Note on the Absorption of Slow Mesotrons in Matter

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A mesotron with the positive (or negative) charge passing through an absorbing medium loses its energy
i) gradually by ionizing atoms
and
ii) sometimes by exciting or disintegrating nuclei, until it disappears either
iii) by changing spontaneously into a positive (or negative) electron and a neutrino (or an antineutrino), or
iv) by being absorbed by a nucleus.

We should like to disregard the process ii) for the time being. The importance of the process iii) relative to iv) depends largely on the density of the medium. Thus, it is expected that the majority of mesotrons, which have entered in a dense medium such as lead or water, are immediately absorbed by nuclei after stopped completely, while those which have entered in a gaseous medium such as air are most likely to disintegrate spontaneously before stopped completely. The latter case was already discussed in detail by Euler and Heisenberg(1), so that we want here to confine our attention to the former case, in which the process iv) is important.

We assume that a mesotron with the velocity $v_0 = c\gamma_0$ enters normally into the absorber. Its velocity $v = c\gamma$ diminishes with the depth, due to the process i), according to the formula

$$-rac{d}{dl} \left( \frac{m_e c^2}{\sqrt{1-\gamma^2}} \right) = \frac{a}{\gamma^2},$$

(1)

where $a$ is a constant characteristic to the medium. (1) can be integrated immediately and we obtain


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\[ l = \frac{m_e c^2}{a} \left( \sqrt{1 - \beta^2} - \frac{1}{\sqrt{1 - \beta^2}} \right). \]  
(2)

The time spent before reaching the depth \( l \) can easily be found to be
\[ t = \frac{m_e c}{a} \left( \frac{\beta}{\sqrt{1 - \beta^2}} - \arcsin \beta \right). \]
(3)

by using the relation \( \frac{dl}{dt} = c \beta \).

Now the probability \( N(l) \) that the mesotron reaches the depth \( l \) satisfies the equation
\[ -\frac{dN(l)}{dl} = N(l) \{ \phi(\beta) + p(\beta) \}, \]
(4)

where \( \phi(\beta) dl \) is the probability that the mesotron with the velocity \( c \beta \) is absorbed by the process iv) between \( l \) and \( l + dl \) and \( p(\beta) dl \) is the corresponding probability that it annihilates spontaneously, \( \beta \) being a function of \( l \) as given by (2). As shown in previous papers\(^5\), \( p(\beta) \) has the form
\[ p(\beta) dl = \frac{w_0 \sqrt{1 - \beta^2}}{c \beta} dl, \]
(5)

where \( w_0 \) is the reciprocal mean life time of the mesotron at rest. The absorption of the mesotron by the nucleus is brought about either iv,) by the process

\[ U^+ + N \text{ (bound)} \rightarrow P \]
( or \( U^- + P \text{ (bound)} \rightarrow N \))

similar to the photoelectric effect as discussed by Sakata and Tanikawa\(^6\) and by Massey and Corben\(^6\), or

iv,) by the process

\[ U^+ + N \text{ (free)} \rightarrow \gamma + P \]
( or \( U^- + P \text{ (free)} \rightarrow \gamma + N \))

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as discussed by Heitler\textsuperscript{(5)}\ Kobayasi, and Okayama\textsuperscript{(6)}. In both cases, the cross-sections for small velocities are proportional to $v^{-1}$, so that $\phi(\beta)$ takes the form

$$\phi(\beta) d\ell = \frac{b}{\beta} d\ell,$$

(6)

where $b$ is a constant characteristic to the medium. By inserting (5) and (6) into (4) and by using the relation (1), we obtain

$$\frac{dN}{N} = \frac{m_v c^2}{a} \left( \frac{\beta^2}{(1-\beta^2)^{3/2}} \left( b + \frac{w_0}{c} \sqrt{1-\beta^2} \right) \right) d\beta,$$

(7)

which leads to

$$N = A \exp \left[ -\frac{m_v c^2}{a} \left\{ b \left( \frac{\beta}{\sqrt{1-\beta^2}} - \sin^{-1} \beta \right) \right. \right.

+ \left. \left. \frac{w_0}{c} \left( -\beta + \frac{1}{2} \log \frac{1+\beta}{1-\beta} \right) \right\} \right]$$

(8)

by an elementary integration, where

$$A = \exp \left[ -\frac{m_v c^2}{a} \left\{ b \left( \frac{\beta_0}{\sqrt{1-\beta_0^2}} - \sin^{-1} \beta_0 \right) \right. \right.

+ \left. \left. \frac{w_0}{c} \left( -\beta_0 + \frac{1}{2} \log \frac{1+\beta_0}{1-\beta_0} \right) \right\} \right]$$

(9)

which is determined by the initial condition $N = 1$ for $\ell = 0$. In the case where the velocity of the mesotron is small compared with $c$, i.e., $\beta \ll \beta_0 \ll 1$, (3) and (8) reduce to

$$t \approx \frac{m_v c}{3a} (\beta_0 - \beta),$$

(10)

$$N \approx \exp \left\{ -\frac{m_v c}{3a} \left( bc + w_0 \right) (\beta_0 - \beta) \right\}$$

(11)

respectively.


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Thus, the probability that a slow mesotron survives after a time $t$ decreases nearly exponentially with $t$.

Now the constant $b$ is given by

$$b \approx \left\{ 4.5 \times 10^{-28} + 4\pi \left( \frac{g\theta}{m_v c^2} \right)^2 \right\} n$$

approximately according to previous calculations, where $n$ is the number of neutrons (or protons) in unit volume of the absorber. The first term in the right hand side of (12) is due to the process $iv_v(\gamma)$, and the second term to $iv_\theta(\gamma)$, where $g$ is the constant characteristic of the mesotron theory of nuclear forces.

On the other hand, $w_0$ has the order of magnitude

$$w_0 \approx 10^6 \text{ sec}^{-1}.$$ 

In the case of dense medium, $bc$ is large compared with $w_0$. For Pb, for example, it has the order of magnitude

$$bc \approx 10^8 \text{ sec}^{-1}.$$ 

Thus the mesotron slowed down in such a medium will be absorbed almost always by the nucleus in a time of the order of $10^{-8}$ sec. It should be noticed that the time required to stop the mesotron is still smaller as can easily be calculated by taking $\beta = 0$ in (3). Thus in the case of Pb, it is $6 \times 10^{-11}$ sec for the initial energy $10^7$ eV and $2 \times 10^{-10}$ sec for $10^8$ eV, so that the majority of mesotrons are captured by nuclei after having stopped completely. These results, which can be anticipated without such a detailed calculation, is in accordance with the experiment of Montgomery and others(10), which shows that there is no delayed emission of annihilation electrons from the absorber of 2 cm Pb after a time of the order of $10^{-6}$ sec.

On the contrary, $bc$ is smaller than $w_0$ in gaseous medium such as air by a factor 10, according to the formula (12), so that we

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(7) Sakata and Tanikawa: loc. cit.
(8) Kobayasi and Okayama: loc. cit.
(9) III and IV; Euler and Heisenberg: loc. cit. See further Yukawa and Sakata: Nature, 143 (1939), 761.

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have only to consider the spontaneous disintegration. Recent experiment of Maier-Leibnitz(11), however, seems to indicate that, even in the gaseous medium, the nuclear processes are important for slow mesotrons. Indeed there are reasons to believe that the cross-sections for the processes iv) should be somewhat larger than those obtained hitherto by simple considerations. Detailed discussions of this point, however, will be given by Kobayasi in another place.


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