Antigravity, Electron-Positron Storage Rings, and the K^0-$\overline{K}^0$ System

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1 Problem

Counterpropagating beams of electrons and positrons can be stored in a single ring of magnets,[1] with storage of the beams for 40 hours reported in the first such device, the AdA electron-positron storage ring (see p. 169 of [3]). Discuss whether this result places a limit on possible differences in the gravitational interaction of electrons and positrons.

Also, discuss how the observed phenomenon of mixing/oscillation in the K^0-$\overline{K}^0$ system does place strong limits on such differences.

2 Solution

2.1 Electron-Positron Storage Rings

We will conclude that the beam lifetime in electron-positron storage rings is not sensitive to possible difference in the gravitational interaction of electrons and positrons. See sec. 2.2 for analysis on such effects the K^0-$\overline{K}^0$ system.

Electron-positron storage rings are a kind of magnetic "trap", with the charged particles in stationary orbits at velocities very close to the speed c of light in vacuum. For such orbits to be stable, some effect must cancel the force of the Earth's gravity on the electrons and positrons. This is accomplished (generally without explicit awareness by the storage-ring designers) by a small, horizontal magnetic field component that is perpendicular to the particles' direction of motion.

The primary (and static) magnetic field in a storage ring is vertical, but in the AdA storage ring the vertical field varied with radius, $B_z(r)$, and hence there existed a radial magnetic field that varied with height, $B_r(z)$, to satisfy Maxwell's equation $\boldsymbol{\nabla} \times \mathbf{B} = 0$ for the static field. Then, $\partial B_r \partial z = \partial B_z/\partial r$ in a cylindrical coordinate system (r, θ, z), so with $z = 0$ as the midplane (symmetry plane) of the magnetic field, in which plane $B_r = 0$,

$$B_r(r, z) = \int_0^z \frac{\partial B_r(r, z')}{\partial z'}\, dz' = \int_0^z \frac{\partial B_z(r, z')}{\partial r}\, dz' \approx z \frac{\partial B_z(r, 0)}{\partial r}\, . \tag{1}$$

This field configuration provided so-called weak focusing of the beams transverse to their directions of motion [4] (see also sec. 2.4 of [5]). Then, the particle trajectories were slightly

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[1] The electron-positron storage-ring concept seems to have been invented by B. Touschek [1, 2], who argued that CPT invariance guarantees that electrons and positrons follow the same trajectory in a magnetic system if the positron's velocity is the reverse of the electron's.

displaced from those that would obtain in zero gravity, such that the particles experienced the required, gravity-compensating vertical force from the horizontal component of the magnetic field.[2]

For example, if the electron (of charge $-e$ and rest mass m) at some point ($r \approx r_0, \theta, z \approx 0$) around the ring moves with velocity $\mathbf{v}_{e^-} = (v_r, v_\theta, v_z)$ with $v_\theta \approx c$, $v_r, v_z \ll c$ and $\gamma = 1/\sqrt{1 - v^2/c^2} \gg 1$, then the magnetic field, $\mathbf{B} \approx z \frac{\partial B_z(r,0)}{\partial r}\,\hat{\mathbf{r}} + B_z(r,z)\,\hat{\mathbf{z}}$, exerts a Lorentz force $\mathbf{F} = -e\mathbf{v}_{e^-} \times \mathbf{B}$ on the electron,

$$\mathbf{F} = -e[v_\theta B_z\,\hat{\mathbf{r}} + (v_z B_r - v_r B_z)\,\hat{\boldsymbol{\theta}} - v_\theta B_r\,\hat{\mathbf{z}}] \approx -ecB_z(r_0,0)\,\hat{\mathbf{r}} + ecz\frac{\partial B_z(r_0,0)}{\partial r}\,\hat{\mathbf{z}}, \qquad (2)$$

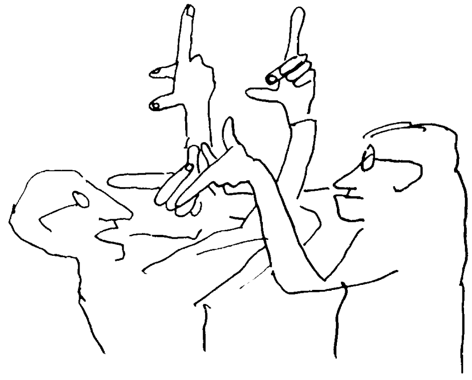
(in SI units) whose vertical component is opposite to the force of gravity, $-2\gamma mg\,\hat{\mathbf{z}}$, on the relativistic electron.[3] An equilibrium circular orbit exists with small vertical displacement z_{0,e_-} from the symmetry plane of the magnetic field,

$$z_{0,e_-} \approx \frac{2\gamma mg}{ec\,\partial B_z(r_0,0)/\partial r} \approx \frac{2 \cdot 500 \cdot 10^{-30} \cdot 10}{1.6 \times 10^{-19} \cdot 3 \times 10^8 \cdot 1.37} \approx 10^{-16}\ \text{m}, \qquad (3)$$

where the numerical example is for 250-MeV electrons, $\gamma \approx 500$, and $\partial B_z(r_0,0)/\partial r = nB_z(r_0,0)/r_0 \approx 1.37$ T/m at the AdA storage ring [8], where $B_z(r_0,0) = 1.44$ T, $r_0 = 0.58$ m, and the weak-focusing parameter n was 0.55.

