

Two-time interpretation of quantum mechanics

Yakir Aharonov and Eyal Y. Gruss*

*School of Physics and Astronomy,
Beverly and Raymond Sackler Faculty of Exact Sciences,
Tel-Aviv University, Tel-Aviv 69978, Israel.*

We suggest an interpretation of quantum mechanics, inspired by the ideas of Aharonov et al. of a time-symmetric description of quantum theory. We show that a special final boundary condition for the Universe, may be consistently defined as to determine single classical-like measurement outcomes, thus solving the “measurement problem”. No other deviation is made from standard quantum mechanics, and the resulting theory is deterministic (in a two-time sense) and local. Quantum mechanical probabilities are recovered in general, but are eliminated from the description of any single measurement. We call this the *Two-time interpretation* of quantum mechanics. We analyze ideal measurements, showing how the quantum superposition is, in effect, dynamically reduced to a single classical state via a “two-time decoherence” process. We discuss some philosophical aspects of the suggested interpretation. We also discuss weak measurements using the two-time formalism, and remark that in these measurement situations, special final boundary conditions for the Universe, might explain some unaccounted for phenomena.

I. INTRODUCTION

The “measurement problem” is a most fundamental problem in the field of the foundations of physics. The problem is that while quantum mechanics allows very accurate calculation of microscopic phenomena, it is not clear how it can fully describe macroscopic measurement processes. Empirically, a measurement performed on a quantum system yields one single outcome, with some probability given by quantum mechanics. By contrast, in the standard unitary quantum mechanics formalism, the quantum state evolves into a linear superposition of the possible measurement outcomes. This discrepancy is a bothersome loophole in the theory, and it can be an actual difficulty when considering “theories of everything” based on quantum mechanics.

To bridge the gap between theory and observed reality, different interpretations of quantum mechanics, or rather, different interpretations of our observations, have been suggested. Some of these interpretations abandon conventional classical concepts such as determinism and locality, which we briefly define.

“Locality”, means that an event cannot have any influence outside its future light-cone (even if such influence does not defy relativistic causality, i.e., does not allow superluminal signalling, it is still forbidden). Since non-locality defies relativistic covariance, it is usually considered an undesirable property of a theory.

“Determinism” means that the physical state at any time is determined completely as a single-valued evolution of the state at some single (usually initial) time. The validity of this principle constitutes a main disagreement point between the different interpretations of quantum mechanics, and therefore we shall classify them by this criterion.

Usually, indeterministic interpretations involve a “collapse” phenomenon. The quantum mechanics formalism is modified by the addition of a wave function collapse rule, responsible for the reduction of the superposition into a single state. This approach has several weaknesses: First, indeterminism is of course counterintuitive to the conventional deterministic perception of nature. Second, the interpretation is nonlocal, as the collapse is assumed to take place instantaneously in all space. This is clearly a noncovariant description, since collapse events that are simultaneous in some frame of reference would not be simultaneous in any boosted frame of reference. Ref. [1] discusses some of the dilemmas of formulating a consistent covariant collapse scheme. Third, most collapse theories lack well defined mechanism and criteria for the collapse process. Such is the case with the orthodox collapse approach, which states merely that a collapse occurs “somewhere” along the line of the measurement process. Nevertheless, more rigorous suggestions do exist, such as the GRW spontaneous localization model and successors thereof [2].

Some deterministic interpretations are based on Bohmian “hidden variables” [4]. These assume the existence of inaccessible local variables with definite values which determine the measurement outcome. Bell’s celebrated inequality [5], and the more recent GHZ argument [6], show that hidden variable theories are inherently nonlocal. A different deterministic interpretation is Everett’s “relative state” formulation [7], also known as the “many worlds” interpretation. Here the state after the measurement is considered to be the full superposition state, where it is assumed that all measurement outcomes in the superposition coexist as separate real world outcomes. The observation of a certain outcome is attributed to the specific state of the observer that is correlated to it in the superposition. Each of the superposition terms constitutes a “branching world”, whereas the overall state evolves unitarily and is deterministic. In this interpretation it is not clear how to recover the

*e-mail: eyalgruss@gmail.com

empirical quantum mechanical probabilities. Both of the above deterministic interpretations require additional entities (wave function plus hidden variables, many worlds) in order to achieve the complete description of physical nature. These entities might be considered an excess by Occam’s razor, the simplicity postulate, which states that “Entities are not to be multiplied beyond necessity”.

We do not consider here epistemological interpretations (including the “Copenhagen” interpretation and some “consistent histories” interpretations), as opposed to the ontological interpretations above. The former pretend only to give a set of logical rules regarding our knowledge of reality, and do not attempt to explain any underlying processes taking place.

In this work we suggest an ontological interpretation, which attempts to overcome the difficulties mentioned above. It is deterministic, local, and simple, and it recovers the empirical probabilities of quantum mechanics.

