PRESENT STATUS OF CP VIOLATION

Lincoln Wolfenstein

Department of Physics, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

CONTENTS

1. INTRODUCTION ................................................................. 137
2. CP VIOLATION IN K° DECAY ............................................. 140
3. CLASSIFICATION OF MODELS OF CP VIOLATION ............ 145
4. KOBAYASHI-MASKAWA MODEL ......................................... 146
   4.1 The KM Matrix ......................................................... 146
   4.2 Calculation of ε and ε' ............................................... 148
   4.3 D and B Systems ...................................................... 152
5. ALTERNATIVE GAUGE MODELS ....................................... 155
   5.1 Superweak Two-Higgs Model ........................................ 156
   5.2 Other Superweak Models ............................................. 157
   5.3 Weinberg Three-Higgs Model ...................................... 157
   5.4 SU(2)_L ∩ SU(2)_R ∩ U(1) Model ................................. 159
   5.5 Supersymmetric Models ............................................. 160
6. OTHER OBSERVABLES ...................................................... 161
   6.1 Neutron Electric Dipole Moment ................................... 161
   6.2 Electric Dipole Moments of Atoms ................................. 162
   6.3 Time-Reversal Violation in Semileptonic Decays ............. 163
   6.4 Nonleptonic Decays .................................................. 164
7. CONCLUSION ...................................................................... 167

1. INTRODUCTION

Symmetry principles have long played a major role in nuclear and particle physics (1, 2). Here we are concerned with three discrete symmetries: P, parity or space inversion, T, time reversal, and C, particle-antiparticle conjugation. Each of these symmetries relates a quantum mechanical state vector or transition amplitude to a unique mirror image. For quantum electrodynamics each of these symmetries follows from the well-established theory. It was natural in formulating theories for strong and weak inter-
actions to assume these symmetries. As is well known, the discovery of
parity violation in the weak interactions forced a reexamination of the
range of validity of each of these symmetries.

It turns out that it is easy to formulate theories in which $P$, $C$, or
$T$ invariance is violated. However, very general principles of relativistic
quantum field theory lead to the conclusion that the product $CPT$ is a
good symmetry. While the validity of $CPT$ invariance also should be tested
experimentally, for the most part in this review we assume its validity. It
follows that, if we consider a Hamiltonian containing a part that is even
(invariant) under $C$, $P$, and $T$ plus terms that are odd under some of these,
there are three odd possibilities:

$$
\begin{align*}
\text{C odd, } P \text{ odd, } CP \text{ and } T \text{ even} & \quad \text{la.} \\
\text{C odd, } P \text{ even, } CP \text{ and } T \text{ odd} & \quad \text{lb.} \\
\text{C even, } P \text{ odd, } CP \text{ and } T \text{ odd} & \quad \text{lc.}
\end{align*}
$$

In particular, any term that violates $CP$ invariance also violates $T$ invariance
so that we customarily group tests of these two together.

After the discovery of parity violation a very successful theory of weak
interactions, the V-A theory, was developed. The V-A Hamiltonian con-
tains an equal mixture of an even part and a part of type 1a above so that
$C$ and $P$ are maximally violated but $CP$ and $T$ remain as good symmetries.
A large number of experiments have demonstrated the validity of the
V-A theory. Among these have been experiments searching for $T$ violation
in beta decay and similar weak processes; to this day, as discussed in
Section 6, all such experiments have produced null results.

