

Nonlocal Aspects of a Quantum Wave

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Various aspects of nonlocality of a quantum wave are discussed. In particular, the question of the possibility of extracting information about the relative phase in a quantum wave is analyzed. It is argued that there is a profound difference in the nonlocal properties of the quantum wave between fermion and boson particles. The phase of the boson quantum state can be found from correlations between results of measurements in separate regions. These correlations are identical to the Einstein-Podolsky-Rosen (EPR) correlations between two entangled systems. An ensemble of results of measurements performed on fermion quantum waves does not exhibit the EPR correlations and the relative phase of fermion quantum waves cannot be found from these results. The existence of a physical variable (the relative phase) which cannot be measured locally is the nonlocality aspect of the quantum wave of a fermion.

I. INTRODUCTION

There are literally thousands of papers about nonlocality in quantum theory. However, there are still some aspects of nonlocality which have not been fully explored and the connection between various aspects have not been clarified. In this paper we will analyze particular nonlocal aspects which are different for quantum waves of bosons and fermions. This is a development of ideas originated in the works of one of us [1,2]. In order to put these nonlocality aspects in the proper perspective we will give a brief review of other aspects of nonlocality of quantum theory.

An important nonlocality aspect which will not be discussed in this paper is related to the concept of *nonlocal variables*. Measurements of nonlocal variables cannot be reduced to measurements of local variables [3]. Probably the simplest example of a nonlocal variable is the sum of spin components of two separated spin- $\frac{1}{2}$ particles, $\sigma_{Az} + \sigma_{Bz}$. According to the postulates of the quantum theory, if a system is in an eigenstate of a measured variable, ideal measurement of this variable should not alter this eigenstate. For example, the singlet state of the two spins, frequently named the Einstein-Podolsky-Rosen (EPR) state,

$$|\Psi\rangle_{EPR} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B), \quad (1)$$

is an eigenstate of the operator $\sigma_{Az} + \sigma_{Bz}$ with an eigenvalue 0. Thus, measurement of $\sigma_{Az} + \sigma_{Bz}$ must leave state (1) unchanged. Note that measurements of local variables, σ_{Az} and σ_{Bz} , invariably change the state.

Some of the eigenstates of the nonlocal variable $\sigma_{Az} + \sigma_{Bz}$ are entangled states. It is interesting that there are nonlocal variables which have only product-state eigenstates. They are nonlocal in the sense of impossibility of their measurement using only local measurements in the space-time regions A and B [4]. Moreover, recently [5] there have been found nonlocal variables with product-state eigenstates which cannot be measured even when measurements are performed at different times in space locations A and B and unlimited classical communication between the sites is allowed.

Very important nonlocal variables are *modular variables* [6]. Many surprising effects related to evolution of spatially separated systems can be effectively analyzed using them. The dynamical equations of modular variables are nonlocal.

However, the nonlocality issues related to nonlocal variables are mingled: it is not easy to separate which part of nonlocality in the dynamics is due to intrinsic nonlocality of the quantum world and which part is due to the nonlocality introduced by the definition of the variable. In this paper we limit ourselves to the analysis of relations between result of measurements of local variables.

The plan of the paper is as follows. In Section II we introduce the basic framework of our analysis. In Sections III-V we discuss three types of nonlocality. This discussion provides the frame of reference for the analysis of nonlocality. In Section VI we give a more detailed explanation of the framework. Following this preparatory introduction, we analyze the nonlocality of the boson quantum wave in Section VII and the nonlocality of the fermion quantum wave in Section VIII. Section IX

is devoted to an apparent causality paradox arising from nonlocality of the boson wave. In Section X we discuss a related issue of collective measurement which is relevant mostly for the fermion quantum wave. Finally, in Section XI we summarize the main results of the paper.

II. THE FRAMEWORK OF THE ANALYSIS

The formalism of non-relativistic quantum theory allows introducing arbitrary Hamiltonians, in particular, Hamiltonians corresponding to nonlocal interactions. However, such interactions have not been observed in experiments. In the framework of our analysis of nonlocality we will assume that the Hamiltonian describes only local interactions. This is a basic assumption of our analysis.

Any wave in space is, in some sense, a nonlocal object. A classical wave, however, can be considered as a collection of local properties. What makes the quantum wave genuinely nonlocal is that it cannot be reduced to a collection of local properties. In order to analyze this aspect of a quantum wave we will concentrate on a particular simple case: a quantum wave which is an equal-weights superposition of two localized wave packets in two separate locations:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|a\rangle + e^{i\phi}|b\rangle). \quad (2)$$

We will analyze various simultaneous (in a particular Lorentz frame) measurements performed in these two locations; see Fig. 1. We will denote by A and B the space-time regions of these measurements. The wave packet $|a\rangle$ is localized inside the spatial region of A and the wave packet $|b\rangle$ is localized inside the spatial region of B.

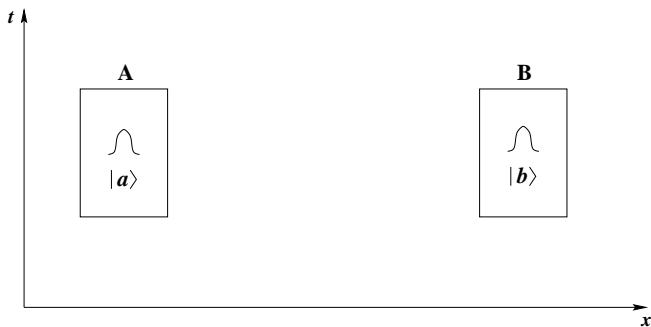


Fig. 1. Space-time diagram of the measurements performed on the quantum wave (2).

In this paper we will show that there is a profound difference in the nonlocal properties of the quantum wave of the form (2) between fermion and boson particles. The boson state leads to statistical correlations between results of measurements in A and in B that *cannot* be explained by local classical physics. The fermion state

does not lead to such correlations but it has a different nonlocality aspect. The fermion quantum state cannot be measured using only local measurements in A and B even if we are given an ensemble of results of measurements performed on identical particles in the state (2). In particular, the relative phase ϕ of the fermion state does not lead to locally measurable effects. This phase has a physical meaning: it influences the result of interference experiments in which the parts of the quantum state in A and in B are brought together. Existence of a physical quantity which does not manifest itself through local measurements is the nonlocality aspect of a fermion wave. In contrast, a boson state can be found from the ensemble of results of local measurements: it can be identified from the nonlocal correlations mentioned above.

