Complementarity between Local and Nonlocal Topological Effects

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In certain topological effects the accumulation of a quantum phase shift is accompanied by a local observable effect. We show that such effects manifest a complementarity between nonlocal and local attributes of the topology, which is reminiscent but different from the usual wave-particle complementarity. This complementarity is not a consequence of noncommutativity, rather it is due to the noncanonical nature of the observables. We suggest that a local/nonlocal complementarity is a general feature of topological effects that are "dual" to the Aharonov-Bohm effect.

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In the Aharonov-Bohm (AB) effect [1,2] a charge moves around a magnetic flux filament in a region with vanishing electromagnetic fields. The charge experiences no electromagnetic forces, yet it accumulates a topological quantum phase shift. Topological effects which are "dual" to the AB effect have been discovered for neutral particles. Aharonov and Casher [3–5] have shown that particles carrying a magnetic moment and moving around a straight wire with uniform charge density will experience no force but acquire a phase shift analogous to the AB phase [6–9]. More recently, it has been shown by He and McKellar [10] and by Wilkens [11] that a neutral particle carrying an electric dipole also exhibits similar dual topological effects [12,13]. Nevertheless, unlike the AB effect, in these dual cases the local fields along the trajectory of the particle do not vanish. Consequently, as was pointed out by Peshkin and Lipkin [14,15], the accumulation of the attendant quantum phase shift may be accompanied by a local observable effect.

In this Letter we suggest that such dual topological effects manifest a complementarity between the nonlocal and the local attributes of the topology, which is reminiscent but yet different from the usual wave-particle complementarity; by measuring a local observable we disturb the nonlocal phase information on the topology. However, the complementarity suggested here is not a consequence of noncommutativity; rather it is due to the noncanonical nature of the corresponding observables [16].

To illustrate this complementarity let us begin with the Aharonov-Casher (AC) effect [3]. In the AC effect the magnetic moment μ interacts with the electric field $\vec{E}$, the vector potential-like term, $\vec{\mu} \times \vec{E}$, which induces a phase

$$\phi_{\rm AC} = \frac{1}{\hbar}\oint \vec{\mu} \times \vec{E} \cdot \vec{dl} = \frac{\mu\lambda}{\hbar}\, n_{\rm AC}\,. \qquad (1)$$

Here μ is the projection of $\vec{\mu}$ in the direction of the line, λ is the charge per unit length, and $n_{\rm AC}$ is the winding number of the magnetic moment's trajectory around the line.

We note that several similarities exist between the AC and AB effects. Since in both cases the force vanishes, they are "force-free" effects [6]. Moreover, in both cases a "vector-potential" coupling gives rise to a topological phase; indeed for a closed trajectory, the AC phase and the AB phase are insensitive to the details of the path and are determined from the winding number alone.

However this similarity breaks down in one important aspect [7]. Since in the AB effect the particle couples to a gauge field, the locally accumulated phase is not gauge invariant. Only the phase accumulated in an interference experiment, on a closed loop, is a gauge invariant quantity. Therefore, the AB effect is sometimes described as being nonlocal. On the other hand, since in the present case the magnetic moment clearly couples directly to the field strengths $\vec{E}$, the locally accumulated phase is a gauge invariant and meaningful observable.

This naturally raises the question of the nonlocality of such effects [17–19]. In particular, Peshkin and Lipkin [15] have noted that a magnetic field is present in the local reference frame of the magnetic moment. Consequently, the magnetic moment vector precesses around the direction of the magnetic field by an angle that turns out to be proportional to the phase shift. Since the accumulation of the quantum phase shift is accompanied by a local precession, and since the latter is locally observable, they concluded that the AC is inherently local.

It was, however, implicitly assumed that the accumulated phase and the precession are two attributes of the effect, which are simultaneously meaningful. Indeed the autocorrelation operator suggested in [15] commutes with the phase operator (that we define below). This apparently implies that we can locally measure the rotation of the magnetic moment without disturbing the accumulated phase. Nevertheless, the precession and the accumulated phase are noncanonical variables [16], and *for noncanonical variables commutativity does not imply mutual observability*. In fact, we will show that by observing the local precession we necessarily induce an uncertainty in the accumulated phase, in a similar way as for ordinary canonical conjugate variables.