In 1964 Christenson et al. discovered evidence that $CP$ was violated
in $K^0$ decay. The $K^0$ particle has an additive quantum number $S = +1$
(called strangeness) and the antiparticle $\bar{K}^0$ has $S = -1$. Strangeness is
conserved in the strong and electromagnetic interactions that produce $K^0$
in the laboratory, but weak interactions allow for the decay of strange
particles. Since $S$ is not a good quantum number of the complete Hamil-
tonian, the particles with definite mass and lifetime are not $K^0$ and $\bar{K}^0$
but rather linear combinations $K_S$ and $K_L$. Indeed experimentally one
observes two $K^0$ decay branches with very different lifetimes and decay
modes (4): $K_S$ has a lifetime of $0.9 \times 10^{-10}$ sec and decays mainly to two
pions while $K_L$ has a lifetime of $5 \times 10^{-8}$ sec and decays mainly into three
pions or semileptonically. If one assumes that $CP$ is an exact symmetry
then one can use $CP$ to classify the eigenstates. This led to the assumption
that $K_S$ was the $CP$-even state (since two pions are $CP$ even) and that $K_L$
was the $CP$-odd state (since three pions with $J = 0$ are $CP$ odd). The
discovery of 1964 was that $K_L$ also decays to the $CP$-even state of two
pions. It follows that either $K_L$ is not an eigenstate of $CP$ or that $CP$ is violated by the decay or both; in any case $CP$ is not an exact symmetry of nature. Alternative explanations have been ruled out (5), particularly by the observation of interference between $K_L$ and $K_S$ in $K^0 \to 2\pi$ (6).

After the discovery of parity violation in nuclear beta decay, many other examples of parity violation were discovered. All the future developments in weak interaction theory built upon this discovery. In contrast, twenty years later no system other than $K^0$ has exhibited $CP$ or $T$ violation. The analysis of all the results on $CP$ violation in $K^0$ decays given in Section 2 shows that they can be explained by a single parameter $|\epsilon|$ already measured in the 1964 experiment. In a sense all subsequent experiments have served to verify the original result and provide null results of uncertain significance. The phenomenon of $CP$ violation has appeared more of a mystery than as a guide to further understanding.

Shortly after the 1964 experiment it was pointed out (7) that $CP$ violation in the $K^0$ system could be explained by a new superweak interaction of a strength $10^{-9}$ times that of the standard weak interaction. The very small mass difference between $K_L$ and $K_S$ makes the $K^0$ system particularly sensitive to a superweak interaction that changes $S$ by two units. The superweak theory predicts that null results will be found in other experiments and all our present knowledge is consistent with the superweak theory. In contrast, as discussed in Section 3, are milliweak theories in which $CP$ violation is incorporated in some way into the weak interaction. Such milliweak theories predict that new $CP$-violating phenomena should be discovered. However, it is not unreasonable to expect such $CP$-violating effects to be of the same order of magnitude as that found in $K^0$ decay, measured by $|\epsilon| = 2 \times 10^{-3}$. In practice it turns out that it is extremely difficult to measure such small $CP$- or $T$-violating effects outside of $K^0$ decay itself. Thus there exist many milliweak models that are quite consistent with present knowledge.

It is possible to incorporate $CP$ violation into the standard Weinberg-Salam electroweak gauge theory with three generations of quarks as first pointed by Kobayashi & Maskawa (8). This simplest of milliweak models is discussed in detail in Section 4. Alternative gauge models that require an extension of the minimal Weinberg-Salam model are discussed in Section 5. To distinguish among models, additional experiments are needed. The most promising of these involves more precise measurements of the $CP$-violating $K_L$ decay to determine the parameter $\epsilon'$ discussed in Section 2. Possible experiments in other systems are discussed in Section 6.

A detailed review of experiments on $CP$ violation has been given by Kleinknecht (6). Here we do not include discussions of the experiments except for significant new results.
2. **CP VIOLATION IN $K^0$ DECAY**

The violation of CP invariance has been observed in three decays of the $K_L$ meson and nowhere else. These observations are summarized in two complex parameters $\eta_{+-}$ and $\eta_{00}$ and the charge asymmetry $\delta$ defined by

$$
\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = |\eta_{+-}| \exp(i\phi_{+-})
$$

$$
\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = |\eta_{00}| \exp(i\phi_{00})
$$

$$
\delta = \frac{\Gamma(K_L \to \pi^- l^+\nu) - \Gamma(K_L \to \pi^+ l^-\bar{\nu})}{\Gamma(K_L \to \pi^- l^+\nu) + \Gamma(K_L \to \pi^+ l^-\bar{\nu})}
$$

where $A$ stands for amplitude and $l$ is either $e$ or $\mu$. The experimental results are

$$
|\eta_{+-}| = (2.274 \pm 0.022) \times 10^{-3}
$$

$$
\phi_{+-} = 44.6 \pm 1.2^\circ
$$

$$
\phi_{00} = 54 \pm 5^\circ
$$

$$
|\eta_{+-}/\eta_{00}| = 0.992 \pm 0.02
$$

$$
\delta = (3.30 \pm 0.12) \times 10^{-3}.
$$

These numbers come from the Particle Data Group averages (9) except for $|\eta_{00}/\eta_{+-}|$, which is discussed below.

