No Bootstrap Spaceships

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1 Introduction

A bootstrap spaceship is an isolated, self-propelled device that does not emit any matter or radiation.[1,2] That is, the center of mass/energy of such a device could accelerate without application of any "external" force, or consumption of any internal propellant that is somehow exhausted.

This note concerns devices that combine matter and electromagnetic fields. Some comments on all-mechanical devices are in Appendix B below.

While most people consider that "bootstrap spaceshops" are impossible, a persistent minority argues that they can exist.[3]

1.1 The Center-of-Energy Theorem

A reason why there are no "bootstrap spaceships" is given by the so-called center-of-energy theorem,[4] that the total linear momentum of any isolated, stationary system is zero if the velocity of its center of mass/energy is zero.

Consider an isolated, system which is a candidate for a "bootstrap spaceship", and is initially stationary. According to the center-of-energy theorem it has zero total linear momentum.

At some time, the system could initiate internal activity that generates quasistatic electromagnetic-field momentum which is not radiated away, but which remains in the vicinity of the matter of the system. For the total momentum of the system to remain zero, there must now be some mechanical momentum in the system. Nominally, such mechanical momentum would imply that the center of mass of the matter of the system is in motion, and would be propelled in some direction.

At a later time, suppose the system stops its internal activity, such that the equal-and-opposite electromagnetic-field momentum and mechanical momentum are constant thereafter. The center of mass of the matter of the system then has a constant velocity in some direction.

If we observe the system in the (inertial) frame with that constant velocity, the system is isolated and stationary. So, according to the center-of energy theorem, the total momentum of the system should be zero in this frame. However, while the mechanical momentum of the system is zero in this frame, its electromagnetic-field momentum is nonzero, and hence the

[1] The term "bootstrap spaceship" was perhaps first used on p. 612 of [67]. See also [80].

[2] A "bootstrap spaceship" would operate via a "reactionless drive",
https://en.wikipedia.org/wiki/Reactionless_drive

[3] Recent examples are [122, 160, 173, 177, 178, 179, 180, 183].

[4] See the Appendix of [50], sec. 2 of [64], and sec. I of [68]. See also Appendix A below.

total momentum of the system is nonzero (in this frame). This contradiction implies that the above scenario is impossible.

1.2 "Hidden" Mechanical Momentum

The resolution of this "paradox" is that if an isolated, stationary system contains nonzero electromagnetic field momentum, the equal-and-opposite mechanical (linear) momentum is not "overt", but rather is "hidden".[5] If an isolated system, that is initially stationary, develops some electromagnetic-field momentum which is not radiated away, the equal-and-opposite (linear) mechanical momentum is not associated with motion of the center of mass of the matter of the system, but rather is "hidden". In Shockley's example, a net momentum is "hidden" in the electrical currents that generate the magnetic field required so that there can be nonzero electromagnetic field momentum.[6]

Versions of this argument have been given since the 1960's, as will be reviewed in Appendix C below, but some people refuse to acknowledge the validity of the center-of-energy theorem, or that "hidden" mechanical momentum can exist, such that they claim what we have called a "bootstrap spaceship" is possible.[7]

A Appendix: Center-of-Energy Theorem

The mechanical behavior of a macroscopic system can be described with the aid of the (symmetric) stress-energy-momentum tensor $T^{\mu\nu}$ of the system. The total energy-momentum 4-vector of the system is given by,

$$U^\mu = (U_{\rm total}, P^i_{\rm total}\, c) = \int T^{0\mu}\, d{\rm Vol}. \tag{1}$$

As first noted by Abraham [23], at the microscopic level the electromagnetic parts of $T^{\mu\nu}$ are,

$$T^{00}_{\rm EM} = \frac{E^2+B^2}{8\pi} = u_{\rm EM}\,, \tag{2}$$

$$T^{0i}_{\rm EM} = \frac{S^i}{c} = p^i_{\rm EM}\, c, \tag{3}$$

$$T^{ij}_{\rm EM} = \frac{E^iE^j+B^iB^j}{4\pi} - \delta^{ij}\frac{E^2+B^2}{8\pi}\,, \tag{4}$$

in terms of the microscopic fields $\mathbf{E}$ and $\mathbf{B}$. In particular, the density of electromagnetic momentum stored in the electromagnetic field is,

$$\mathbf{p}_{\rm EM} = \frac{\mathbf{S}}{c^2} = \frac{\mathbf{E}\times\mathbf{B}}{4\pi c}\,. \tag{5}$$

[5]The term hidden momentum was introduced by Shockley (1967) [61].