The usual understanding is that the force of gravity on antiparticles is the same as that on the corresponding particles; positrons fall downwards at the Earth's surface. Then, if the positrons (of charge e) have velocity $\mathbf{v}_{e^+} = -\mathbf{v}_{e^-}$, the Lorentz force, $e\mathbf{v}_{e^+}/c \times \mathbf{B} \approx 2\gamma mg\,\hat{\mathbf{z}}$, is the same as that on the electrons. That is, in the standard view the Lorentz force due to the horizontal magnetic field component in the storage ring balances the force of gravity on both the positrons and the electrons, for the same small vertical displacement z_0 of their equilibrium orbits.

A cartoon by Touschek (from [3]) illustrating this argument is shown below.



MAGNETIC DISCUSSION

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[2] Contemporary storage rings use so-called strong focusing [6] in which most of the vertical magnetic field is uniform, with small regions of strong quadrupole fields, $\mathbf{B} = K(z\,\hat{\mathbf{r}} + r\,\hat{\mathbf{z}})$, arrayed around the ring to give dynamic stability to the beams.

[3] It is seldom discussed that the acceleration due to gravity on relativistic particles with horizontal motion at the Earth's surface is $2g$, where g is the acceleration due to gravity of a slow-moving particle. For the author's comments on this oft-neglected issue, see [7].

Suppose, however, that the force of gravity on a positron were different from than on an electron, say,

$$\mathbf{F}_{e^+} = k\mathbf{F}_{e^-} = -2k\gamma mg\,\hat{\mathbf{z}}, \tag{4}$$

where $k = -1$ corresponds to antimatter being subject to "antigravity" of the same magnitude, but opposite direction to the effect of gravity on ordinary particles. Since the Lorentz force on the positrons has the same form (2) as that on the (counterpropagating) electrons, the vertical displacements of the equilibrium orbit of the positrons would be

$$z_{0,e_+} \approx \frac{2k\gamma mg}{ec\,\partial B_z(r_0,0)/\partial r} = kz_{0,e_-}. \tag{5}$$

If the vertical displacements of the equilibrium orbits of the electrons and positrons were large enough, collisions between the beams would no occur, contrary to observations. However, the vertical size of the beams in the AdA ring was about 10^{-4} m, so the successful operation of the AdA storage ring as an electron-positron collider places only the extremely weak limit $|k| < 10^{12}$.

As noted by Touschek, the fact that the trajectories of the electrons and positrons are the same in the horizontal plane (in which gravitational effects play no role) provides a verification that the inertial masses of the electron and positron are the same, to accuracy of the relative energy (or momentum) spread of the beams, which was 0.0025 for the AdA ring.

2.2 The K^0-$\overline{K}^0$ System

A claim was made by Good in 1960 [10] that one can infer from observed phenomena in the K^0-$\overline{K}^0$ system that the parameter k of eq. (4) is very small, $|1-k| < 10^{-10}$. This argument for this is somewhat indirect, assumes CP invariance, and invokes the concept of a scalar gravitational potential, which has led some people to question its validity.[4] Here, we give a different argument based on the K^0-$\overline{K}^0$ system, not involving potentials.

Namely, the phenomenon of K^0-$\overline{K}^0$ oscillations has been observed for laboratory times up to 200 ns.[5] Such oscillations can only exist so long as the K^0 and $\overline{K}^0$ wavefunctions overlap, whereas this will cease to occur at large times if the K^0 and $\overline{K}^0$ fall differently due to gravity. A relativistic K^0 particle, produced with horizontal momentum at height $z = 0$ at time $t = 0$ at the Earth's surface, falls with vertical acceleration $2g$ [7] according to $z_{K^0} = -gt^2$.

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[4] There is a long history of proponents of tests of the gravitational interaction of antiparticles denying the relevance of all previous experiments. For recent examples, see [11, 12]. *Caveat emptor*!

One of the slyest arguments is that very little of the mass of antiparticles is due to "bare" antimatter, being rather due to energy of relevant strong, electromagnetic or Higgs fields. In this view, it is almost impossible to measure the gravitational interaction of the "bare" antimatter (and it is essentially irrelevant to observable phenomena what that interaction might be).

[5] See, for example, Figs. 2 and 3 of [13]. Note that proper time of 1 ns corresponds to laboratory time of 200 ns for Kaons of momentum ≈ 90 GeV/c.

We suppose the effect of the Earth's gravity on the antiparticle $\overline{K}^0$ can be parameterized as $z_{\overline{K}^0} = -kgt^2$, where $k = -1$ corresponds to the gravitational mass of the $\overline{K}^0$ being the negative of that of the K^0. The vertical separation of the K^0 and $\overline{K}^0$ would then vary as $\Delta z = |1-k|\, gt^2$. Once this separation exceeded the Compton wavelength of the Kaons, $\lambda\!\!\!^{-}_K = \hbar/m_K c$, the K_0 and $\overline{K}^0$ wavefunctions would not well overlap, and the K^0-$\overline{K}^0$ system would decohere; oscillations would cease.

Experimentally, such decoherence is not observed, with $t \approx 200$ ns being the longest time for which K^0-$\overline{K}^0$ oscillations have been observed. From this we infer,

$$\Delta z = |1-k|\, gt^2 < \frac{\hbar}{m_K c}\,, \qquad |1-k| < \frac{m_e}{m_K}\frac{\lambda\!\!\!^{-}_e}{gt^2} \approx 10^{-3}\frac{4 \times 10^{-13}}{10 \cdot (2 \times 10^{-7})^2} \approx 10^{-3}. \tag{6}$$

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