To complete the required background, we briefly discuss the notions of “classicality” and “decoherence”. After the measurement interaction between the quantum system and the measuring device (of course also a “quantum” system) is complete, and assuming no reduction takes place, we get a superposition of states of possible outcomes. By a “state of a possible outcome” we mean a measuring device state correlated to the corresponding system state. Our experience tells us, these measuring device states belong to a certain definite basis of localized states. No outcome in the form of a superposition of these states can be measured. We regard these states as “classical” states and postulate that it is impossible to interfere or “mix” these states. This is usually assumed to be so due to a decoherence process a la Zurek [8]. In this process the essential part of the measuring device, referred to as the “pointer”, becomes correlated to additional quantum systems, the “environment”. It is assumed that the environment states, due to their interaction with the pointer states, very quickly become and remain nearly orthogonal. The correlation of the pointer with these environment states defines a preferred basis of pointer states, in relation to the original measurement. Further, it is assumed that the environment degrees of freedom are practically inaccessible, and thus they must be ignored. Tracing out these degrees of freedom, we obtain the reduced density matrix, which shows to have negligible off-diagonal interference terms. We thus get a dynamical superselection process, which damps out any superpositions of classical states, isolating the classical states in an effectively irreversible way. In this work, we

characterize ideal measurements as those in which the measurement outcome (or a record thereof) remains classical up to some very far “final time”. Of course, decoherence by itself does not resolve the measurement problem – no reduction to a single classical state occurs. Yet, most importantly, a preferred basis for such a reduction is defined and superselected.

The outline of the article is as follows: We start with a brief review of the two-time, or time-symmetric, formalism of quantum mechanics. We suggest a generalization of this formalism in order to describe the measurement process in closed systems. This is done by the introduction of a special final boundary condition for the Universe, which accounts for observed measurement outcomes. We analyze ideal measurements in the framework of the suggested interpretation, and show how effective reduction arises dynamically due to the final boundary condition, with no additional mechanism. We call this process “two-time decoherence”. Next, we briefly discuss some philosophical aspects of the suggested interpretation. We address the relevance and validity of concepts such as locality, microscopic irreversibility, causality, determinism, freedom of choice, realism and counterfactual definiteness, in the framework of our interpretation. Finally, we discuss the concept of non-ideal “weak measurements”. The outcome of which is a two-time expectation value of the measured operator, the “weak value”. We explain the emergence of the weak value using the two-time formalism, and discuss the possible effect of a special final boundary condition for the Universe, in this context.

II. GENERALIZATION OF THE TWO-TIME FORMALISM TO CLOSED SYSTEMS

In 1964, ABL [9] derived a probability rule concerning measurements performed on systems, with a final state specified in addition to the usual initial state. Such a final state may arise due to a post-selection, that is, performing an additional measurement on the system and considering only the cases with the desired outcome. Alternatively, some systems in nature may have an inherent final boundary condition, just as all systems have an initial boundary condition. Given an initial state Ψ_i and a final state Ψ_f , the probability that an intermediate measurement of the non-degenerate operator A yields an eigenstate a_k is

$$\text{prob}(a_k | \Psi_i, \Psi_f) = \frac{\text{prob}(\Psi_f | a_k) \text{prob}(a_k | \Psi_i)}{\sum_j \text{prob}(\Psi_f | a_j) \text{prob}(a_j | \Psi_i)} = \frac{|\langle \Psi_f | a_k \rangle|^2 |\langle a_k | \Psi_i \rangle|^2}{\sum_j |\langle \Psi_f | a_j \rangle|^2 |\langle a_j | \Psi_i \rangle|^2}. \quad (1)$$

For simplicity, no self-evolution of the states is considered between the measurements. If only an initial state is

specified, (1) should formally reduce to the regular prob-

ability rule:

$$\text{prob}(a_k | \Psi_i) = |\langle a_k | \Psi_i \rangle|^2. \quad (2)$$

This is usually shown by summing over the probabilities of all possible final states, expressing the indifference to the final state. However, (2) can also be obtained in a different way. If the final state would be one of the eigenstates, $\Psi_f = a_k$, then ABL (1) gives probability one for measuring a_k , and probability zero for measuring any orthogonal state. Consider now an ensemble of systems of whom fractions of size $\text{prob}(a_k | \Psi_i)$ happen to have the corresponding final states a_k . The regular probability rule (2) of quantum mechanics is then recovered. But now the probabilities are classical probabilities due to ignorance of the specific final states, and the description is deterministic in a two-time sense. The same would be the result for a corresponding final state of an auxiliary system, such as a measuring device or environment, correlated with the measured system. This reduction of the ABL rule to the regular probability rule is a clue, showing how a selection of appropriate final states can account for the empirical probabilities of quantum measurements.