Let us first show that the above complementarity is required for consistency of the AB effect with ordinary

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wave-particle complementarity [20]. Consider the usual AB interference experiment of charged particles around a solenoid enclosing a magnetic flux. We wish, however, to regard the effect of the charge on the internal degrees of freedom of the fluxon. For simplicity let space be only two dimensional; hence the solenoid is replaced by a magnetic moment $\vec{\mu}$ pointing in the "up" direction. The magnetic moment generates an AB vector potential; hence the charge acquires the usual AB phase shift that can be observed in an interference experiment. On the other hand, consider now the effect of the charged particle on the magnetic moment. In the rest frame of the magnetic moment, the moving charge generates the magnetic field $\vec{B} = \vec{v} \times \vec{E}$, where $\vec{v}$ is the velocity of the charge and $\vec{E}$ is its electric field. As noted above, this magnetic field causes a precession of the magnetic moment $\vec{\mu}$. By evaluating the angle of precession $\delta\varphi$ when the charge q is moving along either the upper or lower side of the magnetic moment we find $\delta\varphi \propto \phi_{\mathrm{AC}}$ (see below). Therefore, by measuring $\delta\varphi$ we can determine on which side of the fluxon the particle moved. Clearly, that contradicts the wave-particle complementarity principle, unless the measurement of precession destroyed the coherence of the two trajectories.

We shall next examine this process in more detail to see how in actuality this loss of coherence happens. The nonrelativistic Hamiltonian in three dimensions [3,8,9]

$$H = \frac{(\vec{P} + \vec{\mu} \times \vec{E})^2}{2m} - \frac{\mu^2 E^2}{2m} \tag{2}$$

describes a spin-half neutral particle carrying magnetic moment $\vec{\mu} = \mu\vec{\sigma}$, which moves in an electric field $\vec{E}$. In the AC effect, the electric field is generated by a straight wire with uniform charge density λ. If the particle moves in the plane orthogonal to the wire, and the momentum in the direction of the wire vanishes, one finds [9] that H effectively reduces to a two-dimensional Hamiltonian. Using polar coordinates $(r(x,y), \theta(x,y))$ where the charged wire is located at $r = 0$, and points in the z direction, we get

$$H = \frac{p_r^2}{2m} + \frac{(p_\theta + \xi\sigma_z)^2}{2mr^2}, \tag{3}$$

where $\xi \equiv \lambda\mu/2\pi$.

The Heisenberg equations of motion for the spin

$$\begin{aligned} \dot{\sigma}_x &= -\frac{2\xi}{\hbar m r^2} p_\theta \sigma_y, \\ \dot{\sigma}_y &= \frac{2\xi}{\hbar m r^2} p_\theta \sigma_x, \\ \dot{\sigma}_z &= 0. \end{aligned} \tag{4}$$

describe a precession of the spin around the z axis. When the magnetic moment moves between time t_0 to time t along a path joining points with angular coordinates $\theta(t_0)$ and $\theta(t)$, we find that (up to the trivial phase $\frac{2\xi}{\hbar m}\int_{t_0}^{t}\frac{dt'}{r^2}$), the precession is generated by the unitary operator

$$U(t,t_0) = e^{-i\frac{2\xi}{\hbar}\int_{t_0}^{t}\sigma_z\dot{\theta}(t')\,dt'}. \tag{5}$$

Indeed, this precession is induced by the magnetic field, $B_z = (\vec{v} \times \vec{E})_z = 2\xi\dot{\theta}$, experienced by the spin in its rest frame. If σ_z is constant, the angle of precession φ is hence $\varphi = 2\xi\sigma_z[\theta(t) - \theta(t_0)]/\hbar$.

Consider now the wave function ψ of the magnetic moment. For the above trajectory, it changes according to

$$\psi \rightarrow U(t,t_0)\psi = e^{-i\delta\phi_{\mathrm{AC}}}\psi, \tag{6}$$

where

$$\delta\phi_{\mathrm{AC}}(t,t_0) = \frac{\xi}{\hbar}\int_{t_0}^{t}\sigma_z\dot{\theta}(dt')\,dt'. \tag{7}$$

In the AC effect, ψ is an eigenstate of σ_z. If $\sigma_z = 1$, $\delta\phi_{\mathrm{AC}}(t,t_0) = \xi[\theta(t) - \theta(t_0)]/\hbar$. We will henceforth refer to $\delta\phi_{\mathrm{AC}}(t,t_0)$ in (7) as the "phase operator." In general it describes the phase accumulated along a definite trajectory for arbitrary σ_z.