[6]For an example in which the "hidden" mechanical momentum is more abstractly related by $\mathbf{P}_{\rm hid,mech} = \mathbf{P}_{\rm mech} - m_{\rm mech}\mathbf{v}_{\rm cm,mech} = -m_{\rm mech}\mathbf{v}_{\rm cm,mech}$, see sec. 2.4 of [116].

[7]Recent arguments against the center-of-mass theorem and "hidden momentum", while being in favor of "bootstrap spaceships", include [160, 183] and [179, 180].

The macroscopic stress tensor $T^{\mu\nu}$ also includes the "mechanical" stresses within the system, which are actually electromagnetic at the atomic level. The form (4) still holds in terms of the macroscopic fields $\mathbf{E}$ and $\mathbf{B}$ in media where $\epsilon = 1 = \mu$ such that strictive effects can be neglected. The macroscopic stresses T^{ij} are related the volume density $\mathbf{f}$ of force on the system according to,

$$f^i = \frac{\partial T^{ij}}{\partial x^j}\,. \tag{6}$$

The stress tensor $T^{\mu\nu}$ obeys the conservation law,

$$\frac{\partial T^{\mu\nu}}{\partial x_\mu} = 0, \tag{7}$$

with $x^\mu = (ct, \mathbf{x})$ and $x_\mu = (ct, -\mathbf{x})$. Once consequence of this is that the total momentum is constant for an isolated, spatially bounded system, *i.e.*,

$$\int \frac{\partial T^{\mu i}}{\partial x_\mu} = 0 = \frac{\partial}{\partial ct}\int T^{0i}\,d\mathrm{Vol} - \int \frac{\partial T^{ji}}{\partial x^j}\,d\mathrm{Vol} = \frac{dP^i_{\mathrm{total}}}{dt} - \int T^{ji}\,d\mathrm{Area}^j = \frac{dP^i_{\mathrm{total}}}{dt}\,. \tag{8}$$

A related result is that the total (relativistic) momentum $\mathbf{P}_{\mathrm{total}}$ of an isolated system is proportional to the velocity $\mathbf{v}_U = d\mathbf{x}_U/dt$ of the center of mass/energy of the system [50, 64, 68],

$$\mathbf{P}_{\mathrm{total}} = \frac{U_{\mathrm{total}}}{c^2}\mathbf{v}_U = \frac{U_{\mathrm{total}}}{c^2}\frac{d\mathbf{x}_U}{dt}\,, \tag{9}$$

where,

$$U_{\mathrm{total}} = \int T^{00}\,d\mathrm{Vol}, \tag{10}$$

$$P^i_{\mathrm{total}} = \frac{1}{c}\int T^{0i}\,d\mathrm{Vol}, \tag{11}$$

$$\mathbf{x}_U = \frac{1}{U_{\mathrm{total}}}\int T^{00}\,\mathbf{x}\,d\mathrm{Vol}. \tag{12}$$

That is, the total momentum of an isolated system is zero in that (inertial) frame in which the center of mass/energy is at rest.

B Appendix: Mechanical Bootstrap Spaceships

There exist numerous suggestions for all-mechanical bootstrap spaceships, although, of course, none has ever been demonstrated to work. An overview of these is given in [136], which emphasizes two types of devices, *oscillation thrusters* such as [41], and *gyroscopic antigravity* such as [102] and the many variants reported at http://www.gyroscopes.org.

These suggestions generally came from outside the academic community, but an exception is Laithwaite, who in 1974 interpreted his experiments with gyroscopes as evidence for both a bootstrap spaceship [69, 102], and antigravity [73]. He gave a famous set of lectures at the Royal Institution in 1974 [71] illustrating the counterintuitive behavior of gyroscopes, perhaps hoping to lead the audience to his interpretation that the demonstrations could not be explained by Newton's laws. He was unable to get his views published in mainstream academic journals, but did publish a few "popular science" articles [70, 72, 73, 77, 79].

