

Green's Function for a Conducting Plane with a Hemispherical Boss

Kirk T. McDonald

Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544

(April 23, 2002)

1 Problem

What is the electric potential in cylindrical coordinates $\phi(r, \theta, z)$ when a charge q is located at $(r_0, z_0 > 0)$ and there is a grounded conducting plane at $z = 0$ that has a (conducting) hemispherical boss of radius $a < b = \sqrt{r_0^2 + z_0^2}$ whose center is at the origin. What is the electrostatic force on the charge q for the case that $r_0 = 0$?

2 Solution

We use the image method.

First, we bring the hemispherical boss to zero potential by imagining that a charge $q' = -qa/b$ is placed at distance a^2/b along the line from the origin to charge q . The cylindrical coordinates of charge q' are

$$\frac{a^2}{b^2}(r_0, 0, z_0). \quad (1)$$

Next, to bring the plane $z = 0$ to zero potential, we add images charges for both q and q' . Namely, we imagine charge $q'' = -q$ at

$$(r_0, 0, -z_0), \quad (2)$$

and $q''' = -q' = qa/b$ at

$$\frac{a^2}{b^2}(r_0, 0, -z_0). \quad (3)$$

Then, both the plane $z = 0$ and the spherical shell of radius a about the origin are at zero potential.

The potential at an arbitrary point (r, θ, z) outside the conductor is therefore

$$\phi(r, \theta, z) = \frac{q}{r_1} - \frac{q}{r_2} - \frac{qab}{r_3} + \frac{qab}{r_4}, \quad (4)$$

where

$$r_{1,2} = \sqrt{r^2 - 2rr_0 \cos \theta + r_0^2 + (z \mp z_0)^2}, \quad (5)$$

and

$$r_{3,4} = \sqrt{b^4 r^2 - 2a^2 b^2 r r_0 \cos \theta + a^4 r_0^2 + (b^2 z \mp a^2 z_0)^2}. \quad (6)$$

When $r_0 = 0$, the force on charge q is in the $-z$ direction, with magnitude

$$F = \frac{q^2}{4z_0^2} + \frac{q^2 a/z_0}{(z_0 - a^2/z_0)^2} - \frac{q^2 a/z_0}{(z_0 + a^2/z_0)^2} = \frac{q^2}{4z_0^2} + \frac{4q^2 a^3 z_0^3}{(z_0^4 - a^4)^2}. \quad (7)$$