Let us next examine what is the effect on a system when the spin precession is measured. The rotation implies that the spin at t_0 is related to the spin at some latter time t. Particularly we have the identity

$$C_\varphi(t,t_0) \equiv U^\dagger(t,t_0)\vec{\sigma}(t_0)U(t,t_0) - \vec{\sigma}(t) = 0, \tag{8}$$

which follows from the equation of motion of the spin. By observing that $C_\varphi(t,t_0)$ indeed vanishes we can verify that the spin rotates. One might think that since

$$[C_\varphi(t,t_0), \delta\phi_{\mathrm{AC}}(t,t_0)] = 0, \tag{9}$$

we should actually be able to observe simultaneously both the precession operator $C_\varphi(t,t_0)$ and the phase operator $\delta\phi_{\mathrm{AC}}(t,t_0)$.

However, the above commutativity is a dynamical result, i.e., it is valid only by virtue of equations of motion (4). To define $C_\varphi(t,t_0)$ and $\phi_{\mathrm{AC}}(t,t_0)$ one has to specify the Hamiltonian (2). For such noncanonical variables [16], commutativity does not imply mutual observability.

To observe $C_\varphi(t,t_0)$, we have to couple to the system twice, first at time t_0 and then at a later time t. Since the coupling of the system to a measuring device at time t_0 changes the Hamiltonian (2) (because we must add to the Hamiltonian new terms describing the interaction of the system with a measuring device), the spin component σ_z will no longer be constant, and the accumulated phase (7) will change by an uncertain amount.

Let us examine in more detail the uncertainty produced in $\delta\phi_{\mathrm{AC}}(t,t_0)$ when $C_\varphi(t,t_0)$ is measured. To this end, we couple at $t = t_0$ to σ_i and at some later time t to the rotated spin $U^\dagger\sigma_i U$. To be able to observe a precession of a single spin, we must be sure that the spin has rotated by a sufficiently large angle, say $\varphi = \pi/2$. Choosing $i = x$, we get for this case

$$C_{\pi/2}(t,t_0) = \sigma_y(t) - \sigma_x(t_0) = 0. \tag{10}$$

Because $C_{\pi/2}(t,t_0)$ is of order unity it must be observed with precision

$$\Delta C_{\pi/2}(t, t_0) \ll 1\,. \tag{11}$$

Hence $\sigma_x(t_0)$ is effectively measured with precision $\Delta\sigma_x \ll 1$. During the time interval (t, t_0), σ_z then becomes uncertain by $\Delta\sigma_z \approx 1$. The consequent uncertainty in the AC phase

$$\Delta\phi_{\rm AC}(t, t_0) = \frac{\xi}{\hbar}[\theta(t) - \theta(t_0)]\Delta\sigma_z \approx \pi/4 \tag{12}$$

is hence sufficiently large to erase the phase information. This achieves our goal of showing that by measuring the precession we destroy the coherence.

More generally, we will be able to infer that $C_\varphi(t, t_0) = 0$ only statistically. Consider, for example, the limiting case that the spin has precessed by only a small angle $\varphi \ll 1$. Let $|x\rangle$ be the eigenstate of σ_x, and denote the rotated eigenstate by $|\varphi\rangle$. Hence $|\langle\varphi\,|\,x\rangle| \simeq 1 - \varphi^2$. To verify the precession, one has to repeat the experiment over a sample of $N \sim \frac{1}{\varphi^2}$ spins, all initially in the same $|x\rangle$ state, and measure separately for each spin the operator $C^i_\varphi(t, t_0)$. The total phase accumulated by the N spins, $\phi^N_{\rm AC} = \sum_{i=1}^N \phi^i_{\rm AC}$, will become uncertain by

$$\Delta\phi^N_{\rm AC} = \frac{\xi}{\hbar}[\theta(t) - \theta(t_0)]\sum_{i=1}^{N}\Delta\sigma^i_z \approx \varphi\sqrt{N}/2 \sim 1/2\,, \tag{13}$$

where the relation, $\phi = 2\xi[\theta(t) - \theta(t_0)]/\hbar$, was used, and we have assumed that the uncertainties $\Delta\sigma^i_z \approx 1$, for each spin, are independent. Therefore, if we verify that the N spins precess, the total accumulated phase becomes uncertain. This verifies our claim also for this case.