III. NONLOCALITY OF THE COLLAPSE OF A QUANTUM STATES

In a situation in which a particle (boson or fermion) is described by the state (2), each region, A or B separately, cannot be described by a pure quantum state. By introducing the *vacuum* states $|0\rangle_A$ and $|0\rangle_B$ which describe the regions A and B without the particle, we can rewrite the state (2) in the following form

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle_A|0\rangle_B + e^{i\phi}|0\rangle_A|1\rangle_B), \quad (3)$$

where $|1\rangle_A \equiv |a\rangle$ and $|1\rangle_B \equiv |b\rangle$. This form allows us to write down the complete quantum description of region B (as well as region A) by means of the density matrix

$$\rho_0 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}. \quad (4)$$

In the framework of standard quantum theory, a measurement instantaneously collapses the quantum state of a system. Thus, an action in A can change the density matrix in B. After a measurement of the projection operator in A, i.e., after observing whether the particle is in A, the density matrix in B is changed instantaneously to the density matrix of one of the pure states:

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad (5)$$

in anti-correlation to the corresponding density matrices in A.

According to the collapse interpretation, the measurement in A changes the state of affairs in B. Before the action in A the outcome of a possible measurement of the projection operator in B was undetermined not only to the observer in B, but to all. Nothing in nature could give an indication about the outcome of the experiment. The outcome is genuinely random with probability $\frac{1}{2}$ both for

finding region B empty and for finding the particle there. After the measurement in A, the observer in B still does not know the outcome, but nature (in particular, the observer in A) has this information: the probabilities for the results of the measurement in B change to either 1 and 0 or to 0 and 1 according to the outcome in A.

There is no other example in physics in which a local action changes the state of affairs in a space-like separated region. Thus, this aspect of nonlocality provides an argument in favor of adopting one of the interpretations which does not have the collapse of a quantum state. We now briefly describe these interpretations.

According to the pragmatic approach [7], quantum theory is limited to providing a recipe for predicting probabilities in quantum experiments, i.e. frequencies of the outcomes in the experiments. In this approach the density matrix is a statistical concept. An observer in B, who does not know which outcome is obtained in A, considers the mixture of the two possibilities (5) as described by the statistical density matrix ρ_0 even after the measurement in A.

The causal interpretation of Bohm [9,10] has no collapse and therefore it lacks the nonlocality aspect of instantaneous change of a quantum state. The result of the measurement of projection operators on region B is predetermined by a ‘‘Bohmian position’’ and, therefore, the measurement in A changes nothing in B. For a single particle, Bohmian theory is a local hidden variable theory which completes quantum mechanics without contradicting statistical predictions of the latter. However, for systems consisting of more than one particle, the evolution of ‘‘Bohmian positions’’ of the particles is nonlocal. The Bohmian theory is nonlocal in a robust sense: action in A can change the outcome in B. For example, consider the EPR state of two spin- $\frac{1}{2}$ particles (1). Consider Bohmian positions which are such that if a particular σ_z measurement is performed on either particle, it must yield $\sigma_z = 1$. However, if these σ_z measurements are performed on both particles, the results will be different: the earlier measurement of σ_z in A will change the outcome of the consequent measurement in B to $\sigma_z = -1$. The details of this example are given in Ref. [8].

The non-collapse interpretation which one of us (L.V.) finds most appealing [11] is the many-worlds interpretation (MWI) [12]. In the physical universe, due to the measurement in A, the quantum state of the two particles and the measuring device in A changes in the following way:

$$\frac{1}{\sqrt{2}}(|1\rangle_A|0\rangle_B - |0\rangle_A|1\rangle_B)|\text{ready}\rangle_{MDA} \rightarrow \frac{1}{\sqrt{2}}(|1\rangle_A|0\rangle_B|\text{click}\rangle_{MDA} - |0\rangle_A|1\rangle_B|\text{no click}\rangle_{MDA}), \quad (6)$$

but the density matrix in B is still ρ_0 . Note, that relative to an observer in A, who belongs to a world with

a particular reading of the measuring device, the density matrix of the particle in B is that of one of the pure states (5). Only from the point of view of an external observer, who is not correlated to a particular outcome in A, the density matrix in A is unchanged.

If we now add an observer in B who measures the projection operator there, then in A there is a mixture of two worlds with and without the particle in A and, similarly, in B there is a mixture of two worlds with and without the particle in B. These mixtures were created locally by the decisions of the observers to make these particular measurements. What remains nonlocal in this picture are the ‘‘worlds’’: the observer in A who found the particle, in his travel to B, will meet there the observer that has not found the particle, and vice versa in the other world.

One of us (Y.A.) strongly prefers an interpretation which does not require a multitude of worlds. The two-state vector formalism of quantum theory [13,14] allows covariant description of the collapse. This picture suggests radical change in the concept of time which will avoid statements made above such as: ‘‘According to the collapse interpretation, the measurement in A changes the state of affairs in B.’’ These ideas will be presented elsewhere.

IV. NONLOCALITY OF CORRELATIONS

In the framework of standard quantum theory the (anti)correlations between finding particles in the two regions A and B described above are nonlocal in the sense that the theory does not yield a causal explanation for them. The complete quantum description does not specify the results of measurements and it does not yield a local causal explanation for this correlations. One might imagine that the quantum theory can be completed by a deeper theory which will provide a local causal explanation for the results of measurements. In fact, the Bohmian theory mentioned above provides a local explanation for the anti-correlations in finding the particle in the regions A and B, but for some other experiments performed in these space-like separated regions it is impossible to find a local hidden variable theory. In particular, statistics of the results of spin measurements performed on two separated spin- $\frac{1}{2}$ particles in a singlet state (the setup for which Bohmian theory is not local) cannot be explained by a local hidden variables theory. This is the content of the celebrated Bell-inequalities paper [15].

There are numerous proofs that quantum correlations cannot have local causes. We present here one more argument of this kind inspired by the work of Mermin [16]. However, the reader can choose any other proof of this statement in order to proceed with the line of argumentation of this paper.

The argument presented here assumes the principle of counterfactual definiteness [17], i.e., that in any physical situation the result of any experiment which can be performed has a definite value. We will analyze again the EPR state (1). Consider measurements of the spin components in $N+1$ directions for the particle in A and in N different directions for the particle in B. These directions are in the $\hat{x} - \hat{z}$ plane and they are characterized by the angle θ_i with respect to the \hat{z} axis,

$$\theta_i \equiv \frac{i\pi}{2N}, \quad i = 0, 1, \dots, 2N. \quad (7)$$

Note that the measurement in the direction $\theta_{2N}(= \pi)$ is physically equivalent to the measurement in the direction $\theta_0(= 0)$, but the result has to be multiplied by -1 , i.e., $\sigma(\pi) = -\sigma(0)$.