From a phenomenological view CP violation may occur either in the mass matrix of the $K^0$-$\bar{K}^0$ system or in the decay amplitude. Because CP invariance is only broken a little, it is convenient to start with the CP eigenstates

$$
K_1 = (K^0 + \bar{K}^0)/\sqrt{2}, \quad CP = +
$$

$$
K_2 = (K^0 - \bar{K}^0)/\sqrt{2}, \quad CP = -
$$

where $\bar{K}^0 = (CP)K^0$. In the $K_1$-$K_2$ representation the complex mass matrix takes the form

$$
M - i\frac{\Gamma}{2} = \begin{pmatrix}
M_1 & -\frac{\Gamma}{2} - i\gamma \\
-\frac{\Gamma}{2} + i\gamma & M_2
\end{pmatrix}
$$

The off-diagonal terms that mix $K_1$ and $K_2$ are the result of CP violation.
Since these are small, to a good approximation the diagonal values are equal to the eigenvalues
\[ M_1 - M_2 + i(\gamma_1 - \gamma_2)/2 = (M_S - M_L) + i(\Gamma_S - \Gamma_L)/2 = -\Delta M + i\Gamma_S/2, \]
where \( \Delta M \) is the mass difference between \( K_L \) and \( K_S \) and in the last approximation we use \( \Gamma_L \ll \Gamma_S \). The experimental data (9) on \( \Delta M \) and the widths are
\[ \Gamma_S^{-1} = (0.892 \pm 0.022) \times 10^{-10} \text{ sec} \]
\[ \Gamma_S/\Gamma_L = 581 \pm 4.5 \]
\[ \Delta M/\Gamma_S = 0.477 \pm 0.003. \]

The factor \( i \) and the antisymmetry indicate that the term \( m' \) violates not only CP but also \( T \) as expected from the CPT theorem. On the other hand, the term \( \delta' \) is \( T \) invariant and violates CPT; we set it equal to zero for this reason. The decay amplitudes of main interest are \( K^0 \to 2\pi \) written as
\[
A[K^0 \to \pi \pi(I)] = A_I \exp(i\delta_I)
\]
\[
A[\bar{K}^0 \to \pi \pi(I)] = A_I^* \exp(i\delta_I)
\]
where \( I \) is the \( \pi \pi \) isospin and \( \delta_I \) is the corresponding \( \pi \pi \) phase shift for \( s = M_K^2 \). From CPT invariance and unitarity one can show that \( A_I \) is real if CP is not violated and also that \( \gamma' \) in Equation 2 is given to a good approximation by
\[
\frac{i\gamma'}{\Gamma_S} = \frac{A(K_2 \to \pi \pi)A(K_1 \to \pi \pi)}{|A(K_1 \to \pi \pi)|^2} = \frac{i \Im A_0}{\Re A_0}. \]

The approximation involves a neglect of all final states except the two-pion state with \( I = 0 \). A detailed analysis including other intermediate states and the possibility of CPT violation is given by Barmin et al (10).

