermi’s constant $G$ is about 4 times larger than that inferred in 1949 from muon decay, $\mu \rightarrow e\nu\bar{\nu}$, presumably because it was thought earlier that the final-state neutrinos could each have two spin states, rather than being only left-handed as in the $V-A$ theory. See T.D. Lee, M. Rosenbluth and C.N. Yang, Interaction of Mesons with Nucleons and Light Particles, Phys. Rev. 75, 905 (1949), http://physics.princeton.edu/~mcdonald/examples/EP/lee_pr_75_905_49.pdf.

Some people (for example, [5]) define the Majorana neutrino chirality states to be self-conjugate, with the consequence that such states contain all four of $u_R, u_L, v_R$ and $v_L$. However, in the $V-A$ theory of the weak interaction only the $u_L$ and $v_R$ participate, such that these self-conjugate states are not well “matched” to the formalism of the weak interaction.

Some people emphasize the Majorana helicity states $\psi_\pm = \bar{\psi}_\pm = (u_\pm + v_\pm)/\sqrt{2}$ where $v$ is the antiparticle of $u$. Then, in the $V-A$ theory of the weak interaction, only the states $(1 - \gamma^5)\psi_\pm/2$ participate. This leads to the same conclusions as in the text, but the arguments are slightly longer.


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In a model of neutrinoless double-beta decay via the exchange of a virtual, left-handed Majorana neutrino, the spin-1/2 propagator is combined with two factors of $1 - \gamma^5$, leading to a factor in the matrix element of $(1 - \gamma^5)(\not\!q - m_\nu)(1 - \gamma^5) = -2m_\nu(1 - \gamma^5)$. Hence, the rate includes a factor of $(m_\nu/E_\nu)^2$, where $m_\nu = \sum_i U_{ei}^2 m_i$ in a 3-neutrino scenario with neutrino mass states $m_i$ and MNS mixing matrix $U_{\alpha i}$, and $E_\nu$ is a characteristic energy of the virtual neutrino. This factor is small, which would heavily suppress the rate of neutrinoless double-beta decay compared to the early estimate of Furry [8] if Majorana neutrinos existed.


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