Spin measurement of one particle in a given direction (effectively) collapses the spin state of the other particle to the opposite direction and, therefore, quantum theory predicts the same probability for all the following relations between the results of measurements, if performed:

$$\sigma_A(\theta_{2n}) = -\sigma_B(\theta_{2n+1}), \quad (8)$$

$$\sigma_A(\theta_{2n+2}) = -\sigma_B(\theta_{2n+1}), \quad (9)$$

where $n = 0, 1, \dots, N-1$. The probability is

$$p = \cos^2\left(\frac{\theta_{i+1} - \theta_i}{2}\right) = \cos^2\left(\frac{\pi}{4N}\right). \quad (10)$$

From the principle of counterfactual definiteness and the locality assumption, according to which local measurements yield the same outcomes independently of what has been measured in the other location, it follows that identical expressions in the equations (8) and (9) must correspond to equal values. Thus, we can use all these $2N$ equations together. The correctness of all the equation leads to a contradiction. Indeed, we obtain $\sigma_A(\theta_0) = \sigma_A(\theta_{2N})$ contrary to the fact that these expressions represent the same measurement in opposite directions: $\sigma_A(0) = -\sigma_A(\pi)$. Therefore, at least one out of $2N$ equations (8) and (9) *must* fail to be satisfied. On the other hand, irrespective of what correlations (compatible with quantum mechanics) follow from a hidden variable theory, the probability that at least one of these relations fails to be satisfied cannot be more than the probability that one fails multiplied by the number of equations:

$$\text{prob(fail)} \leq 2N(1-p) = 2N\left(1 - \cos^2\left(\frac{\pi}{4N}\right)\right). \quad (11)$$

This expression, however, is smaller than 1 even for $N=2$ and for large N it goes to zero as $\frac{\pi^2}{8N}$.

Recently, Greenberger, Horne and Zeilinger (GHZ) [18] have found an even more robust example (improved by Mermin [19]) of such nonlocality. While in our example

we have several relations which have to be true according to quantum theory with high probability, in spite of the fact that they all cannot be true, in the GHZ example we have four relations which must be true with probability 1, but, nevertheless, they cannot all be true together. However, in the GHZ(Mermin) example we have to consider three, instead of two, space-like separated regions.

Note that the Bell and the GHZ arguments do not hold without the principle of counterfactual definiteness, i.e., it is not applicable in the framework of the many-worlds interpretation in which, in general, quantum measurements do not possess single outcomes.

V. THE AHARONOV-BOHM TYPE NONLOCALITY

Another nonlocality aspect of quantum theory is related to the Aharonov-Bohm (AB) effect. The effect has a topological basis. The wave-function of a particle enters two space regions tracing out trajectories in space-time which start and end together. An interference pattern which depends upon a field is observed in spite of the fact that locally, inside these regions, it is impossible to make measurements which can specify the result of the interference experiment. The main aspect of the effect is that it exists even when there is no field inside the regions during the whole time of the experiment.

In this paper we consider measurements in two space-time regions. This is different from the AB effect for which a closed trajectory in space is required. What is relevant to our discussion is the feature of a particle inside the two space-time regions A and B which will eventually be manifested in the results of the interference experiment. The AB nonlocality is the existence of a physical property (a property which has observable consequences) which does not have any manifestation in local measurements.

A simple example is a particle wave-packet which splits into a superposition of two wave packets (2) and later brought back again to the same region for an interference experiment. This can be achieved in a one dimensional model of a wave packet arriving at a barrier at time t_0 ; see Fig. 2a. The barrier is such that the particle has the probability $\frac{1}{2}$ to pass through and the probability $\frac{1}{2}$ to be reflected. Two reflecting walls at equal distance from the barrier return the two wave packets back to the barrier at the same time and the result of the interference experiment is observed by finding the particle on the left or on the right side of the barrier at a later time. The time-dependent (scalar) AB effect is obtained by changing the relative potential between the two parts of the wave during the time they are separated. For a charged particle this can be achieved by moving two large oppositely charged parallel plates located between the wave

packets; see Fig. 2b). The two plates are placed originally one on top of the other, i.e., there is no charge distribution and, therefore, there is no electric field anywhere. The plates are then moved a short distance apart and then they are brought back. We will call such an operation “opening a condenser”.

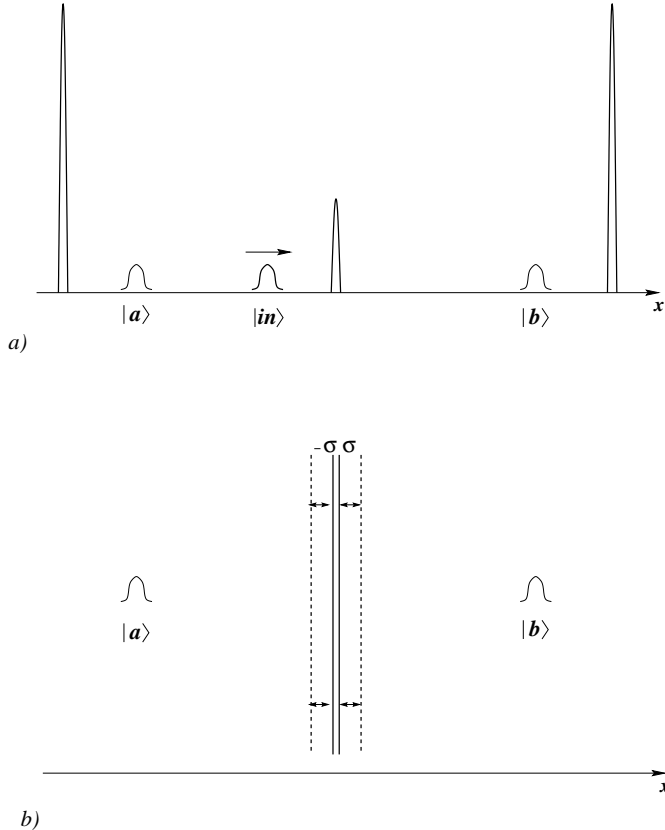


Fig. 2. Scalar Aharonov-Bohm effect interference experiment. *a*). One-dimensional interference experiment. The particle in the wave packet $|in\rangle$ splits at the barrier into a superposition of the two wave packets, $|a\rangle$ and $|b\rangle$, which are reflected from the walls and reunited to interfere at the barrier. *b*). Parallel-plate condenser with charged plates, originally one on top of the other, is opened (by moving the plates apart) for a short time while the wave packets $|a\rangle$ and $|b\rangle$ are far apart. This operation introduces change in the electric potentials between the locations of $|a\rangle$ and $|b\rangle$ which generates the AB phase.