Next consider a special case where the spinor nature has a special role. Suppose that the spin rotates around the $\hat{z}$ axis, by either $\varphi = +\pi$ or $\varphi = -\pi$. In both cases, of either a clockwise or a counterclockwise rotation, σ_x changes to $-\sigma_x$ and

$$C_\pi = \sigma_x(t) + \sigma_x(t_0) = 0\,. \tag{14}$$

In space-time these two alternatives correspond to a magnetic moment moving along either a clockwise or a counterclockwise path around the charged wire with $\theta(t) - \theta(t_0) = \pm\hbar\pi/2\xi$. Since both paths give rise to the same rotation of the spin, they cannot be distinguished by measuring $C_\pi(t, t_0)$. Therefore, in this particular case, consistency with ordinary wave-particle complementarity does not require that coherence must be lost. So it may appear that this provides a counterexample to our claim.

However, by observing C_π, we are still unable to distinguish between a nontrivial or a trivial phase in an interference experiment, i.e., we cannot detect a nontrivial topological effect. To see that, let us compare two cases: first consider an AC effect where the charged line generates the phases $\phi_{\rm AC} = \pi/2$ on one path and $-\pi/2$ for the other. This yields a relative nontrivial π phase. In the second setup we arrange a special charge distribution which generates the *same* $\pi/2$ phase for *both* paths. The relative phase in the second case is trivial, however, since the spin rotates by π, $C_\pi = 0$ in both cases, and we cannot distinguish by performing this measurement between the cases of a topological and a nontopological effect.

Similar reasoning applies for the case of a magnetic moment moving through a region with a homogeneous but time dependent magnetic field $B(t)$. The corresponding phase shift

$$\phi_{\rm AC} = \frac{1}{\hbar}\int \vec{\mu}\cdot\vec{B}(t')\,dt' \tag{15}$$

was observed by Allman *et al.* [5]. This effect is sometimes referred to in the literature as the Scalar-AB effect, because the interaction term $\vec{\mu}\cdot\vec{B}$ in (2) is analogous to the qV term in the AB Hamiltonian $(p - qA)^2/2m + qV$, which gives rise to the potential-AB effect. Since with the inclusion of a magnetic field the AC Hamiltonian is $H_{\rm AC} = (\vec{p} + \vec{\mu}\times\vec{E})^2/2m - \vec{\mu}\cdot\vec{B}$, perhaps it is more natural to identify this phase as a "potential-AC effect." It can be readily shown that our arguments for the AC effect follow, by replacing the particle's rest-frame magnetic field, $\dot{\theta}(t)$, with $B(t)$.

So far we have shown a complementarity relation in the cases of the AC and the potential AC (or Scalar-AB) effects. Nevertheless, the gedanken experiment suggested earlier indicates that a similar complementarity relation exists for other dual effects. Such is the topological effect for electric dipoles [10,11,13], which is manifested via a vector potential $\vec{d}\times\vec{B}$ or a "potential" $\vec{d}\cdot\vec{E}$ [11]. It was noted that the effect for electric dipoles can be obtained from the AC setup by a Maxwell duality transformation [12]. To connect our gedanken experiment with these cases, we will, however, make use of a different type of duality [3]

$$\text{charge} \leftrightarrow \text{magnetic moment} \tag{16}$$

which transforms the magnetic filament and the charge in the AB effect to a charged wire and a magnetic moment in the AC effect and vice versa. This duality is closely related to the Galilean invariance of the nonrelativistic charge-magnetic moment system. The total accumulated phase depends only on the relative motion of the charge and the magnetic moment. Hence by a duality transformation we transform from the rest frame of the magnetic moment (in the AB effect) to the rest frame of the charge (in the AC effect). As we already have shown, since the phase is "common" to the charge and the magnetic moment, the consistency of the AB effect with ordinary wave-particle complementarity necessitates a local/nonlocal complementarity for the "companion" dual effect.

We next note that the potential electric-dipole effect generated by the $\vec{d}\cdot\vec{E}$ term [11] is dual to the nonlocal potential AB effect: let two charged plates of a "capacitor" initially overlap each other. A potential difference between the two sides of the capacitor is then formed when a charged particle passes close to the capacitor (so that $\vec{E} \simeq 0$), by changing temporarily the distance between the plates. The charge then experiences no force but accumulates the AB phase $\frac{q}{\hbar} \int V(t')\, dt'$.