It is clear from the ABL rule that the final state of a system may be of importance. Weak measurements, which are discussed later, are one example. Following [10], we reformulate quantum mechanics to be time-symmetric, in the sense that it will take into account both initial and final boundary conditions. The Schrödinger equation is linear in the time derivative, therefore only one temporal boundary condition may be consistently specified for the wave function. If both initial and final boundary conditions exist, we must have two wave functions, one for each boundary condition. The first is the regular wave function, or state vector, evolving forward in time from the initial boundary condition. We call it the “history vector”, $\Psi_{\text{HIS}}(t)$. The second is a different wave function evolving from the final boundary condition backwards in time, which we call the “destiny vector”, $\Psi_{\text{DES}}(t)$. A measurement (including post-selection) will later be shown to constitute an effective boundary condition for both wave functions. We postulate that the complete description of a closed system is given by the two wave functions. These may be combined into operator form by defining the “two-state”:

$$\rho(t) \equiv \frac{|\Psi_{\text{HIS}}(t)\rangle\langle\Psi_{\text{DES}}(t)|}{\langle\Psi_{\text{DES}}(t)|\Psi_{\text{HIS}}(t)\rangle}, \quad (3)$$

where orthogonal history and destiny vectors at any time t are forbidden. For a given Hamiltonian $H(t)$, the two-state evolves from time t_1 to t_2 according to

$$\rho(t_2) = U(t_2, t_1)\rho(t_1)U(t_1, t_2), \quad (4)$$

where $U(t_2, t_1)$ is the regular evolution operator:

$$U(t_2, t_1) = T \exp\left(-i/\hbar \int_{t_1}^{t_2} H(\tau)d\tau\right) \quad (5)$$

(T signifies the time ordered expansion). The reduced two-state describing a subsystem, is obtained by tracing out the irrelevant degrees of freedom.

In standard quantum mechanics we may also use operator form similar to the above, replacing the state vector $\Psi_{\text{HIS}}(t)$ with the density matrix:

$$\rho^{\text{density}}(t) \equiv |\Psi_{\text{HIS}}(t)\rangle\langle\Psi_{\text{HIS}}(t)|. \quad (6)$$

The density matrix again evolves by

$$\rho^{\text{density}}(t_2) = U(t_2, t_1)\rho^{\text{density}}(t_1)U(t_1, t_2), \quad (7)$$

and again the reduced density matrix for a subsystem, is obtained by tracing out the irrelevant degrees of freedom. Assuming unitary evolution (no measurements), the density matrix is a complete description of systems which, as is usually assumed, evolve from some initial state and (apparently) have no definite final boundary condition. For such systems, the two-time formalism should reduce to the standard one. This will be so, if the destiny vector is equal to the history vector at any time,

$$|\Psi_{\text{DES}}(t)\rangle = |\Psi_{\text{HIS}}(t)\rangle. \quad (8)$$

This is true for any system with a trivial final boundary condition that is just the initial state evolved unitarily from the initial time t_i to the final time t_f ,

$$|\Psi_f\rangle = U(t_f, t_i)|\Psi_i\rangle. \quad (9)$$

Therefore, apart from the measurement problem, it is clear that the standard formalism is a special case of the two-time formalism. We trivially take (9) as a zero order approximation of the final boundary condition. The ABL rule (1) is not relevant here (it would give a square of the regular probability), since we are not yet considering measurements. Now by considering final boundary conditions deviating from the above, we may introduce a richer state structure into quantum theory. When would the final boundary condition and two-time formalism show to affect the dynamics? It would do so if the reduced two-state describes a subsystem for which the ignored degrees of freedom do not satisfy (9). Then, the reduced two-state should replace the density matrix which is no longer a reliable description of the state of the system.

A measurement, as empirically observed, generally yields a new outcome state of the quantum system and the measuring device. This state may be treated as an effective boundary condition for both future, and as indicated by the ABL rule, past events. We suggest that it is not the case that a new boundary condition is generated at each measurement event, by some unclear mechanism. Rather, only a final boundary condition needs to be given for the measuring device, as part of a final boundary condition of the Universe. In the following section we shall demonstrate how an effective boundary condition then arises at the time of measurement due to a two-time decoherence effect. ABL (1) of course agrees that in the classical basis (determined by decoherence), the outcome

of the measurement can only be the single classical state corresponding to the final boundary condition.

We thus suggest a special final boundary condition for the Universe, in which each classical system (measuring device) has a final boundary condition equal to one of its possible classical states (evolved to the final time). We further assume that these final states have an appropriate distribution so as to recover the empirical quantum mechanical probabilities for large ensembles. The final boundary condition is a boundary condition for the destiny vector (not the history vector). Since the description is unitary, the destiny vector data may be given at any time, and we may ignore the question of the actual cosmological final state of the Universe. A scheme to choose the final boundary condition would be as follows: Given the initial boundary condition of the Universe and the Hamiltonian, calculate the trivial final boundary condition as the initial boundary condition evolved unitarily to the final time. Next, identify the classical systems and their classical basis, due to the effect of decoherence. Change the trivial final boundary condition, so that the final boundary condition of each classical system (and systems correlated with it) is a single normalized term of the calculated final superposition state written in the classical basis. The choice of the specific state should be random, with a probability proportional to the squared amplitude of the corresponding term in the superposition. With this choice, the two-time formalism of [10] is generalized to fully describe the measurement process in closed systems, and there is no longer a “measurement problem”. The measurement outcome is selected, as we shall show, by the dynamics due to unitary Schrödinger evolution alone. Note that the measurement outcome will be realized in a subsystem which is “open”, but a complete description may be given for any closed system, namely for the closed Universe.