A naive answer to the question, “What is the nonlocal feature of the two regions A and B?” (the feature of the two parts of the wave after they are separated) would be the quantum phase ϕ appearing in the equation (2). Indeed, we will argue, discussing fermions in Section VIII, that in certain circumstances the quantum phase is a non-local feature in the sense that it cannot be found through local experiments in A and in B. However, the statement is not correct for bosons. Moreover, the phase is not a

gauge invariant concept. The physical effect of interference is of course gauge invariant since it is a topological property of the whole trajectory. Still, there is a property of the system in A and B which specifies the final outcome of the interference experiment given fixed circumstances. The quantum phase does characterize this property provided we are careful enough to fix the gauge in the problem.

This and preceding sections described nonlocality aspects which are very different: here we discuss an observable property of a system in two locations which does not have any local manifestation, while in the previous section we discussed results of local measurements which do not allow local-cause explanation. It is possible to perform analysis of these nonlocalities using different terms, such as *local action*, *separability*, etc. Then the differences between the nonlocalities discussed in the two sections might not be as sharp as stated above [20]. However, such analysis strongly depends on the interpretation of quantum theory and is less helpful for the purpose of the present paper.

VI. THE DETAILED FRAMEWORK OF THE ANALYSIS

Our goal in this paper is to perform an analysis of nonlocal aspects of the quantum state (2). The main question is: “What are the physical consequences of the presence of this quantum wave in the space-time regions A and B?” One of the questions is: “Can we find the quantum phase ϕ through local measurements in A and B?” In order to be able to make such analysis we have to specify exactly the meaning of space-time regions A and B. Are the positions of A and B fixed relative to each other or are they fixed relative to an external reference frame? Are there fixed directions in A and B such that measuring devices can be aligned according to them? Is the time in A and B defined relative to local clocks, or relative to an external clock? What are the measuring devices which are available in A and B? All these questions are relevant. We have to specify what is given in A and B prior to bringing the quantum wave there in order to distinguish effects related to the quantum wave from the effects arising from our preparation and/or definition of the sites A and B.

We make the following assumptions:

- (i) There is an external inertial frame which is massive enough so that it can be considered classical.
- (ii) There is no prior entanglement of physical systems between the sites A and B. The two laboratories in A and B are also massive enough so that the measurements performed on the quantum wave can be considered measurements performed with classical apparatuses. However, for various aspects of our analysis we will have to consider the two laboratories as quantum systems. We as-

sume that relative to the external reference frame the two laboratories are initially described by a product quantum state $|\Psi_A\rangle|\Psi_B\rangle$.

(iii) There is no entanglement between location of the apparatuses in A and the wave packet $|a\rangle$ (as well as between location of the apparatuses in B and the wave packet $|b\rangle$). Instead, the fact that apparatus A measures $|a\rangle$ and apparatus B measures $|b\rangle$ is achieved via localization relative to the external frame. The measuring devices and the wave packets are well localized at the same place. This can be expressed in the equations

$$\langle a|\hat{x}|a\rangle = \langle \hat{x}_{MDA}\rangle, \quad (12)$$

$$\langle b|\hat{x}|b\rangle = \langle \hat{x}_{MDB}\rangle, \quad (13)$$

where x_{MDA} (x_{MDB}) is the variable which describes the location of the interaction region of the measuring devices in A (in B). It is assumed that the wave packet $|a\rangle$ remains in the space region A (and $|b\rangle$ remains in B) during the time of measurements.

(iv) Measurements in A and in B are performed by local measuring devices activated by *local* clocks, say, at the internal time $\tau_A = \tau_B = 0$. The clocks are well synchronized with the time t of the external (classical) clock:

$$\langle \tau_A(t)\rangle = \langle \tau_B(t)\rangle = t, \quad (14)$$

and the spreads of the clock pointer variables $\Delta\tau_A$, $\Delta\tau_B$ are small during the experiment. Again, as stated in (ii), there is no entanglement between clocks in A and in B.

The assumptions can be summarized as follows: a measurement in A, the space-time point relative to an external classical frame, means a measurement performed by local apparatuses in A triggered by the local clock. The apparatuses and clocks in A are not entangled with the apparatuses and the clocks in B.

Given all apparatuses in A and B, but without the quantum particle (2), it is impossible to observe the nonlocality of the collapse described in Section III. Since the quantum state of all systems (measuring apparatuses, clocks, etc.) is the product state of a quantum state in spatial location A times a quantum state in spatial location B, there are no correlations between the results of measurements in A and in B. This requirement need not be so strong: the crucial feature is the absence of quantum correlations (following from entanglement between the systems in A and in B). Here, for simplicity of the analysis we forbid any initial correlation between measuring devices in the two sites.

There is a somewhat more complicated situation in relation to the nonlocality discussed in Section V. Clearly there is no quantum phase which characterizes the devices in A and B: these systems are in the product state. But the operational definition of the AB nonlocality of Section V was a feature which cannot be found through local experiments in A and B, the feature which leads to observable effects when the systems from locations A

and B are brought together. If we restrict ourselves to measurements using local measuring devices, then there are many features which cannot be found locally, for example, the relative orientation of the measuring devices in A and in B. The observer in A (or in B) making measurements using local devices cannot find out his (or her) orientation. However, if we have other observers in the product state in regions A and B with well defined *known* orientation, they can measure locally the orientation of the system in A and orientation of the system in B. The question of what can and what cannot be measured from within the system itself is interesting [21,22], but we will not discuss it here. Here we allow all possible measuring devices provided that they do not possess entanglement between A and B.

In our discussion we assume that measurements are performed on a single system. But, for the question of finding the phase, the question of obtaining non-classical correlations, etc., we assume that we have an ensemble of experiments on identical single systems. Collective measurements on the ensemble of particles are not allowed: clearly, the results of such experiments can manifest properties of the composite system of many particles which are not intrinsic properties of each particle. (We will briefly discuss collective measurements in Section X.)

After stating here precisely the “rules of the game” we now proceed to discuss the nonlocality of the quantum wave (2) for various particles.

VII. SINGLE-PHOTON NONLOCALITY

Let us start with considering a photon in a state (2). There have been several proposals [23–26] how to obtain quantum correlations based on such and similar systems [27]. The photon in a state (2) exhibits nonlocality of the EPR correlations described in Section IV. The state of the photon, if we write it in the form (3), is isomorphic to the EPR state (1).

