deduced from an appropriate Lagrangian and are
\[ \left\{ -\frac{\partial^2}{\partial \delta x_\mu \partial x_\mu} + \lambda \right\} \varphi(x, \mu) = -\frac{\partial^\alpha}{\partial x^\alpha} (X^\mu + \gamma^\alpha \gamma_\mu \varphi(x^\mu - \gamma^\alpha \mu)), \]

\[ \gamma^\alpha \gamma_\mu + M(x) = -\int \varphi(x') \varphi(x')^* dx'^\mu dx'^\nu, \]

We insert (5) into (6), multiply both sides by the complex conjugate \( x_\mu^*(x') \) and integrate over the four-dimensional space of \( r_1, r_2, r_3, r_4 \)
\[ r_0 = -\gamma_0. \] The result is
\[ \left( \frac{\partial^2}{\partial \delta x_\mu \partial x_\mu} + m_\mu^2 \right) \varphi(x') = \int \varphi(x') \varphi(x')^* \tilde{C}(x') dx'^\mu dx'^\nu, \]

where
\[ \varphi(x', x'^\mu) = \gamma^\alpha \gamma_\mu \tilde{C}(x') \varphi(x')^* dx'^\mu dx'^\nu. \]

Similarly, we obtain from (7) the equation
\[ \gamma^\alpha \gamma_\mu + M(x) = -\int \varphi(x', x'^\mu) \varphi(x'^\mu)^* dx'^\mu dx'^\nu. \]

If we compare these equations with the corresponding equations (19) of Møller and Kristensen in the theory of nonlocal interaction between a scalar (or pseudoscalar) field and a local spinor field, we notice that the internal eigenfunction \( x_\mu^*(x') \) plays the role of a convergence factor. There is, however, an essential difference between their equations and ours. Namely, in our theory, we are obliged to take into account simultaneously all the particles with different masses \( m_\mu \) which were derived from an eigenvalue problem. Furthermore, the form function for each of these particles is uniquely determined by the same eigenvalue problem.

In the following letter, the above general considerations will be illustrated and further details will be examined.

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**Structure and Mass Spectrum of Elementary Particles. II. Oscillator Model**

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A n illustration of the general considerations on nonlocal fields in the preceding letter, let us assume that the operator \( F \) has a very simple form
\[ F = -\frac{\partial^2}{\partial \delta x_\mu \partial x_\mu} + \lambda \frac{\partial^2}{\partial \gamma^\alpha \partial x_\mu} \left( \lambda \gamma_\mu \varphi \right)^2, \]

where \( \lambda \) is a small constant with the dimension of length. One may call this the four-dimensional oscillator model for the elementary particle, which was considered first by Born in connection with his idea of self-reciprocity. However, our model differs from his model in that we have introduced internal degrees of freedom of the particles which are related to the nonlocalizability of the field itself. The internal eigenfunctions in our case are
\[ x_\mu^*(x') = \int H_\mu(r_1) H_\nu(r_2) H_\lambda(r_3) \hat{x} \hat{H}_\mu(r_1) \exp \left\{ -\left( r_1^2 + r_2^2 + r_3^2 + \gamma^\alpha r_\lambda^\alpha \right)/\left( 2 \lambda \right) \right\}, \]

and the corresponding eigenvalues for the mass become
\[ m_\mu = \sqrt{2}/\lambda \left| n_1 + n_2 + n_3 - n_0 + 1 \right|, \]

where \( r_0 = -\gamma_0 \) is a real variable and \( n_1, n_2, n_3, n_0 \) are quantum numbers which can take only zero or positive integer values. \( H_\mu \) denotes the Hermite polynomial of \( x \) of degree \( n \). All these eigenfunctions (2) decrease rapidly in any direction whatsoever in the four-dimensional \( r \) space. Furthermore, the Fourier transform of each of these eigenfunctions has exactly the same form as the original function due to the self-reciprocity. Thus, the form function (9) in the preceding letter seems to be sufficient to cut off high energy-momentum intermediate states in such a way that each term corresponding to each Feynman diagram in the expansion of the nonlocal \( S \)-matrix according to the Bloch-Kristensen-Møller formulation is convergent. However, since we have to take into account all of infinitely many of different mass states of the nonlocal system, the number of terms in the \( S \) matrix increases very rapidly with the increasing power of the coupling constant, so that we can claim nothing for the moment concerning the convergence or divergence of the \( S \) matrix as a whole.

The totality of the internal eigenfunctions (2) constitutes a complete set of orthogonal and quadratically integrable functions in the four-dimensional \( r \) space and can be regarded as the eigenvectors for an infinite-dimensional unitary representation of the Lorentz group. The eigenvalues (3) for the mass are all infinitely degenerate. For instance, all those values of \( n \) which satisfy \( n_1 + n_2 + n_3 - n_0 = 0 \) give the same mass, \( m_\mu = \sqrt{2}/\lambda \). This is not a peculiar feature of the oscillator model; it is common to all those models for which the operator \( F \) is separable, because there can be no unitary representation of finite dimensions for the Lorentz group. Presumably, such an undesired degeneracy could be removed either by introducing interaction with other fields or by first introducing the coupling between the external and internal degrees of freedom. The latter possibility can be illustrated by the addition of the coupling term,

\[ -\beta \varphi \left\{ -\frac{\partial^2}{\partial x_\mu \partial x_\mu} + \frac{1}{\lambda^2} \frac{\partial^2}{\partial \gamma_\mu \partial x_\mu} \right\}, \]

to the expression (1) for \( F \), where \( \beta \) is a dimensionless real constant. The free field equation becomes

\[ \left( \frac{\partial^2}{\partial x_\mu \partial x_\mu} + \frac{1}{\lambda^2} \frac{\partial^2}{\partial \gamma_\mu \partial x_\mu} \right) \varphi(x, \mu) = 0, \]

in the eight-dimensional space of \( \varphi \) and \( \mu \), where \( \varphi(x, \mu) \) is the Fourier transform of \( \varphi(x_\mu^*(x')) \) as defined by

\[ \varphi(x_\mu^*(x')) = \int \exp \left( \frac{i}{\hbar} (p x_\mu) \lambda \right) (p_\mu), \]