After the completion of this work, we found a paper by Davidon [11] dated 1976, which suggests a description similar to ours.

III. IDEAL MEASUREMENTS

Consider an experiment performed on a spin- $\frac{1}{2}$ particle, in order to measure its spin component along some axis. Let the initial state of the particle be $(a \uparrow + b \downarrow)$, and denote the initial state of the measuring device *READY*, and the initial state of the environment ε_0 . The initial state, or the history vector, of the composite system at the initial time t_0 is

$$|\Psi_{\text{HIS}}(t_0)\rangle = (a|\uparrow\rangle + b|\downarrow\rangle) \otimes |\text{READY}\rangle \otimes |\varepsilon_0\rangle. \quad (10)$$

Assume an interaction between the particle and the measuring device takes place until time t_1 , such that if the device measures \uparrow , it evolves into the state *UP*, and if it measures \downarrow , it evolves into the state *DOWN*. The

composite system then evolves to the state:

$$|\Psi_{\text{HIS}}(t_1)\rangle = (a|\uparrow\rangle \otimes |\text{UP}\rangle + b|\downarrow\rangle \otimes |\text{DOWN}\rangle) \otimes |\varepsilon_0\rangle. \quad (11)$$

Assume that after this time, a decoherence process takes place, in which the measuring device interacts with the environment, giving after a short decoherence time t_d ,

$$\begin{aligned} |\Psi_{\text{HIS}}(t > t_1 + t_d)\rangle &= a|\uparrow\rangle \otimes |\text{UP}\rangle \otimes |\varepsilon_{\text{up}}\rangle + \\ &+ b|\downarrow\rangle \otimes |\text{DOWN}\rangle \otimes |\varepsilon_{\text{down}}\rangle, \end{aligned} \quad (12)$$

where ε_{up} and $\varepsilon_{\text{down}}$ are nearly orthogonal environment states, which induce the superselection of *UP* and *DOWN* as the preferred basis of pointer states. By our definition of an ideal measurement, decoherence is assumed to cause these states to remain classical up to some far “final time”. For the time being, assume that after the measurement interaction is over, the measuring device is left idle and its state remains unchanged. We now introduce the novel key element of the suggested interpretation: We assume a specific final boundary condition for the measuring device and correlated environment at the above final time. Let this be

$$\langle \dots | \otimes \langle \text{UP} | \otimes \langle \varepsilon_{\text{up}} |, \quad (13)$$

where “...” represents the state of any other system correlated with the measuring device at the final time. The final boundary condition was chosen as one specific state of the preferred basis. This is legitimate since decoherence prevents any interference with this classical state up to the final time. The particle, by contrast, remains in its measured state only until some other measurement is performed on it, preparing it in a new arbitrary state:

$$|\phi\rangle = c|\uparrow\rangle + d|\downarrow\rangle. \quad (14)$$

For the time being, assume that this second measurement takes place instantaneously at time t_2 , producing the backward-evolving state ϕ . This takes the role of a final boundary condition for our particle. We can write the effective final boundary condition, or the destiny vector, of the composite system at time t_2 as

$$\langle \Psi_{\text{DES}}(t_2) | = \langle \phi | \otimes \langle \text{UP} | \otimes \langle \varepsilon_{\text{up}} |. \quad (15)$$

At any time t : $t_1 + t_d < t < t_2$ the complete description of the composite system is given by the two-state (up to normalization):

$$\begin{aligned} \rho(t) = |\Psi_{\text{HIS}}(t)\rangle \langle \Psi_{\text{DES}}(t)| &= \\ &= a|\uparrow\rangle \otimes |\text{UP}\rangle \otimes |\varepsilon_{\text{up}}\rangle \langle \phi | \otimes \langle \text{UP} | \otimes \langle \varepsilon_{\text{up}} | + \\ &+ b|\downarrow\rangle \otimes |\text{DOWN}\rangle \otimes |\varepsilon_{\text{down}}\rangle \langle \phi | \otimes \langle \text{UP} | \otimes \langle \varepsilon_{\text{up}} |. \end{aligned} \quad (16)$$

Tracing out the environment degrees of freedom, we obtain the reduced two-state describing the particle-measuring device subsystem:

$$\rho_{\text{reduced}}(t) = \text{tr}_{\varepsilon} \rho(t) \approx |\uparrow\rangle \langle \phi | \otimes |\text{UP}\rangle \langle \text{UP} |. \quad (17)$$

One may say that an effective reduction has occurred, yielding the single measurement outcome: “UP”. The reduction is the result of a two-time environment induced superselection process. We call this process, which dictates the classical behavior of the measuring device, “two-time decoherence”, in analogy with the regular notion.