In order to get the EPR-type correlations we must be able to perform measurements on the photon analogous to the spin measurements in arbitrary direction. The analog of the spin measurement in the \hat{z} direction is trivial: it is observing the presence of the photon in a particular location. A gedanken experiment yielding the analog of the spin measurements on the EPR pair in arbitrary directions is as follows [28]. Let us consider, in addition to the photon, a pair of spin- $\frac{1}{2}$ particles, one located in A and one in B; see Fig. 3. Both particles are originally in a spin “down” state in the \hat{z} direction. In the locations A and B there are magnetic fields in the \hat{z} direction such that the energy difference between the “up” and “down” states equals exactly the energy of the photon. Then we construct a physical mechanism of absorption and emission of the photon by the spin which is described by the unitary transformation in each site:

$$\begin{aligned}
|1\rangle|\downarrow\rangle &\leftrightarrow |0\rangle|\uparrow\rangle, \\
|1\rangle|\uparrow\rangle &\leftrightarrow |1\rangle|\uparrow\rangle, \\
|0\rangle|\downarrow\rangle &\leftrightarrow |0\rangle|\downarrow\rangle.
\end{aligned}
\tag{15}$$

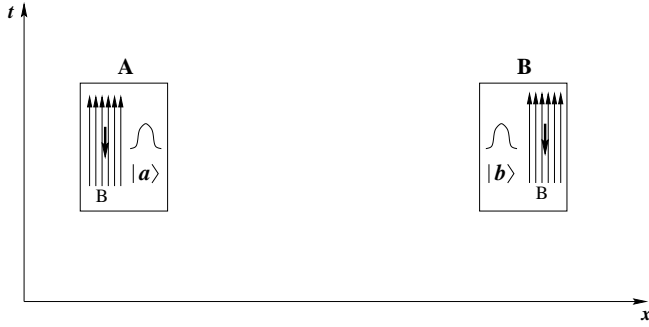


Fig. 3. Swapping of the single-photon state with the entangled state of two spin- $\frac{1}{2}$ particles.

This transformation swaps the quantum state of the photon and the quantum state of the pair of spin- $\frac{1}{2}$ particles as follows:

$$\begin{aligned}
\frac{1}{\sqrt{2}} (|1\rangle_A|0\rangle_B + e^{i\phi} |0\rangle_A|1\rangle_B) |\downarrow\rangle_A|\downarrow\rangle_B \rightarrow \\
\frac{1}{\sqrt{2}} |0\rangle_A|0\rangle_B (|\uparrow\rangle_A|\downarrow\rangle_B + e^{i\phi} |\downarrow\rangle_A|\uparrow\rangle_B).
\end{aligned}
\tag{16}$$

Thus, we can obtain nonlocal correlations of the EPR state starting with a single photon, swapping its state to the state of the pair of spin- $\frac{1}{2}$ particles, and then making appropriate spin component measurements. Statistical analysis of the correlations between the results of spin measurements in A and in B allows us to find the phase ϕ . For example, the probabilities for coincidence and anti-coincidence in the x spin measurements are given by

$$\text{prob}(|\uparrow_x\rangle|\uparrow_x\rangle) = \text{prob}(|\downarrow_x\rangle|\downarrow_x\rangle) = \frac{1}{4}|1 + e^{i\phi}|^2, \tag{17}$$

$$\text{prob}(|\uparrow_x\rangle|\downarrow_x\rangle) = \text{prob}(|\downarrow_x\rangle|\uparrow_x\rangle) = \frac{1}{4}|1 - e^{i\phi}|^2. \tag{18}$$

We have shown that, in principal, the nonlocality of a single photon is equivalent to the nonlocality of the EPR pair. Now we will turn to the discussion of the possibilities of manifestation of this nonlocality in real experiments and will try to explore the nature of this equivalence.

We are not aware of experiments in which a spin in a magnetic field absorbs a photon with high efficiency. However, there is an equivalent operation which is performed in laboratories. Recently there has been a very significant progress in microwave cavity technology and there are experiments in which Rydberg atoms which operate as two-level systems absorb and emit photons into

a microwave cavity with a very high efficiency [30]. The excited state $|e\rangle$ and the ground state $|g\rangle$ of the atom are isomorphic to $|\uparrow\rangle$ and $|\downarrow\rangle$ states of a spin- $\frac{1}{2}$ particle. For the atom, measuring the analog of the z spin component is trivial: it is the test whether the atom is in the excited state or the ground state. For measurements analogous to the spin measurements in other directions there is an experimental solution too. Using appropriate laser pulses the atom state can be “rotated” in the two dimensional Hilbert space of ground and excited states in any desired way. Thus, any two orthogonal states can be rotated to the $|e\rangle$ and $|g\rangle$ states and, then, a measurement which distinguishes between the ground and excited states distinguishes, in fact, between the original orthogonal states.

The Hamiltonian which leads to the required interactions can be written in the following form:

$$H = a^\dagger|g\rangle\langle e| + a|e\rangle\langle g|, \tag{19}$$

where a^\dagger , a are creation and annihilation operators of the photon. This Hamiltonian is responsible for the two needed operations. First, such coupling between the photon in the cavity in A and the atom in A together with similar coupling in B swaps the state (3) to the state of two Rydberg atoms:

$$\begin{aligned}
\frac{1}{\sqrt{2}} (|1\rangle_A|0\rangle_B + e^{i\phi} |0\rangle_A|1\rangle_B) |g\rangle_A|g\rangle_B \rightarrow \\
\frac{1}{\sqrt{2}} |0\rangle_A|0\rangle_B (|e\rangle_A|g\rangle_B + e^{i\phi} |g\rangle_A|e\rangle_B).
\end{aligned}
\tag{20}$$

The same Hamiltonian can also lead to an arbitrary rotation of the atomic state. To this end the atom has to be coupled to a cavity with a *coherent state* of photons,

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \tag{21}$$

The phase of α specifies the axis of rotation and the absolute value of α specifies the rate of rotation. For example, the time evolution of an atom starting at $t = 0$ in the ground state is:

$$|\Psi(t)\rangle = \cos(|\alpha|t) |g\rangle + \frac{\alpha}{i|\alpha|} \sin(|\alpha|t) |e\rangle. \tag{22}$$

This is correct when we make the approximation $a^\dagger|\alpha\rangle \simeq \alpha^*|\alpha\rangle$ which is precise in the limit of large $|\alpha|$. The Hamiltonian (19) is actually implemented in laser-aided manipulations of Rydberg atoms passing through microwave cavities.