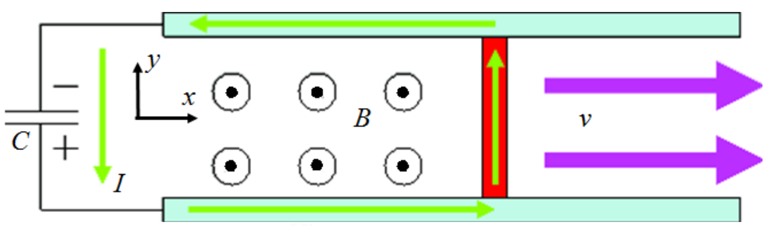
C Appendix: History of Electromagnetic Bootstrap Spaceships

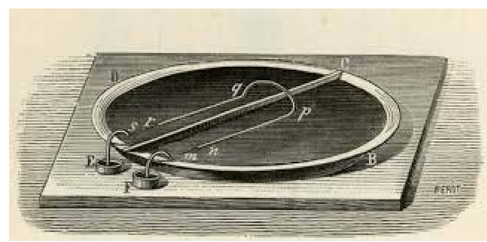
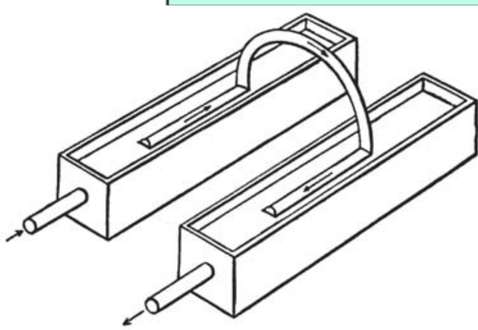
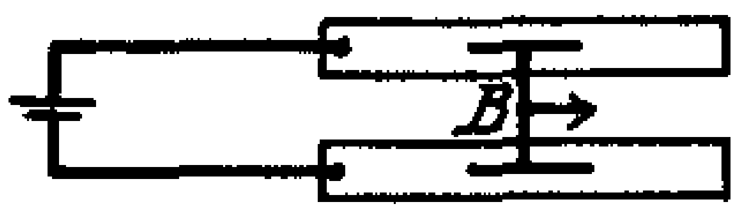
C.1 Electrostatic and Magnetostatic Repulsion

An electric charge exerts a repulsive force on another like charge (and a “pole” of a magnet exerts are repulsive force on a like “pole” of another magnet). Such a system is not a bootstrap spaceship in that the center of mass of an isolated pair of like charges would remain at rest as the two charges move away from one another.

C.2 Ampère

After the discovery by Oersted [3, 4] that an electric current can exert a force on a permanent magnet, the possibility of electromagnetic “spaceships” arose. In 1822, Ampère and de la Rive [6] demonstrated an intriguing effect of a bent wire (“hairpin”) whose two “legs” floated in separate trough of mercury, such that when the latter were connected the a battery the “hairpin” was propelled along the troughs, as sketched below.



The figure on the lower left above is from Art. 687 of Maxwell’s *Treatise* [13], and the figure on the lower right is from Hering [31].

Ampère considered that this demonstration supported his theory of magnetic forces, in which collinear current elements with the same sense repel one another. The present view, based on the so-called Biot-Savart-Lorentz force law, $d\mathbf{F} = I\, d\mathbf{l} \times \mathbf{B}/c$ predicts that the force on the crosspiece of the “hairpin” is largely due to the magnetic field of the currents in the portions of the “hairpin” in the mercury troughs.[8] This explanation indicates that “magnetic forces can do work”, and that the motion of the “hairpin” is due to a force of one portion of the “hairpin” on another.

[8] In Art. 687 of [13], Maxwell remarked that Ampère’s experiment involves a closed circuit, and so cannot distinguish between Ampère’s force law and that of Biot-Savart-Lorentz (which Maxwell attributed to Grassmann [10]).

The latter effect suggests that Ampère's device is a "bootstrap spaceship", which bothers many people,[9] some of whom argue that the Lorentz force law is wrong, and Ampère's original force law is the correct one. This sage is reviewed by the author in [175].

The author's view is that the Lorentz force law is valid, and that Ampère's device is not a "bootstrap spaceship" in that the force on the crosspiece of the "hairpin" is due to the electric current in the other portions of the "hairpin", and this current is not strictly an aspect of the "hairpin" as a rigid body, but is an aspect of the larger electric circuit. That is, the "hairpin" considered as a rigid body does not exert a force on itself

Ampère's experiment was the precursor of the electromagnetic railgun, on which the literature is now vast.[10]

C.3 Thomson

The earliest discussion of a possible bootstrap spaceship was by J.J. Thomson (1904) [25].[11]

C.3.1 Electric Charge + Magnetic Monopole

Thomson considered the electromagnetic field momentum, $\mathbf{P}_{\rm E} = \int \mathbf{E} \times \mathbf{B}\, d{\rm Vol}/4\pi c$ in Gaussian units,[12] of an electric charge q and a (Gilbertian) magnetic (mono)pole p, both at rest, and found this to be zero on p. 333 of [25].