We have shown how effective reduction can take place subsequent to the measurement process. The process is actually time-symmetric. Remember we have treated the measurement following our measurement (the one that yielded ϕ) as instantaneous, in order to formulate the backward-evolving state. We now relax this assumption by showing how effective reduction occurs for, and determines, the backward-evolving state of the particle. Evolving the destiny vector backward in time to t_1 , we obtain:

$$\langle \Psi_{\text{DES}}(t_1) | = \langle \phi | \otimes \langle \text{UP} | \otimes \langle \varepsilon_0 |, \quad (18)$$

and at t_0 :

$$\begin{aligned} \langle \Psi_{\text{DES}}(t_0) | &= (c \langle \uparrow | \otimes \langle \text{READY} | + d \langle \downarrow | \otimes \langle \text{ORTHO} |) \otimes \\ &\otimes \langle \varepsilon_0 |, \end{aligned} \quad (19)$$

where the time-reversed interaction between the measuring device and the particle, causes a device in the final state *UP* to evolve backwards into the state *READY*, if the particle is in the state \uparrow , and into the orthogonal state *ORTHO*, if the particle is in the state \downarrow . Again we assume that an environment induced decoherence process takes place (here backwards in time, but the microscopic physics is time-symmetric), singling out *READY* and *ORTHO* as the preferred basis of pointer states for the destiny vector. Then the reduced two-state at times $t < t_0 - t_d$ (where t_d is the decoherence time) is

$$\begin{aligned} \rho_{\text{reduced}}(t < t_0 - t_d) &= (a | \uparrow \rangle + b | \downarrow \rangle) \langle \uparrow | \otimes \\ &\otimes | \text{READY} \rangle \langle \text{READY} |. \end{aligned} \quad (20)$$

Now the backward-evolving state of the particle is \uparrow , as is expected to evolve backwards in time from our “UP” measurement. This sets a final boundary condition for any previous measurement performed on the particle. Since the information for the reduction of the backward-evolving state is carried by the measuring device destiny vector, we see that a final boundary condition is required for the measuring device itself. A final boundary condition for the environment alone, as one might suggest, would be insufficient. The forward-evolving state of the particle before the time of the measurement, is of course not affected by the final boundary condition.

When considering backward-in-time evolution, one may be concerned about issues of stability. Refer to the environment states ε_{up} and ε_{down} used above, at the decoherence time $t_1 + t_d$ (which will serve as the initial time for the current example). Assume these states consist of clusters of N particles, concentrated respectively in the upper and bottom tenth of an isolated one-dimensional box of one meter length. The particles of each macrostate

evolve as to scatter and spread out in the box, the two macrostates remaining nearly orthogonal to each other. As described before, a final boundary condition is assumed in the form of a measuring device pointer state coupled to, and superselected by, one of these environment macrostates, say ε_{up} (all states in their form at the final time). This destiny vector evolves backwards in time to determine the outcome in the measuring device. Now, let the system not be completely isolated, and allow external quantum disturbances which interfere with the evolution in a causally indeterministic and thus irreversible way. While the naked eye might not notice the effect on the late time spread out macrostates, a random disturbance of the time-reversed evolution would obviously give rise to initial states very different from the original concentrated states. This is the instability of the time-reversed evolution, a process of decreasing entropy due to the second law of thermodynamics. The question then rises, what are the implications on the suggested interpretation, where the backward-evolving state determines the measurement outcome. The threat will come from state changing quantum measurements performed on the environment particles. Consider therefore, a position measurement performed on n of the particles, with a precision of one angstrom, at a time when the particles are spread out in the box. The new disturbed macrostates: ε'_{up} and ε'_{down} , have projection amplitudes: $\langle \varepsilon'_{up} (down) | \varepsilon_{up} (down) \rangle = 10^{-10n}$. Assuming our environment is large enough in order to constitute a good environment, the disturbed macrostates are still nearly orthogonal, and the decoherence process remains intact. We now have to consider the implication of having ε'_{up} instead of ε_{up} in the destiny vector, evolving backwards from the disturbance measurement. The above requirement for the intactness of decoherence, implies that the disturbed state is still nearly orthogonal to ε_{down} . On the other hand, the scalar product with ε_{up} , on which the disturbed state was projected, is of course finite (10^{-10n}). For a crude upper limit on the scalar product with ε_{down} , one may take 10^{-N} , accounting for the spread out state of the ε'_{up} at the initial time, at which ε_{down} is concentrated at the bottom tenth of the box. In order for the two-time decoherence scheme to still work, the latter number should be negligible relative to the former, that is $N \gg 10n$. This is a requirement on the size of the environment relative to the disturbance, which is presumably connected to the requirement for decoherence.