Conceptually, the above scheme can be applied to any type of bosons (instead of photons), even charged bosons. An example of a (gedanken) Hamiltonian for this case describes a proton $|p\rangle$ which creates a neutron $|n\rangle$ by absorbing a negatively charged meson:

$$H = a_m^\dagger |p\rangle \langle n| + a_m |n\rangle \langle p|, \quad (23)$$

where a_m^\dagger , a_m are creation and annihilation operators of the meson. This Hamiltonian swaps the state of the meson (now written in the form (3)) and the state of the nucleon pair:

$$\frac{1}{\sqrt{2}} (|1\rangle_A |0\rangle_B + e^{i\phi} |0\rangle_A |1\rangle_B) |p\rangle_A |p\rangle_B \rightarrow \frac{1}{\sqrt{2}} |0\rangle_A |0\rangle_B (|n\rangle_A |p\rangle_B + e^{i\phi} |p\rangle_A |n\rangle_B). \quad (24)$$

Since there is no direct measurement of a superposition of proton and neutron, we need again a procedure which rotates the superposition states of a nucleon to neutron or proton state. This rotation requires coherent states of mesons which would be, in this case, a coherent superposition of states with different charge. Due to strong electro-magnetic interaction the coherent state will decohere very fast. This is essentially an environmentally induced “charge super-selection rule” which prevents stable coherent superpositions of states with different charge. It is important that there is no *exact* charge super-selection rule which would prevent, in principle, performing the experimental scheme presented above. Indeed, Aharonov and Susskind (AS) [31] proposed a method for measuring the relative phase between states with different charge, thus showing that there is no exact charge super-selection rule. In their method one can measure the phase even if the whole system (the observed particle and the measuring device) is in an eigenstate of charge. This corresponds to initial entanglement between measuring devices in A and B and thus will not be suitable for the present procedure. Here we assume existence of superpositions of different charge states: only then it is possible that the quantum state of measuring devices in A and B is a product state.

There are some arguments that the total charge of the universe is zero and therefore, we cannot have a product of coherent states of charged particles in A and in B. More sophisticated analysis has to be performed: since the observable variables are only relative variables, the final conclusion will be as in the AS paper [31]: conceptually, there is no constraint on a measurement of the relative phase of a charged boson, but decoherence will prevent construction of any realistic experiment. See also very different arguments against exact super-selection rule by Giulini [33].

VIII. NONLOCALITY OF A FERMION QUANTUM WAVE

As we have shown above, the nonlocality properties of the boson quantum state (2) are equivalent to the nonlocality of the EPR pair. In contrast, the nonlocality

properties of the fermion quantum state (2) are very different from those of the EPR pair. We cannot generate quantum correlations between results of local measurements performed in A and in B, the correlations which violate Bell inequalities.

The reason why the method which was applicable to bosons fails for fermions is that there is no coherent state of fermions. The number state $|n\rangle$ exists only for $n = 0$ and $n = 1$ [32].

The intuitive understanding of the role of the coherent state is as follows. If, in addition to the measuring devices, there is an auxiliary identical particle in a *known* superposition of localized wave packets in A and B, then the phase ϕ can be found using local measurements. We consider the superposition of $|a'\rangle$ and $|b'\rangle$ positioned near $|a\rangle$ and $|b\rangle$ respectively; see Fig. 4. We choose the phase of the auxiliary particle to be equal zero,

$$|\Psi'\rangle = \frac{1}{\sqrt{2}} (|a'\rangle + |b'\rangle), \quad (25)$$

i.e., we have a composite system of two identical particles in the state

$$|\Psi\rangle |\Psi'\rangle = \frac{1}{2} (|a\rangle + e^{i\phi} |b\rangle) (|a'\rangle + |b'\rangle). \quad (26)$$

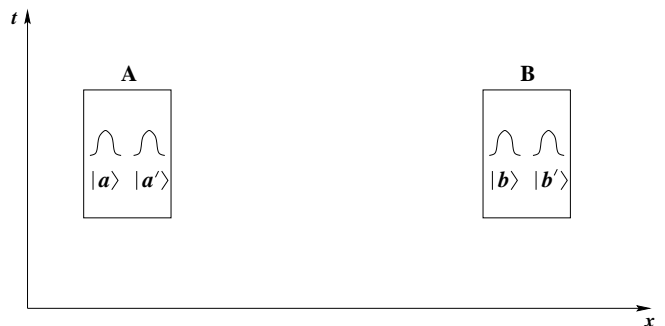


Fig. 4. Space-time diagram of local measurements which allow finding the phase ϕ of a quantum wave when an auxiliary identical particle with known phase is given.

The phase ϕ controls the rate of coincidence counting in the measurements of a local variable in A with eigenstates

$$|a_+\rangle \equiv \frac{1}{\sqrt{2}} (|a\rangle + |a'\rangle), \quad |a_-\rangle \equiv \frac{1}{\sqrt{2}} (|a\rangle - |a'\rangle), \quad (27)$$

and a local variable in B with eigenstates

$$|b_+\rangle \equiv \frac{1}{\sqrt{2}} (|b\rangle + |b'\rangle), \quad |b_-\rangle \equiv \frac{1}{\sqrt{2}} (|b\rangle - |b'\rangle). \quad (28)$$

In the case that one particle was found on each side, the probabilities are (compare with (17)):

$$\text{prob}(|a_+\rangle |b_+\rangle) = \text{prob}(|a_-\rangle |b_-\rangle) = \frac{1}{4} |1 + e^{i\phi}|^2, \quad (29)$$

$$\text{prob}(|a_+\rangle |b_-\rangle) = \text{prob}(|a_-\rangle |b_+\rangle) = \frac{1}{4} |1 - e^{i\phi}|^2. \quad (30)$$

The method described in the previous paragraph is applicable both for bosons and fermions. However, the existence of a particle described by (25) as a part of our measuring devices contradicts our assumption that sites A and B do not possess an entangled physical system prior to bringing in the test particle. For bosons we can consider a coherent state of particles described by state (25); it is equal to the product of local coherent states of bosons in A and in B:

$$e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \frac{1}{\sqrt{2}^n} (|a'\rangle + |b'\rangle)^n = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |a'\rangle e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |b'\rangle. \quad (31)$$

Thus, this state has no entanglement between the sites but it provides the reference for measuring the phase ϕ of the state (2) via methods described in the previous section.

Again, if we assume that there is no prior entanglement between the sites A and B, the phase ϕ of the fermion quantum state (2) cannot be measured locally. Quantum correlations which break Bell's inequality cannot be obtained. The only type of nonlocality for a fermion wave (except the collapse nonlocality) is the AB nonlocality. The quantum phase manifests itself only in the interference experiments in which the wave packets $|a\rangle$ and $|b\rangle$ are brought together.

The impossibility of local measurement of the phase ϕ is due to anti-commutation of fermion operators: the operator $a_A^\dagger + a_A$ does not commute with the operator $a_B^\dagger + a_B$. The eigenstates of the operator $a^\dagger + a$ are $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$; we have used measurements of such operator for finding out the phase ϕ of the boson wave in Section VII. A measurement in site A of $a_A^\dagger + a_A$ leads to an observable change in the results of measurement of $a_B^\dagger + a_B$, where $a_A^\dagger, a_A, a_B^\dagger, a_B$ are creation and annihilation operators of the fermion in A and in B, respectively. This means that the possibility of such measurements would lead to superluminal communication.