The phenomenology is seen to contain three CP-violating quantities \( m' \), \( \Im A_0 \), and \( \Im A_2 \). We now express the observables in terms of these. As a result of the CP violation in the mass matrix the mass eigenstates differ from the CP eigenstates
\[
K_S = \frac{(K_1 + \bar{\xi}K_2)/(|1 + |\bar{\xi}|^2)}\]
\[
K_L = \frac{(K_2 + \bar{\xi}K_1)/(|1 + |\bar{\xi}|^2)}\]
\[
\bar{\xi} = i \frac{m' - i\gamma'/2}{\Delta M + i\Gamma_S/2}. \]
After a little algebra one finds

\[ \eta_{+-} = \varepsilon + \varepsilon'/(1 + \text{Re } A_0) \]

\[ \eta_{00} = \varepsilon - 2\varepsilon'/(1 - \sqrt{2} \text{Re } A_0) \]

\[ \varepsilon = \bar{\varepsilon} + i(\text{Im } A_0/\text{Re } A_0) \approx \frac{1}{\sqrt{2}} e^{i\theta} \left( \frac{m'}{\Delta M} + \frac{\text{Im } A_0}{\text{Re } A_0} \right) \]

\[ \varepsilon' = \frac{1}{\sqrt{2}} e^{i\varphi} w \left( \frac{\text{Im } A_2}{\text{Re } A_2} - \frac{\text{Im } A_0}{\text{Re } A_0} \right) \]

\[ w = \text{Re } A_2/\text{Re } A_0 = 0.045 \]

\[ \theta = \tan^{-1} \left( \frac{2\Delta M}{\Gamma_0} \right) = 43.67 \pm 0.14^\circ \]

\[ \theta' = \delta_2 - \delta_0 + \frac{\pi}{2} = 48 \pm 8^\circ, \]

and the numerical values are experimental results. The value of the \( \pi \pi \) phase shifts \( \delta_2, \delta_0 \) are extracted from the review by Devlin & Dickey (11). Assuming the \( \Delta Q = \Delta S \) rule, we obtain the asymmetry parameter

\[ \delta = 2 \text{ Re } \bar{\varepsilon} = 2 \text{ Re } \varepsilon. \]

From the experimental values of Equation 2 we find

\[ |\varepsilon| = (2.27 \pm 0.02) \times 10^{-3} \]

\[ \varepsilon'/\varepsilon = \left( \frac{\eta_{+-}}{\eta_{00}} - 1 \right)/3 = (-3 \pm 6) \times 10^{-3}, \]

where in Equation 14b we have used the approximation \( \cos(\theta - \varphi) = 1 \) based on Equation 12. Given the small value of \( (\varepsilon'/\varepsilon) \), the phase of \( \varepsilon \) is essentially the phase of \( \eta_{+-} \) and we see from Equation 2 that the experimental value of \( \phi_{+-} \) agrees perfectly with the value \( \theta \) given in Equation 12. While this prediction for \( \phi_{+-} \) was first made in the superweak theory, we see here that it follows to a good approximation (the main approximation is Equation 6) from CPT invariance once we know \( \varepsilon'/\varepsilon \) is small. We also expect the phase \( \phi_{00} \) to be almost the same as \( \phi_{+-} \); the experimental value taken literally cannot be understood without invoking CPT violation but allowing for a two-standard-derivation error it is consistent with \( \phi_{+-} \). The experimental value of \( \delta \) agrees perfectly with Equation 13.

While our results were expressed in terms of \( m', \text{Im } A_0, \) and \( \text{Im } A_2 \) only the combinations \( \varepsilon \) and \( \varepsilon' \) enter and the phases of each of these is determined. In fact, as first emphasized by Wu & Yang (12), the parameters \( m', \text{Im } A_0, \) and \( \text{Im } A_2 \) cannot be determined unambiguously because it is
possible to make a transformation of the phase of the s quark: $s \rightarrow se^{-i\alpha}$
or, infinitesimally

$$s \rightarrow s(1 - i\alpha).$$

As a result

$$\text{Im } A_I \rightarrow \text{Im } A_I - \alpha \text{ Re } A_I$$

$$m' \rightarrow m' + \alpha(\Delta M)$$

$$\bar{\epsilon} \rightarrow \bar{\epsilon} + i\alpha.$$ 

Thus any of the three original parameters may be set equal to zero; Wu & Yang chose $\text{Im } A_0 = 0$. Most theoretical models are expressed in terms of a convenient phase convention such that in general none of the three turns out to be zero.