Our example can readily be extended to more complex systems. For instance, we have considered only a single measurement process per measuring device, where it is of course possible to perform more than one measurement with the same device. Since the measuring device has a single specific final state (assuming it has only one degree of freedom), where does the information for the many measurement outcomes reside? The answer is that a device that is used for multiple measurements, must be initialized between the measurements. The unitary initialization interaction transfers the information of the

previous state of the measuring device to other systems, no information being lost. In any case, a multiple-time generalization of the two-time description, with a multitude of intermediate boundary conditions (such as the picture in “consistent histories” interpretations) is not required in order to account for multiple measurements.

IV. PHILOSOPHICAL ASPECTS

It was already assumed, that subsequent to the measurement interaction, decoherence causes an effectively irreversible branching of the superposition into isolated terms. Therefore, no inconsistencies can arise from the existence of a special final boundary condition of the form described before, which simply causes the selection of a single specific branch from the many worlds picture. For this reason, the locality property of standard quantum mechanics remains valid in the suggested interpretation.

Also for the above reason, the measurement process does not increase the measure of irreversibility beyond that of regular thermodynamics. That is, the suggested interpretation does not suggest a microscopic quantum mechanical arrow of time. It does however assume asymmetric initial and final boundary conditions.

It must be emphasized that the states in the final boundary condition, which we have taken to be specific in the examples, are generally unknown prior to the completion of the measurement. Classically, a priori knowledge of the future is of course an acausal state of affairs. The following example shows that it is also a problem due to quantum mechanics itself. Assume there are two entangled spin- $\frac{1}{2}$ particles located at two far away locations, in the initial state $\uparrow_A\uparrow_B + \downarrow_A\downarrow_B$. A denotes the particle at Alice’s location, and B – the particle at Bob’s location. Assume that Bob knows the final state to be $\uparrow_A\uparrow_B + \uparrow_A\downarrow_B$. Now, Alice may or may not perform a unitary rotation on her particle, of the form $\uparrow_A \rightarrow \downarrow_A$ and $\downarrow_A \rightarrow \uparrow_A$, leaving the initial composite state as it was, or transforming it into the state $\downarrow_A\uparrow_B + \uparrow_A\downarrow_B$. Bob, measures the spin of his particle, obtaining \downarrow_B or \uparrow_B , according to the action or non-action of Alice. In this manner, Alice may allegedly transmit signals to Bob at an instant. A procedure like this would be possible for many arbitrary choices of initial and final states. Only when identical measurements are performed in sequence, can a final state (or a measurement outcome) be predicted with certainty in advance.

Therefore, like in hidden variables theories, the parameters determining the measurement outcome are inaccessible. Still, like in those theories, the evolution may be considered deterministic (though unpredictable). As mentioned before, determinism is valid if considered in a broader two-time context, where the evolution is determined not only by an initial boundary condition, but also by a final boundary condition. The latter dynamically determines the measurement outcomes, such that in all intermediate times the physical states are deter-

mined by (two) unitary Schrödinger evolution. Given the boundary conditions and a Hamiltonian, one may reconstruct the whole evolution history, no random dice need be tossed. Alternatively, the state of the system at any time, is completely determined by its two-state at any single time.

The existence of a future boundary condition, and its deterministic effect, do not deny our apparent freedom of choice. The latter is allowed due to the inaccessibility of the data (which was a requirement of causality). Imagine an external observer with a reversed arrow of time, classically (or weakly) monitoring and recording our future measurements, without disturbing them. As long as he does not pass to us any beforehand information, the picture is consistent by definition. His records, analogous to a future boundary condition, cannot be spoiled by (nor do they prohibit) a “change of mind” on our behalf, which would already be taken into account. “Foreknowledge no more “forces” the future to be a certain way, than true reports in history books “force” the past to have been a certain way”[12]. Perhaps this is close to the omniscient approach expressed by the old Hebrew sages: “All is foreseen and the choice is given”[13].

So, our apparent freedom of choice is not at danger. But is a genuine freedom of choice really possible? It is not easy to see how such a concept could even be well defined in physical terms. Nevertheless, consider the case that the extra degree of freedom of an agent-system, the destiny vector, corresponds to what we refer to as the “free choice of the agent”. This determines the outcomes of (cerebral) quantum measurements, in the manner described in this work. We have a description where the choice is “causa sui” (cause of itself), while still being the choice of the agent-system. This would constitute a unique realization of the concept of genuine free will.

The last property we wish to address is that of “realism” or “objectivity”. These refer to the classical concept that the existence of physical properties is independent of observations of these properties. EPR [14] define realism by the following counterfactual:

If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

This would require the existence of some additional (possibly “hidden”) variables, which determine the outcomes of the measurements. As mentioned before, hidden variable theories are inherently nonlocal [5],[6], and the possibility of local realism is excluded. This line of reasoning requires the validity of “counterfactual definiteness”. The meaning of which is that it is meaningful to ask hypothetical “what-if” questions. If it is not, the EPR definition of realism is irrelevant. Such is the case with the many worlds interpretation, where each measurement yields all possible outcomes. This is also the case with

the suggested interpretation, were it is assumed that a final boundary condition is predetermined according to the measurements that actually take place. The destiny vector constitutes a special element of reality or hidden variable, which answers only the question that is being asked.