One can solve Eq. (5) in the coordinate system in which only one component of the wave vector is different from zero. Thus, one obtains the mass spectrum

\[ m_\mu = \sqrt{2} \left| n_1 + n_2 + n_3 - n_0 + 1 \right|, \]

where \( n_0 \) is restricted by the condition

\[ n_0 < \frac{1}{2} \left( \frac{\lambda}{\beta} - 1 \right). \]

If we take, for instance, \( \beta = 1/2 \), only \( n_0 = 0 \) is allowed and the mass spectrum reduces to

\[ m_\mu = (2/\lambda) \left| n_1 + n_2 + n_3 + 1 \right|, \]

and the degree of degeneracy of the mass eigenvalues is now finite. In particular, the lowest mass, \( m_\mu = 2/\lambda \), is free from degeneracy and the corresponding solution of (5) is given by

\[ \chi_{\mu_0}(p_\mu) \right) = \left( \frac{1}{2 \lambda} \right)^{\frac{1}{2}} \exp \left\{ -\frac{1}{2} \lambda \left( r_1^2 + r_2^2 + r_3^2 \right)/m_\mu \right\}, \]

in an arbitrary coordinate system, where \( \hbar \mu_0 = -m_\mu^2 \).

The above advantage of introducing the coupling between external and internal degrees of freedom is offset, however, by a
A study of the shapes of these curves indicates that the \( \text{Cu}^{64}(\text{He}^3, 2a) \) curve fits smoothly a theoretical curve involving compound nucleus formation, as expected. The high-energy part of the experimental curve for the \( \text{Cu}^{64}(\text{He}^3, \alpha) \) reaction can be similarly fitted to a theoretical curve down to about 9.0 Mev. For lower energies the shape of the curve changes in a marked way. The cross section is less energy-sensitive and the reaction can be traced to as low as 5.4 Mev where essentially no penetration of the potential barrier would be expected.

The experimental data appear to indicate that the inverse \( O-P \) phenomenon comes into evidence from very low energies up to an energy where compound nucleus formation mechanism takes over and, from there on, the latter plays the major role in the \( \text{Cu}^6(\text{He}^3, \alpha) \) reaction.

It may be pointed out that the 5.10-minute \( \text{Cu}^6 \) activity was not produced in measurable amount by the reaction \( \text{Cu}^6(\text{He}^3, 2p) \). The similar reaction \( \text{Cu}^{64}(\text{He}^3, 2p) \) has a \( Q \) value of only \( +0.18 \) Mev as compared with the \( Q \) value of \( +10.7 \) Mev for the reaction \( \text{Cu}^6(\text{He}^3, \alpha) \text{Cu}^6 \). Analogous to the latter is the reaction \( \text{Cu}^6(\text{He}^3, \alpha) \) which gave a substantial yield of the 9.9-minute \( \text{Cu}^6 \) activity.

**Bremssstrahlung at High Energies**

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Two of us\(^1\) have published a formula for the cross section for bremssstrahlung which was derived without the use of the Born approximation. The matrix element was taken to be

\[
M = \int \psi^* \alpha \frac{\phi e^{-ik \cdot r}}{r} \, dr,
\]

with \( \alpha \) the Dirac matrix in the direction of polarization of the emitted quantum, and \( \psi \) and \( \phi \) wave functions of the electron in the Coulomb field of a nucleus.

Like most authors before us\(^2\) we took both electron wave functions to be plane waves plus outgoing spherical waves. This is of course correct for the initial state \( \psi_i \), but it is wrong for the final state \( \psi_f \); the latter must be taken as a plane wave plus ingoing spherical waves.

To show this, we note that in the process of determining a differential cross section, we observe (e.g., by means of a counter) an electron moving in direction \( p_i \), some time after the radiation process has taken place. This observation is described by a plane wave packet concentrated around the point \( v_f \); this wave packet is moving away from the nucleus and is not accompanied by any other waves at this late time. We now follow the development of this wave packet backwards in time by use of the Dirac equation. As long as the packet is outside the field of the nucleus, it remains a plane wave packet. When the time \( t \) approaches zero (from above), the Coulomb field will scatter the wave packet, and for \( t < 0 \) the plane wave packet will be accompanied by scattered spherical waves. As \( t \) becomes more and more negative, both the plane wave and the spherical waves will move farther away from the nucleus. But in the usual language in which one describes the motion with increasing time, both waves will, for \( t < 0 \), move towards the nucleus. This shows that we have indeed ingoing spherical waves associated with the plane wave, and this result can be carried over into the time-independent formalism.

In fact, the use of converging spherical waves in \( \psi \) is the natural counterpart of the use of outgoing spherical waves in \( \psi \). Here the observation of an electron going in the direction \( p_i \) is made in the beginning; therefore the electron will be described by a pure plane wave until it hits the nucleus, and by a plane wave plus outgoing spherical waves thereafter.