Another question which can be asked is: "Can we measure locally the phase ϕ of a superposition of a pair of fermions?" The quantum state is:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|2\rangle_A |0\rangle_B + e^{i\phi} |0\rangle_A |2\rangle_B), \quad (32)$$

where, for example, $|2\rangle_A$ might represent two electrons in identical spatial state inside A being in a singlet spin state. Since $a_{\uparrow A}^\dagger a_{\downarrow A}^\dagger + a_{\uparrow A} a_{\downarrow A}$ commutes with $a_{\uparrow B}^\dagger a_{\downarrow B}^\dagger + a_{\uparrow B} a_{\downarrow B}$, the argument presented in the preceding paragraph for unmeasurability of the phase of a superposition of single-fermion wave packets does not hold in this case. In fact, a pair of fermions is, in a sense, a boson. We can construct a procedure for measuring phase ϕ of the state (32) similar to the procedure which

was previously described (for a photon (19)-(22) and for a charged meson (23), (24)) in Section VII. A difficulty is that the coherent state of pairs of fermions which is required for our procedure can only be constructed approximately.

IX. IS IT POSSIBLE TO CHANGE THE PHASE IN A NONLOCAL WAY?

The main message of Section VII is that the phase ϕ for boson state (2) is locally measurable. Given an ensemble of bosons with identical phase ϕ we can generate a set of numbers (results of measurements) in A and another set of numbers in B, such that the two sets together yield ϕ . This sounds paradoxical, in particular, because ϕ is not a gauge invariant parameter.

Moreover, it seems that this phase can be changed non-locally. Indeed, it has been described in Section VIII how opening a condenser for a period of time in the space between the locations of a charged particle, A and B, changes the phase: this is the scalar AB effect. Thus, it seems that by an action in a localized region we can send information to a space-like separated region. Opening or not opening a condenser apparently changes correlations in the results of measurements in A and B; see Fig. 5.

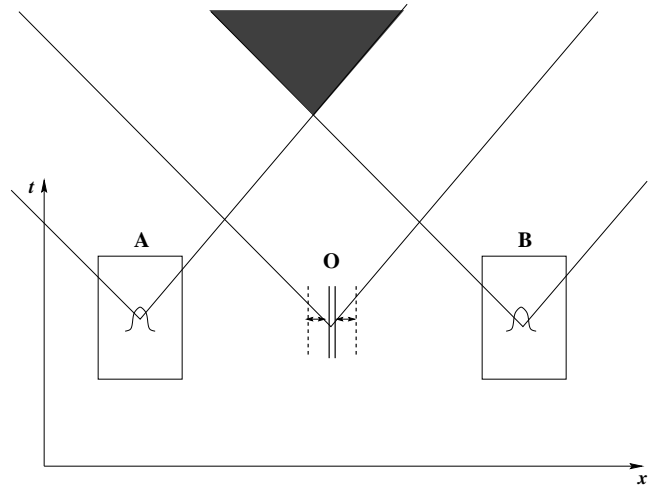


Fig. 5. Apparent sending signals to a space-like separated region. Operation in O, opening the condenser for a period of time, apparently changes the correlations between measurements in A and B. No signal is sent from O, neither to A nor to B, but the signal *is* sent to the union of A and B. The intersection of light cones originated at A and at B lies inside the light cone originated at O. Therefore, the action of the condenser falls into the category of “jammers” considered in Ref. [34].

It has been shown [34] that such an action, if possible, cannot lead to a paradoxical causal loop similar to

the one generated by a possibility of sending signals from one localized space-time region to another space-like separated *local* region. In our case the region to which we send the information consists of two space-like separated regions. There is no local observer who receives superluminal signals.

In spite of the fact that we cannot reach a causality paradox if such operation is possible, it clearly contradicts the spirit, if not the letter, of special relativity. And, in fact, it is impossible. It is incorrect that the opening of a condenser will change correlations between results of measurements in A and B. It must be incorrect because we should be able to use a covariant gauge in which changes in the potentials take place only inside the light cone. However, we can explain this phenomena also in a standard (Coulomb) gauge. In our scheme the measurements in local sites include interactions with coherent states of auxiliary particles, particles which are identical to the particle in a superposition. Therefore, if the particle in question is charged, the auxiliary particles are also charged and opening the condenser changes the phase of the coherent state in such a way that the correlations are not changed. The gauge which we choose changes the description of auxiliary particles too, so that the probabilities for results of measurements remain gauge invariant.

Consider now a neutral boson state. A massive plate in between the regions A and B which we move or not move toward one of the sites will introduce the phase shift in complete analogy with the scalar AB effect. (The difference here is that the gravitational fields in the regions A and B are not zero, but the fields are not affected by the motion of the plate.) In a scenario where the boson is absorbed by spins in a magnetic field and the correlations are obtained from the spin measurements, it is not obvious how the measuring devices will be influenced by the movement of the massive plate. The resolution of the paradox in this case is similar to the resolution of Einstein's paradox of an exact energy of an exact clock [35]. The explanation is that the pointers of the local clocks are shifted. Simultaneity between A and B is altered due to the action of the massive plate. Since in our case local clocks activate the measurements, the shift in the pointer will lead to a change. This change compensates exactly the phase change of the boson.

X. COLLECTIVE MEASUREMENTS

In this paper we have considered the results of measurements on an ensemble of identical particles in an unknown state. We allow measurements to be performed only on single members of the ensemble, so that we will have an ensemble of results of measurements performed on single particles. We believe that this is the proper approach for the analysis of the nature of a quantum wave of a particle; however, it might be interesting to consider

a related question: "Are there any changes to the questions posed in this paper if collective measurements are allowed?" Note that there is a recent result showing that collective measurements do make a difference for similar questions regarding the nonlocality of an ensemble of pairs of spin- $\frac{1}{2}$ particles in a particular mixed state [36].

For bosons we do not expect any difference because, even for single-particle measurements, we got the answers to our questions: (i) statistical analysis of the results of measurements allows us to find the phase ϕ and (ii) there are measurements in A and in B such that the results are characterized by correlations which can not have local causes. For single-particle measurements on fermions both (i) and (ii) are not true and thus raises the question of the status of (i) and (ii) when collective measurements are allowed.

Let us start this analysis by assuming that our particle is an electron and, contrary to the assumption of no prior entanglement, we now have an auxiliary particle, a positron, in a known superposition in A and B, say, of the form (25). In this case both (i) and (ii) are true: the fermion state *is* measurable via local measurements, and some measurements in A and B exhibit correlations which have no local causes.