C.3.2 Electric Charge + Gilbertian Magnetic Dipole

On p. 334 Thomson noted that the field momentum of a single electric charge and any number of magnetic poles is also zero, which includes the case of an electric charge and a (Gilbertian) magnetic dipole $\mathbf{m}$.

C.3.3 Electric Charge + Ampèrian Magnetic Dipole

On p. 347 of [25], Thomson noted that the external magnetic field of a Gilbertian magnetic dipole is the same as that of an Ampèrian dipole, so the field momentum of the latter (in the presence of an electric charge) is just the momentum associated with the "interior" of the dipole. If the magnetic dipole is realized by a coil of area A and length l with N turns that carry current I, then the interior axial field is $B_{\rm in} \approx (4\pi/c)NI/l = (4\pi/c)NIA/{\rm Vol}_{\rm coil} = 4\pi m/{\rm Vol}_{\rm coil}$, where the magnetic moment of the coil is $m = NIA/c$. Hence, the field mo-

[9]Ampère would not have considered his device to be a "bootstrap spaceship". In the 20$^{\rm th}$ century, Ampère's force law was championed most notably by Hering [31] and by Graneau [84].

[10]A railgun was patented by Birkeland in 1902 [22]. A sample of two more recent articles on railguns is [76, 147].

[11]This paper is also notable for containing the first recognition that the electromagnetic field could carry angular momentum. See also [163, 172].

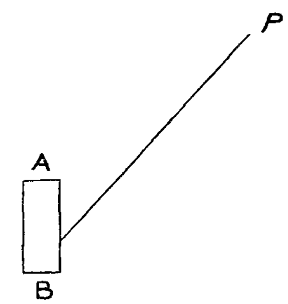
[12]Thomson had invented the concept of electromagnetic field momentum in 1891 [18], and related it to the Poynting vector [17] on p. 9 of [19].

mentum inside the coil (and also the total field momentum of the system) is[13]

$$\mathbf{P}_{\rm EM} = \frac{\mathbf{E}\times\mathbf{B}_{\rm in}{\rm Vol}_{\rm coil}}{4\pi c} = \frac{\mathbf{E}\times\mathbf{m}}{c}, \tag{13}$$

where $\mathbf{E}$ is the electric field of charge q at the magnetic dipole $\mathbf{m}$.

In the figure below, from [25], the electric charge is at P, and the small solenoid is AB.



On p. 348 of [25] Thomsom argued that the field momentum is associated with the electric charge, and that if the Ampèrian magnetic dipole were a small permanent magnet (in the field of an electric charge), and this magnet were demagnetized by "tapping", the electric charge would acquire the initial field momentum (13).

That is, Thomson's argument, if correct, would imply that the system of an electric charge and a small magnet is a "bootstrap spaceship".

C.3.4 Trammel

Thomson's example of a charge plus (long) solenoid magnet was considered (without reference to Thomson) by Aharonov and Bohm (1959) in their well-known paper [43]. This led Trammel (1964) [47] to remark that while the magnet exerts negligible force on the (moving) charge (if the charge remains always outside the magnet),[14] the charge exerts a force on the magnet. That is, this system appears to be a kind of "bootstrap spaceship".

The paper of Trammel may have been the immediate cause of the effort in the mid 1960's that led to the concept of "hidden" mechanical momentum.

C.3.5 Calkin

Thomson's example was considered in 1966 by Calkin [56] (who attributed it to Cullwick, sec. C.6 below, rather than to Thomson), in a manner that supposed it to be a "bootstrap spaceship".[15]

In 1970, Calkin [68] discussed "hidden" mechanical momentum in such examples, with the implication that Thomson's example is not a "bootstrap spaceship".

[13]The difference between the magnetic fields of "point" Ampèrian and Gilbertian magnetic dipoles is $4\pi\,\mathbf{m}\,\delta^3(\mathbf{r})$ (see, for example, sec. 5.6 of [111]), which also leads to eq. (13).