Consider now the duality transformation

$$\text{charge} \leftrightarrow \text{electric dipole} \qquad (17)$$

which replaces the capacitor (viewed as a planar density of electric dipoles) by a homogeneously charged plate, and the moving charge by a time dependent electric dipole $\vec{d}(t)$. In a sense this again corresponds to a transformation from the capacitor rest frame to that of the charge. Hence the electric dipole effect is the dual companion of the potential AB effect. The electric dipole experiences no forces, yet it acquires the phase $\phi_D = \frac{1}{\hbar} \int \vec{d}(t') \cdot \vec{E}\, dt'$. However, viewing the dipole as formed by an extended (time dependent) charge distribution in an external electric field, it will induce a corresponding time dependent nonvanishing internal stress. The consistency of the potential AB effect then requires that the local internal effects should be complementary to the accumulated phase. It would be interesting to understand the details of the complementarity relation for this case and the related vector-potential dipole effect.

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Comment on "Complementarity between Local and Nonlocal Topological Effects"

In Ref. [1], Aharonov and Reznik suggest a complementarity occurring in the Aharonov-Casher (AC) effect [2] between the accumulated phase shift, which they regard as a nonlocal quantity, and the local precession of the spin state due to a magnetic field in the rest frame of the magnetic moment.

We point out that there can be no such complementarity since only one of the two attributes occurs for a given initial spin state. This is related to the fact that there will be a local torque on the particle due to the magnetic field from the charged line in the rest frame of the magnetic dipole, causing a phase shift in the ideal case, where the spin state of the magnetic moment is an eigenstate of $\hat{\sigma}_z$, and spin precession in the nonideal case. Thus, in the ideal case the phase shift has to be considered a local attribute of the same origin as the precession in the nonideal case. It is accompanied by local angular momentum fluctuations of the electromagnetic field, observable in principle for a finite field [3].

In order to clarify the relation between phase shift and precession, we solve Eqs. (4) in [1] as

$$\sigma_x^H = \sigma_x^S \cos\gamma - \sigma_y^S \sin\gamma, \qquad \sigma_y^H = \sigma_x^S \sin\gamma + \sigma_y^S \cos\gamma,$$

$$\sigma_z^H = \sigma_z^S, \qquad \gamma = \frac{2\xi[\theta(t) - \theta(t_0)]}{\hbar},$$

where the superscripts H and S denote operators in the Heisenberg and Schrödinger pictures, respectively. It can be seen that $\boldsymbol{\sigma}^H = \mathcal{U}^\dagger \boldsymbol{\sigma}^S \mathcal{U}$ with $\mathcal{U} = \exp[-i\sigma_z^S \gamma/2]$ the spin rotation operator around the z axis, being the direction of the charged line. In the Schrödinger picture, the operator $\mathcal{U}$ corresponds to the part of the time evolution operator related to spin and should act on the initial state ket $|\Psi_0\rangle$. Thus, in a single beam, the torque causes a phase shift in the ideal AC case and spin precession in the nonideal case. However, a phase *difference* between two beams $|\Psi_1\rangle$ and $|\Psi_2\rangle$ passing the line charge on opposite sides can be defined in both cases by the Pancharatnam connection $\phi = \arg\langle\Psi_1 \mid \Psi_2\rangle$ [4], leading to $\phi = -\arctan[\cos\beta \tan\phi_{\rm AC}]$, where β is the angle between the direction of the spin state and the z axis, and $\phi_{\rm AC} = 2\pi\xi/\hbar$. The phase difference and the visibility $|\langle\Psi_1 \mid \Psi_2\rangle| = (1 - \sin^2\beta \sin^2\phi_{\rm AC})^{1/2}$ can be measured by adding an extra U(1) phase shift to one arm of the interferometer [5]. In the nonideal case this measurement performed with, e.g., a triple-Laue neutron interferometer AC setup [6] extended to polarized neutrons, also tests the precession as it determines the two parameters β and $2\phi_{\rm AC}$, where the latter in this case corresponds to the precession angle around the interferometer loop.

A measurement of the precession in the same setup is restricted by the fact that a spin measurement in a beam in a direction other than the direction of the spin state disturbs the Pancharatnam phase difference between the two interfering beams. However, if the local spin state is a $+$ state in a certain direction $\mathbf{n}$, then a local measurement of the projector $\frac{1}{2}(1 - \sigma_{\mathbf{n}})$ would be a null measurement that verifies the state without affecting it, provided that the AC precession can be neglected during the measurement. When performing such a null measurement in each arm of the interferometer, the interference should not be disturbed and precession can simultaneously be confirmed by comparing the directions of the two devices that measure the local projectors.