The result of the analysis can be summarized as follows: given CPT invariance and the $\Delta S = \Delta Q$ rule, all the present observations on CP violation in the $K^0$ system depend to a good approximation on two parameters, which may be chosen as $|\epsilon|$ and $|\epsilon'|$. The parameter $|\epsilon'|$, which unambiguously depends on CP violation in the decay amplitude, is consistent with zero. Our only measure of CP violation therefore is $|\epsilon|$, which could arise solely from the $K^0$ mass matrix term $m'$; $|\epsilon|$ also contains a contribution from the decay amplitude $A_0$ but these two contributions cannot be unambiguously separated because of the phase ambiguity given by the transformation of Equations 15.

Here we have assumed CPT invariance. A detailed analysis of the data has been given without this assumption (10) in order to provide empirical limits on CPT violation. In our opinion the most interesting possibility of CPT violation comes in the mass matrix because very small entries here can have a significant effect. The term $\delta'$ in Equation 2 corresponds to a mass difference between $K^0$ and $\bar{K}^0$. Adding this term to the analysis has the effect of changing the phase of $\epsilon$ and thus (since $\epsilon'$ is empirically small) of changing the phase of $\phi_{+-}$ and $\phi_{00}$ keeping $\phi_{+-} = \phi_{00}$. If we neglect the deviation in $\phi_{00}$, then the agreement between the measured $\phi_{+-}$ and the theoretical value of Equation 12 determines that

$$|M(K^0) - M(\bar{K}^0)|/M(K^0) \leq 10^{-18}.  

This represents the best test by far of CPT invariance. If one tries to explain the deviation in $\phi_{00}$ it is necessary to have CPT-violating terms in the decay amplitude and in the mass matrix that conspire so as to give the CPT-invariant prediction for $\phi_{+-}$.

Because a nonzero value of $|\epsilon'|$ would demonstrate that CP violation is
not confined to the mass matrix, a great deal of experimental effort has been devoted to its measurement. Accepting the theoretical phases of Equation 12, one can deduce the value of $|\epsilon'|$ directly from the measurement of $|\eta_+/\eta_0|$. This corresponds to a measurement of a ratio of ratios

$$
\frac{|\eta_+/\eta_0|^2}{\frac{\Gamma(K_L \rightarrow \pi^+\pi^-)/\Gamma(K_S \rightarrow \pi^+\pi^-)}{\Gamma(K_L \rightarrow \pi^0\pi^0)/\Gamma(K_S \rightarrow \pi^0\pi^0)}}.
$$

The most precise experiment (13) so far is the Chicago-Saclay collaboration, Fermilab E617. In order to reduce systematic errors, two side-by-side $K_L$ beams were employed, in one of which a regenerator was inserted to provide the $K_S$ component. Thus $K_L$ and $K_S$ decays were measured simultaneously. The greatest difficulty lies in measuring the decay rate $K_L \rightarrow \pi^0\pi^0$ in the background of the $CP$-allowed decay $K_L \rightarrow 3\pi^0$. Another difficulty comes from the problem of separating coherent from incoherent regeneration. To reconstruct the $2\pi^0$ state one of the four $\gamma$s was converted with the resultant $e^+e^-$ pair tracked with a spectrometer. This pair, together with the other three gammas, was measured in a large lead glass block array. The number of real events and background events observed is shown in Table 1. After background subtractions, the data are simultaneously fitted to $\epsilon'$ and the regeneration amplitude, with the result $\epsilon'/\delta = -0.0046 \pm 0.0053 ({\text{stat}}) \pm 0.0024 ({\text{syst}})$.

A somewhat similar experiment (14) was carried out at Brookhaven National Laboratory with observations on $K_L$ and $K_S$ alternating in time instead of being observed simultaneously. Their event rates and background are also shown in Table 1. Their result, also consistent with zero,