[14]A "paradox" involving a charge that somehow passes through the coil of an infinite solenoid was posed in [51], and discussed in [55, 59, 130].

[15]Trammel and Calkin discussed an "infinite" solenoid, which has certain delicacies that must be treated with care. See, for example [168].

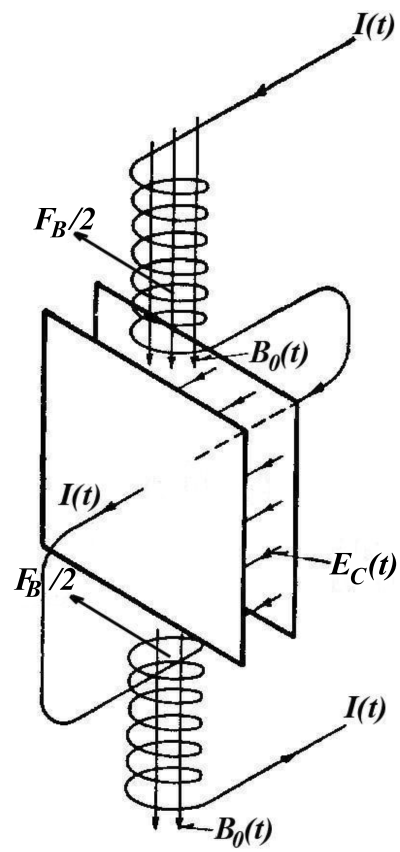
C.4 Brown

In the 1920's, Brown [35] claimed that a system of two electrodes with an electrostatic potential difference exerted a self force in the direction from the larger electrode to the smaller, if they were not the same size. Despite a general consensus that the force is due to corona discharge ("ion wind")[16] [118, 117, 126, 161, 162], and hence is not a self force, papers continue to be published arguing that the phenomenom includes a small self force [123, 174].

The claims of self propulsion by an asymmetric rf cavity, sec. C.10 below, are a variant on this theme.

C.5 Slepian

In the late 1940's J. Slepian, a senior engineer at Westinghouse, posed a series of delightful pedagogic puzzles in the popular journal *Electrical Engineering*. One of these concerned how a capacitor in a cylindrical magnetic field might or might not be used to provide a form of rocket propulsion [38].



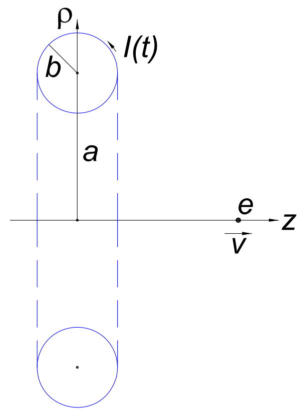
[16]That electrified objects can emit a kind of "wind" was perhaps first noted in 1709 by Hauksbee, pp. 46-47 of [1], and these *electrick vapours* were briefly discussed by Newton, p. 315 of [2]. This phenomenon was called the *brush discharge* by Faraday [9].

For a historical review, see [45]. Propulsion by ions in static electromagnetic fields finds application in some satellites. See, for example, [58, 140].

The current in Slepian's example is sinusoidal at a low enough frequency that radiation is negligible, so that system can be regarded as quasistatic. In this case, the electromagnetic field momentum is always equal and opposite to the "hidden" mechanical momentum, according to a general result of sec. 4.1.4 of [155]. Consequently, the Lorentz force on the system associated with the **E** and **B** field induced by the oscillating **B** and **E** fields are always equal and opposite to the "hidden" momentum forces associated with the oscillatory "hidden" momentum, and the total momentum of the system remains constant (no rocket propulsion).[17]

C.6 Cullwick

Cullwick (1952) [40, 42] noted that an electric charge moving along the axis of a constant-current toroidal coil is paradoxical because no force is exerted on the moving charge,[18] but the moving charge exerts a nonzero force on the toroid.[19]



In the quasistatic limit, Cullwick's paradox is resolved by noting that the unbalanced force is equal and opposite to the time rate of change of the field momentum [133].

For a "spaceship" based on Cullwick's paradox, suppose the toroid and the electric charge form an isolated system. Initially the electric charge is at rest at a nonzero value of z along the axis of the toroid, which latter supports an initially steady current I that creates a steady magnetic field B inside the toroid. For current in the sense shown in the figure, the system has nonzero electromagnetic field momentum (see, for example, sec. 2.1.1 of [133]),

$$\mathbf{P}_{\rm EM} = \frac{\pi b^2 I e}{c^2} \frac{a}{(z^2 + a^2)^{3/2}}\, \hat{\mathbf{z}}\,, \tag{14}$$

The system is initially "at rest," and according to the center-of-energy theorem its total, initial momentum is zero. The momentum equal and opposite to the initial field momentum is the "hidden" mechanical momentum associated with the current in the toroid.