We note that the same remarks on the local/nonlocal complementarity apply to the scalar Aharonov-Bohm effect [7], as well as to the He-McKellar-Wilkens effect [8].

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Aharonov and Reznik Reply: The two main claims raised by Hyllus and Sjöqvist [1] (HS) are that: (a) The local effect on the magnetic moment and the nonlocal phase shift are attributed to a single degree of freedom, and therefore such complementarity [2] cannot exist. (b) To demonstrate this, they argue that a verification measurement of the local effect does not destroy the interference. While we cannot agree with the first claim, we agree that under certain circumstances (which we discuss), a verification measurement is possible. We argue that this does not void the suggested concept of complementarity, but does require a more careful definition of the complementarity. To clarify these issues, it will be helpful to consider first the complementarity for the case of the Aharonov-Bohm (AB) [3] effect.

Consider the AB interference experiment in a two-dimensional setup. An electron moves along two circular paths around a fluxon whose magnetic field is oriented in a direction orthogonal to the plane of motion. Suppose that the fluxon is generated by a spin carrying a magnetic moment. Now the moving electron generates a magnetic field $\pm B_z$ at the location of the spin, where the $\pm$ depends on its trajectory. In order to distinguish between "right" and "left" trajectories, one can measure the magnetic field by observing the precession of the spin. But standard wave-particle complementarity tells us that such a measurement must destroy the interference. This shows how the local and nonlocal complementarity is manifested in the AB effect. The local effect on the spin is complementary to the interference effect of the charge. By interchanging the roles of the electron and the spin, this problem is mapped to the Aharonov-Casher (AC) [4] setup. The relative velocities are unchanged by this transformation, and therefore, the local spin precession as well as the accumulated relative phase are identical.

In both cases, of the AB and AC effects, we must in addition to the spin precession consider the spatial degrees of freedom of the interfering particle, i.e., the position, on a quantum mechanical level. Otherwise, no interference effect can be observed at all. The motion of the center of mass of the interfering particle is identical in both effects, and the accumulated phase can be derived from an operator depending on the spatial degrees of freedom alone. For instance, we can consider the expectation value of the modular velocity operator $\cos(mvL/\hbar)$, with $v = (p - A)/m$ the velocity perpendicular to the direction of motion and L the distance between the trajectories. It yields the cosine of the AB or AC phases. This modular operator dependents on the position degree of freedom. Hence, we argue that, in both cases, the interference effect cannot be an attribute of spin alone.

Next, consider the second claim of HS. A measurement of the precession alone can distinguish which path the interfering particle follows and, hence, destroys the interference. But one can consider a measurement involving *both* the position and the spin that nevertheless does not distinguish between the paths [5]. In the AB case, consider a "controlled" measurement: only if the electron follows the right path we measure $U_R^\dagger(T)\sigma_x(0)U_R(T) - \sigma_x(0)$ where $U_R(T)$ generates the rotation during the time interval T, when the electron is at the right path. For that we employ a von-Neumann coupling twice: at $t = 0$ we couple to the spin $\sigma_x(0)$, and later at time $t = T$ we couple to the rotated spin $U_R^\dagger(T)\sigma_x U_R(T)$. The outcome is recorded at time $t > T$. A straightforward computation yields for this outcome zero, whether or not the electron is on the right side. Furthermore, at $t > T$ the system returns to its unperturbed state. Hence, the verification measurement does not destroy the interference. In the AB case such a measurement requires a nonlocal coupling. In the AC effect the spin degree of freedom is carried by an interfering particle. Hence, the later "null" measurement may be performed locally along one of the interference arms.

If the location of the trajectory (in the AB effect) or the location of the charged source (AC effect) is fixed, we know in advance the conditional precession effect on the spin. Hence, a null verification experiment is possible. The possibility of a verification experiment still does not void the idea of complementarity, because in the general case, one would like to use either the interference effect or the local precession in order to measure an *unknown* topological phase. For example, suppose that the value of the source or that the precise location of the charged source are not known. For these cases, the value of the local electric field in the AC effect (or the induced magnetic field in the AB effect) are not fixed, and a nondisturbing null verification experiment is not possible.

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