V. WEAK MEASUREMENTS

Aharonov et al. have introduced the concept of weak measurements and weak values [15]. The idea is to perform a measurement, in which the interaction is weak enough, in some sense, leaving the two-state (both history and destiny vectors) of the measured system almost undisturbed. The weakness of the interaction, implies that the state of the measuring device cannot be sharp but rather has a broad spread (large uncertainty) in its pointer variable. The weighted superposition of the shifted broad pointer states, that are obtained by the interaction, adds up to a single state centered about a two-time expectation value of the measured operator. This is the weak value of the measured operator, which is determined by both initial and final states of the measured system. For a system with initial state Ψ_i and final state Ψ_f , the weak value of operator A is given by

$$A_w \equiv \frac{\langle \Psi_f | A | \Psi_i \rangle}{\langle \Psi_f | \Psi_i \rangle}. \quad (21)$$

In general A_w may be far from any eigenvalue of A . The measurement is concluded with an observation of the measuring device, effectively reducing its state to give a sharp outcome according to its probability distribution. Weak values may play an important role in the interpretation of certain phenomena such as tunnelling [16], trans-Planckian frequencies in the Hawking radiation from a black hole [17] and Cherenkov radiation of superluminal particles [18]. A formal reduction to the single-time case is made by considering a final state equal the initial state, $\psi_f = \psi_i$. This is just the trivial boundary condition (9), assuming that the measured system is almost undisturbed by the measurement, and assuming for simplicity no self-evolution during the measurement process. The weak value then reduces to the regular expectation value of the measured operator A ,

$$\langle A \rangle \equiv \langle \Psi_i | A | \Psi_i \rangle = \sum_k a_k \text{prob}(a_k | \Psi_i). \quad (22)$$

Let us follow the weak measurement process using the two-time formalism. Consider a many level quantum system in the basis $\{a_k\}$, and the operator A defined as

$$A = \sum_k a_k |a_k\rangle\langle a_k|. \quad (23)$$

We take the initial state of the system to be

$$|\phi_1\rangle = \sum_k c_k |a_k\rangle. \quad (24)$$

The measuring device is described by a pointer state $Q(q)$, which is a Gaussian-like function centered around the pointer variable q . The initial state of the measuring device is $Q(0)$, and the initial state of the composite system is

$$|\Psi_i(0)\rangle = |\phi_1\rangle \otimes |Q(0)\rangle. \quad (25)$$

A measurement of A is performed by an interaction between the system and the measuring device, such that if the device measures a_k , it evolves into the state $Q(a_k)$. The state of the composite system at time t_1 after the measurement interaction is

$$|\Psi_i(t_1)\rangle = \sum_k c_k |a_k\rangle \otimes |Q(a_k)\rangle. \quad (26)$$

At a later time t_2 we perform an ideal post-selection measurement on the quantum system obtaining the result:

$$|\phi_2\rangle = \sum_k c'_k |a_k\rangle, \quad (27)$$

which serves as a final boundary condition for the measured system. The final composite state is thus the projection on ϕ_2 :

$$|\Psi_f(t_2)\rangle = |\phi_2\rangle\langle\phi_2| |\Psi_i(t_1)\rangle = |\phi_2\rangle \otimes \sum_k c_k c'_k{}^* |Q(a_k)\rangle. \quad (28)$$

The condition for the weakness of the measurement is that the Gaussians in the last term are sufficiently broad so that the relation

$$\sum_k c_k c'_k{}^* |Q(a_k)\rangle \approx \sum_k c_k c'_k{}^* |Q(a')\rangle \equiv |\hat{Q}(a')\rangle, \quad (29)$$

holds for some a' . It can be shown [15] that a' is just A_w , the weak value of A ,

$$A_w = \frac{\langle \phi_1 | A | \phi_2 \rangle}{\langle \phi_1 | \phi_2 \rangle} = \frac{\sum_k c_k c'_k{}^* a_k}{\sum_k c_k c'_k{}^*}. \quad (30)$$

Therefore the measuring device reading is centered on the real part of the weak value. It is clear that after post-selection, the weak value is obtained as a consequence of the projection onto the new quantum state ϕ_2 . Looking at the reduced two-state of the measuring device, it is easy to see how the weak value actually emerges before the post-selection measurement is performed. The two-state of the composite system at a time t between t_1 and t_2 (after the weak interaction is complete and before post-selection) is given (up to normalization) by

$$\rho(t) = |\Psi_i(t_1)\rangle\langle\Psi_f(t_2)|. \quad (31)$$

Tracing out the measured system degree of freedom, the measuring device already reads the weak value:

$$\begin{aligned} \rho_{pointer}(t) &= \text{tr}_\phi \rho(t) = \sum_{k,j} c_k c'_k{}^* c'_j c'_j{}^* |Q(a_k)\rangle\langle Q(a_j)| \approx \\ &\approx |\hat{Q}(A_w)\rangle\langle\hat{Q}(A_w)|. \end{aligned} \quad (32)$$

Considering a complete set of post-selections, the probability distributions of the corresponding weak measuring device states, add up to recover the regular probability distribution without post-selection. The probability distribution of any specific state of the former is always smaller than the latter, and the post-selection is consistent with the standard quantum mechanical probabilities (where the “weak” result may always be considered a “measurement error”). Also, the weakness condition must imply that the initial state of the measuring device is, in general, an analytic function in a strip around the real axis of the pointer variable. Thus any local region contains the information of the shape of the entire wave function even before the interaction takes place, and the process is one of amplification rather than information transfer. The above two properties of weak measurements ensure that the procedure is always causally consistent.