If at some later time the current goes to zero, then an electric field is induced, which transfers the initial field momentum into the final "mechanical" momentum of the electric

[17]For additional discussion, see [134]. Slepian also described an "electrostatic spaceship" at [39].

[18]To avoid consideration of electrostatic forces associated with charges induced on the conductor of the toroid, one can suppose its current is due to pairs of counter-rotating, oppositely charged disks.

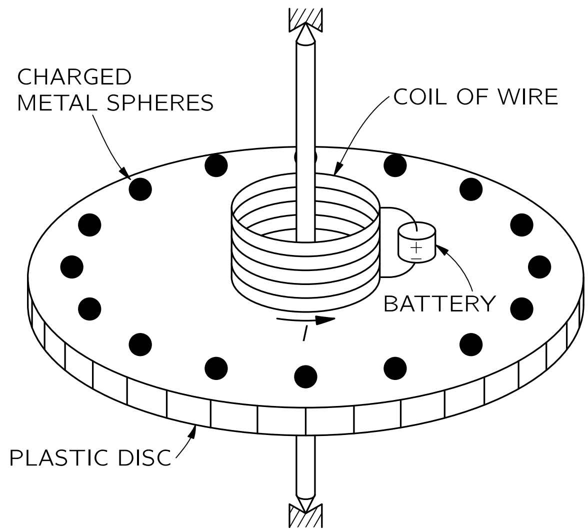
[19]This paradox was revived in [47, 50, 119], without reference to Cullwick.

charge.[20,21] Meanwhile, the now-moving charge exerts a force on the toroid as long as the current is nonzero, such that the final mechanical momentum of the toroid is equal and opposite to the final "mechanical" momentum of the electric charge.

The total momentum of the system is zero at all times. The system does not constitute a "bootstrap spaceship".

C.7 Feynman

In 1963, Feynman posed the now-famous disk paradox related to field angular momentum in sec. 17-4 of [49]. This paradox was perhaps inspired by a comment of J.J. Thomson, p. 348 of [25], and has led to extensive additional commentary, including [55, 57, 59, 78, 81, 83, 85, 86, 87, 89, 90, 91, 92, 93, 95, 96, 100, 105].



An insulating disk has electric charge around its rim and is initially at rest. This disk is coaxial with a solenoid magnet that initially has nonzero current, and the disk is free to rotate with respect to the solenoid.

If at some time the current in the solenoid goes to zero, the decreasing magnetic flux in the solenoid induces an azimuthal electric field that causes the charged disk to rotate. The "paradox" is that this behavior appears to violate conservation of angular momentum.

At the end of sec. 27-6 of [49], Feynman gave a verbal resolution of the paradox: *Do you remember the paradox we described in Section 17-4 about a solenoid and some charges mounted on a disc? It seemed that when the current turned off, the whole disc should start to turn. The puzzle was: Where did the angular momentum come from? The answer is that if you have a magnetic field and some charges, there will be some angular momentum in the field. It must have been put there when the field was built up. When the field is turned off, the angular momentum is given back. So the disc in the paradox would start rotating.*

[20]This process is the principle of the *induction linac*, invented in 1964 [48].

[21]We consider the momentum of the self-field of the electric charge to be part of its "mechanical" momentum". One could also take the view that the momentum in the initial static fields of the system has been transferred into the momentum of the self-fields of the moving electric charge.