Indeed, we can apply an interaction such that the positron and the electron located in the same site annihilate and create a photon. Such interaction will lead to the following transformation

$$\frac{1}{\sqrt{2}}(|e^- \rangle_A + e^{i\phi}|e^- \rangle_B) \frac{1}{\sqrt{2}}(|e^+ \rangle_A + |e^+ \rangle_B) \rightarrow \frac{1}{2}(|e^- \rangle_A |e^+ \rangle_B + e^{i\phi}|e^- \rangle_B |e^+ \rangle_A + |\gamma \rangle_A + e^{i\phi}|\gamma \rangle_B). \quad (33)$$

After testing and *not* finding the electron and the positron in the sites A and B the remaining state will be [37]:

$$\frac{1}{\sqrt{2}}(|\gamma \rangle_A + e^{i\phi}|\gamma \rangle_B), \quad (34)$$

which is a different notation for a single-photon state of the form (2). For a single photon we know that (i) and (ii) are true: the phase of a single-photon state (which is the original phase ϕ of the fermion) can be found, and quantum correlations breaking Bell inequalities can be obtained.

However, we do not have a positron in a state (25). Instead, we have an ensemble of electrons in a state (2). So, the first step is to *swap* the state of the electron with the state of a positron [38]. If we have an entangled state of a composite system which has two parts, one in A and another in B, such as the EPR state of two spin- $\frac{1}{2}$ particles located in A and B, and we want to transfer this entangled state to another pair of particles in A and B, then all we have to do is to perform local operation in each site which swaps the local quantum states of one

particle from one pair with one particle from the other pair located in the same site [3]. Linearity of quantum mechanics will ensure that swapping of local states, i.e. the states of parts of the systems, will lead to swapping of the quantum state of the whole systems.

In this paper we are interested in the swapping of a nonlocal state of a single particle to another single particle. The method described above cannot be applied directly because it is assumed that we have no another particle in a superposition of being in A and B (this is entanglement). Therefore, the other particle is not present in at least one of the sites and consequently, the “local swapping interaction” with this particle is meaningless. However, if the particles are bosons, then the swapping operation is possible. It can be done by transferring the quantum state to the entangled state of a composite system: a single-photon state can be transferred to two spin- $\frac{1}{2}$ particles in a magnetic field in the gedanken scenario described in Section VII or to two atoms in a real experiment using microwave cavities. After that, the quantum state can be swapped back to “another” photon.

Let us come back to the question of transferring the quantum state of the electron to a positron. Again, since we assumed no prior entanglement, the positron cannot be in a superposition of being in A and in B. Therefore, we will consider a situation in which there are two positrons one in A and another in B. We apply an interaction such that the positron and the electron which are in the same site annihilate and create a photon. This is described by the equation

$$\frac{1}{\sqrt{2}}(|e^- \rangle_A + e^{i\phi}|e^- \rangle_B) |e^+ \rangle_A |e^+ \rangle_B \rightarrow \frac{1}{\sqrt{2}}(|\gamma \rangle_A |e^+ \rangle_B + e^{i\phi}|\gamma \rangle_B |e^+ \rangle_A). \quad (35)$$

Now, the procedure described in Section VII allows measurements of local superpositions of the vacuum and single-photon states. In particular, there is a nonzero probability to find the state $\frac{1}{\sqrt{2}}(|0 \rangle_A + |\gamma \rangle_A)$ in A and a similar state $\frac{1}{\sqrt{2}}(|0 \rangle_B + |\gamma \rangle_B)$ in B. When this occurs, the final situation is that the electron and one of the positrons are annihilated and a positron appears in a superposition of being in two places

$$\frac{1}{\sqrt{2}}(|e^+ \rangle_B + e^{i\phi}|e^+ \rangle_A). \quad (36)$$

Thus, we can obtain a positron in superposition from an electron in a superposition. If we are allowed to perform collective measurements we now can annihilate this positron with another electron in the ensemble:

$$\frac{1}{\sqrt{2}}(|e^- \rangle_A + e^{i\phi}|e^- \rangle_B) \frac{1}{\sqrt{2}}(|e^+ \rangle_A + |e^+ \rangle_B) \rightarrow \frac{1}{2}(|e^- \rangle_A |e^+ \rangle_B + e^{i2\phi}|e^- \rangle_B |e^+ \rangle_A + e^{i\phi}|\gamma \rangle_A + e^{i\phi}|\gamma \rangle_B) \quad (37)$$

We do not obtain relative phase between photon wave-packets in two places which would allow us to find the phase ϕ , but we do obtain a superposition of a photon in A and B with known (zero) phase. This superposition can generate quantum correlations without local causes as described above.

If we are allowed to perform collective measurements, we can consider measurements on the pairs of fermions from our ensemble. The phase of pairs of fermions is 2ϕ and, in general, it can be found by the method described in Section VII. However, as we mentioned above, all statements about measurability using collective measurements do not describe the nature of a quantum wave of a single particle.

XI. CONCLUSIONS

In this paper we have analyzed nonlocal aspects of a simple quantum wave which is an equal-weights superposition (2) of wave packets in A and in B. For this analysis we assumed that we are given non-entangled laboratories in A and B which are described quantum mechanically by a product state of systems in A and systems in B.

We have shown that presence of an ensemble of bosons in a superposition $\frac{1}{\sqrt{2}}(|a \rangle + e^{i\phi}|b \rangle)$ leads to correlations in the results of single-particle local measurements in A and in B which break Bell’s inequality. These results, collected from a large ensemble allows us to find the phase ϕ . Thus, the boson quantum wave exhibit the EPR-type nonlocality. For a photon state this is not just a theoretical statement: the EPR nonlocality can be observed in an ensemble of measurements carried out on single photons. In principle, the statement applies to any boson state. However, environmentally induced super-selection rule prevents such experiment with charged bosons. Also, experiments with neutral massive bosons do not seem to be feasible.

The presence of an ensemble of fermions in a superposition $\frac{1}{\sqrt{2}}(|a \rangle + e^{i\phi}|b \rangle)$ with the restriction that we perform separate measurements on each fermion does not lead to correlations in the results of the local measurements in A and in B which violate Bell’s inequality. We do get correlations between the results of local measurements in A and B, but these correlations are of the kind which allow local causal explanation. These results do not allow us to find the phase ϕ . The phase ϕ has observable consequences in interference experiments. A fermion quantum wave exhibits the AB nonlocality which is the unobservability of this phase via local single-particle measurements.

ACKNOWLEDGMENTS

It is a pleasure to thank Lior Goldenberg, Jacob Grunhaus, Benni Reznik, Sandu Popescu, Asher Peres and

especially Philip Pearle for helpful discussions. This research was supported in part by grant 471/98 of the Basic Research Foundation (administered by the Israel Academy of Sciences and Humanities) and NSF grant PHY9601280. Part of this work was done during the 1999 ESF-Newton Institute Conference in Cambridge.

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