Measurements performed on very large macroscopic systems are essentially weak measurements, which examine the system’s past and future boundary conditions. Consider, for example, measurements of galactic properties such as mass or angular momentum. A consistent “strange” measurement outcome, might arise due to a special final boundary condition of the measured system, different from the trivial boundary condition (9). This would result in the observation of a weak value which may be far from the expectation value. In contrast to the case of a post-selection measurement, the effect of a special natural final boundary condition would seem to us as new fundamental laws of nature, here already

breaking the framework of standard quantum mechanics. We may speculate, that certain unaccounted for phenomena might be an indication for, and might be explained by, such a special boundary condition. For example, we may mention the inconsistencies in measurements of cosmological parameters by different methods, namely the dark matter puzzle.

VI. SUMMARY

We have suggested a *Two-time interpretation* of quantum mechanics. We have shown how a special final boundary condition for the Universe, can effectively account for the observed wave function reduction, while being consistent with standard quantum mechanics formalism. The suggested interpretation is therefore one which brings together quantum mechanics’ empirical predictions and formalism, thus solving the “measurement problem”. What seems a mystery to the single-time observer may come to a simple resolution looking at our Universe from a two-time perspective.

Acknowledgments

We would like to thank Shmuel Nussinov, Benny Reznik, Daniel Rohrlich and Lev Vaidman for reviewing this work. This research was supported in part by grant 62/01-1 of the Israel Science Foundation, established by the Israel Academy of Sciences and Humanities, NSF grant PHY-9971005, and ONR grant N00014-00-0383.

-
- [1] Y. Aharonov and D.Z. Albert, Phys. Rev. D *24*, 359 (1981); *29*, 228 (1984).
- [2] G.C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D *34*, 470 (1986); P. Pearle, Phys. Rev. A *39*, 2277 (1989); G.C. Ghirardi, P. Pearle, and A. Rimini, Phys. Rev. A *42*, 78 (1990).
- [3] *Quantum Theory and Measurement*, edited by J.A. Wheeler and W.H. Zurek (Princeton University Press, Princeton, NJ, 1983).
- [4] D. Bohm, Phys. Rev. *85*, 166 (1952); *85*, 180 (1952), [reprinted in [3]].
- [5] J.S. Bell, Physics *1*, 195 (1964), [reprinted in [3]].
- [6] D.M. Greenberger, M.A. Horne, and A. Zeilinger, in *Bell’s Theorem, Quantum Theory and Conceptions of the Universe*, edited by M. Kafatos (Kluwer Academic, Dordrecht, 1989), p. 69; N.D. Mermin, Phys. Rev. Lett. *65*, 3373 (1990).
- [7] H. Everett, Rev. Mod. Phys. *29*, 454 (1957), [reprinted in [3]].
- [8] W.H. Zurek, Phys. Rev. D *24*, 1516 (1981); *26*, 1862 (1982).
- [9] Y. Aharonov, P.G. Bergmann, and J.L. Lebowitz, Phys. Rev. *134*, B1410 (1964).
- [10] B. Reznik and Y. Aharonov, Phys. Rev. A *52*, 2538 (1995) [arXiv:quant-ph/9501011].
- [11] W. C. Davidon, Il Nuovo Cimento *36B*, 34 (1976).
- [12] N. Swartz, *Lecture Notes on Free Will and Determinism*, part 3 (1997), <http://www.sfu.ca/philosophy/swartz/freewill1.htm>.
- [13] *Mishna, Pirkei Avot*, chap. 3, verse 15.
- [14] A. Einstein, B. Podolsky and N. Rosen, Phys Rev. *47*, 777 (1935).
- [15] Y. Aharonov, D.Z. Albert, and L. Vaidman, Phys. Rev. Lett. *60*, 1351 (1988); Y. Aharonov and L. Vaidman, Phys. Rev. A *41*, 11 (1990).
- [16] Y. Aharonov, S. Popescu, D. Rohrlich, and L. Vaidman, Phys. Rev. A *48*, 4084 (1993).
- [17] B. Reznik, Phys. Rev. D *55*, 2152 (1997) [arXiv:gr-qc/9606083].
- [18] D. Rohrlich and Y. Aharonov, Phys. Rev. A *66*, 042102 (2002) [arXiv:quant-ph/0107025].