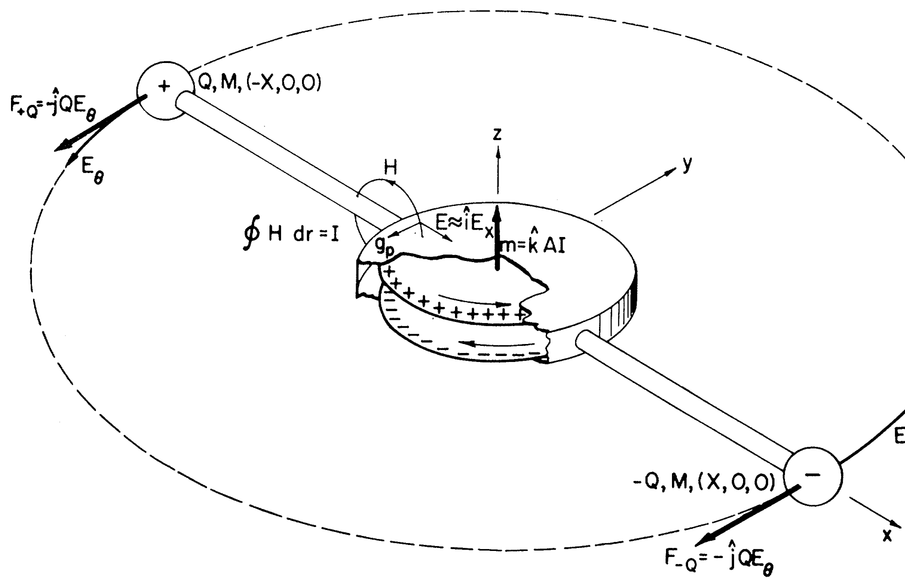
This mystic circulating flow of energy,[22] *which at first seemed so ridiculous, is absolutely necessary. There is really a momentum flow. It is needed to maintain the conservation of angular momentum in the whole world.*

The Feynman disk apparatus was called a "bootstrap merry-go-round" in [67].

The total field momentum is zero in the Feynman disk apparatus, as also is the total mechanical momentum. This is, of course, some mechanical momentum associated with the electric current, but this is not a "hidden" mechanical momentum in the sense of the definition advocated at [152, 155].

C.8 Shockley

In 1967, Shockley and James [61] considered a variant of the Feynman disk apparatus, as shown below.



The (isolated) system is initially at rest, with the two oppositely charged disks rotating with opposite senses. Charges $\pm Q$ are attached to a nonconducting rod that is connected to the axle of the oppositely charged disks. A nonconducting "pillbox" surrounds the latter.

The rotation of the disks might later be slowed to zero, during which time forces would be exerted on the charges $\pm Q$ due to the electric field $\mathbf{E}_\theta$ induced by the decreasing magnetic field of the magnetic dipole $\mathbf{m}(t)$, as shown in the figure. This suggests that the final center of mass of the system would be displaced in the y-direction compared to its initial value.

Shockley considered this to be impossible (without formally invoking the center-of-energy theorem), and that the apparatus would not move (even if not held in place by external forces), *i.e.*, it is not a "bootstrap spaceship". Rather, he aruged[23] that the forces identified above are opposed by "pseudoforces" associated with changes in "hidden" mechanical momentum inside the apparatus.[24]

[22]Feynman referred here to the nonzero Poynting vector [17], $\mathbf{S} = (c/4\pi)\mathbf{E} \times \mathbf{B}$, that can exist in static electromagnetic configurations, such as the present example.

[23]For more details of Shockely's argument, see the companion note [184].

[24]This was the first use of the term *hidden momentum*.

He also noted that when the disks are rotating, the system possesses nonzero electromagnetic field momentum, $\mathbf{E} \times \mathbf{m}/c$ in the $-y$ direction,[25] where $\mathbf{E}$ is the electric field due to the charge $\pm Q$ at the origin, and that this field momentum is equal and opposite to the "hidden" mechanical momentum. Not only is the total momentum of the system zero at all times, its center of mass/energy remains always at rest.

No "bootstrap spaceships".

C.9 Boyer

Beginning in 2001, Boyer has written a series of papers [115, 121, 124, 125, 127, 137, 138, 138, 164, 165] on the theme of "hidden" momentum, with varying attitudes as to whether or not this entity exists in Nature.[26] Boyer's hope is perhaps that a microscopic analysis based on a Darwin lagrangian will avoid the need to consider "hidden" momentum, but he seems to accept that in macroscopic analyses there is a role for this concept.

His comments are generally consistent with the view that there are no "bootstrap spaceships".

C.10 Shawyer

Beginning in 2005, Shawyer [122, 177] has claimed that experiments with an rf cavity in the form of a truncated cone show and extremely time propulsive force.[27] This claim has been supported by a few others, such as [146, 178].

These authors also claim that the propulsive force is due to the pressure of the electromagnetic cavity fields on the cavity, which would be a "self force". This is a misunderstanding of the consequences of Maxwell's equations, as reviewed in [181].

While there is still no crisp explanation for the tiny forces seen in various experiments, the present author considers that they are likely to be thermal effects, as in the famous case of the Crookes radiometer.[28]

C.11 Mansuripur

The most well-publicized objection to "hidden" momentum is perhaps that of Mansuripur (2012) [145], who argued that the Lorentz force law is wrong, and when corrected, the notion of "hidden" momentum is no longer needed. His argument was, however, not related to issues of "bootstrap spaceships".[29]

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[25] This field momentum for a magnet plus electric charges was first computed on p. 347 of [25].

[26] These papers are part of a larger series that purports to show that much of quantum theory is actually "classical".

[27] See also, http://www.emdrive.com/

[28] In 1876, Crookes demonstrated his famous radiometer (aka "light mill"), and speculated that it was driven by the pressure of light [12]. However, it was observed by Schuster [14, 53] that the rotation of the radiometer was opposite to that consistent with radiation pressure, and instead was due to thermal effects in the residual gas inside the device.

For a review of experiments that eventually demonstrated the radiation pressure of light, see [82].

[29] A long comment by the author on Mansuripur's views is at [148], which includes references to several other such comments.

C.12 Franklin

In 2013, Franklin [160] claimed that the center-of-energy theorem does not hold, and there is no need for "hidden" momentum. In the Abstract, Franklin stated about examples like that of Shockley (sec. C.8 above); *The external force required to keep matter at rest during the production of the final static configuration produces the electromagnetic momentum.*[30] That is, Franklin considered that in the absence of any external force, such examples are "bootstrap spaceships".

He seemed to argue that the (Lorentz) force on the charges $\pm Q$ in Shockley's example does not change any mechanical momentum in the system if an external force holds the system "at rest", but rather changes the field momentum (which he claimed is then the only momentum in the system).

On the other hand, if no external force were present, and the initial momentum of the (isolated) system were zero, Franklin implied that the system would be propelled to some velocity, presumably by the Lorentz force, such that (he argued) the mechanical momentum would be equal and opposite to the field momentum (which was somehow created by other action than the Lorentz force in this case). Then, the total momentum would still be zero, although the system were now in motion.

In effect, Franklin claimed that the Bio-Savart-Lorentz force, $\oint I\, d\mathbf{l} \times \mathbf{B}/c$, may or may act on the current I, depending on the character of other forces in the problem, such that analyses of experiments on magnetism dating back the time of Ampère have been incorrect.

Franklin recognized that his scenario violates the center-of-energy theorem, which he reviewed for *static* (and isolated) systems in eqs. (43)-(45) of his sec. IV. He also noted that in *static* systems which contain both matter (electric currents) and electromagnetic fields, the electromagnetic field momentum can be nonzero. He argued that this shows the center-of-energy theorem not to hold (rather than that the system must contain some other momentum equal and opposite to the field momentum), which conclusion could follow only if he thought the momentum $\mathbf{P}$ of his eq. (45) were the field momentum and not the total momentum).[31]

Franklin also promoted these arguments in a more recent paper [183].

C.13 Tuval and Yahalom

Tuval and Yahalom have recently advocated two "electromagnetic spaceships" [159, 173]. The first of these is based on two coupled AC circuits, and could exhibit weak propulsion in reaction to electromagnetic momentum radiated by the circuits.[32] The second is based on a single circuit plus a permanent magnet, and it turns out that such a system cannot radiate net momentum [176]

That is, neither of the examples of Tuval and Yahalom are "bootstrap spaceships".

[30] Similarly, the final sentence of [160] reads: *The external force needed to keep matter at rest during the creation of the charge-current distribution goes directly into EM momentum without moving any matter or hiding any momentum.*

[31] That is, Franklin's claims seem to this author to follow from a basic misunderstanding of force, momentum, and the center-of-energy theorem.

[32] For a review of schemes for rocket propulsion based on laser beams, see [135].

C.14 Redfern

Two recent papers by Redfern [179, 180] argued that "hidden" momentum does not exist, and, in effect, that it is perfectly acceptable for the example of Shockley (sec. C.8 above) to be a "bootstrap spaceship" (which if not held in place, would move in the $-y$ direction as the system is assembled).

While Redfern makes extensive reference to the paper of Coleman and Van Vleck [64], he does not seem to be aware that his views are inconsistent with the center-of-energy theorem, which is the keystone of that paper.

Additional comments by the author on Redfern's paper [179] are at [184].

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