<table>
<thead>
<tr>
<th>Mode</th>
<th>Events after subtraction</th>
<th>Background excluding incoherent $K \rightarrow \pi\pi$</th>
<th>Incoherent $K \rightarrow \pi\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_L \rightarrow \pi^0\pi^0$</td>
<td>3152 ± 61</td>
<td>266.0 ± 13.0</td>
<td>90.7 ± 9.5</td>
</tr>
<tr>
<td>$K_S \rightarrow \pi^0\pi^0$</td>
<td>5663 ± 84</td>
<td>35.3 ± 5.9</td>
<td>825.7 ± 28.7</td>
</tr>
<tr>
<td>$K_L \rightarrow \pi^+\pi^-$</td>
<td>10638 ± 106</td>
<td>324.9 ± 18.0</td>
<td>42.5 ± 6.5</td>
</tr>
<tr>
<td>$K_S \rightarrow \pi^+\pi^-$</td>
<td>25751 ± 163</td>
<td>44.8 ± 6.7</td>
<td>439.2 ± 21.0</td>
</tr>
</tbody>
</table>

Brookhaven experiment (14)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Events after subtraction</th>
<th>Background excluding incoherent $K_L \rightarrow \pi\pi$</th>
<th>Incoherent $K_L \rightarrow \pi\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_L \rightarrow \pi^0\pi^0$</td>
<td>1122</td>
<td>239 ± 41a</td>
<td></td>
</tr>
<tr>
<td>$K_S \rightarrow \pi^0\pi^0$</td>
<td>3267</td>
<td>40 ± 7a</td>
<td>50 ± 17</td>
</tr>
<tr>
<td>$K_L \rightarrow \pi^+\pi^-$</td>
<td>8081</td>
<td>599 ± 123a</td>
<td></td>
</tr>
<tr>
<td>$K_S \rightarrow \pi^+\pi^-$</td>
<td>20921</td>
<td>42</td>
<td></td>
</tr>
</tbody>
</table>

a Background excluding incoherent $K_L$. 


is $\varepsilon' / \varepsilon = +0.0017 \pm 0.007{\text{(stat)}} \pm 0.004{\text{(syst)}}$. Equation 14b gives an average of these two results. A comparison of the two experiments has been given by Gollin (15), from which Table 1 is derived.

Two new experiments now in progress aim to reduce the error on $\varepsilon' / \varepsilon$ to about $\pm 0.001$. Fermilab experiment E731 employs the same method as E617 with an improved beam and better acceptance. At CERN the experiment NA31 employs a very different method, the systematic errors of which should be complementary to those of E731. No regenerator is used but rather the target is moved so that $K_S$ and $K_L$ decays are both observed directly from the same target. The $\pi^+ \pi^-$ detector is nonmagnetic and the $\pi^0 \pi^0$ detector is a large liquid-argon calorimeter so that no converter is used. A comparison of the two experiments is given by Weinstein (16).

3. CLASSIFICATION OF MODELS OF CP VIOLATION

Models that explain the CP violation observed in $K_L$ decay can be classified as follows:

1. Millistrong. CP violation occurs in the parity-conserving $\Delta S = 0$ part of the Hamiltonian, that is, the part that has the selection rules of strong and electromagnetic interactions. We use the prefix milli to indicate that there is an effective factor of $10^{-3}$ in the CP-violating term relative to the normal strong interaction. The result of such a term would be to induce CP-violating effects of the order $10^{-3}$ in all processes involving hadrons: strong, electromagnetic, and weak. Our present theoretical picture in which strong interactions are governed by QCD makes this model unattractive. Many experiments have searched for $T$ violation in strong and electromagnetic processes (2, 6) without success. Nevertheless it is hard to rule out definitively this class of model without a specific theory with which to compare experiments.

   We note in passing that there exists a possibility of a $T$-violating strong interaction in QCD that is also $P$ violating (type 1c). The strength of such an interaction is experimentally limited from the electric dipole moment of the neutron (see Section 6.1); the limit is so low that this interaction cannot play a role in explaining CP violation in $K^0$ decay. We do not discuss CP violation in strong or electromagnetic interactions further in this review.

   2. Milliweak. CP violation occurs in the weak interactions that allow $\Delta S = 0$ and $\Delta S = 1$ but not $\Delta S = 2$. Again the prefix milli indicates that effectively (at least for $K^0$ physics) the CP-violating term is down by a
factor $10^{-3}$. As first pointed out by Kobayashi & Maskawa (KM) (8) and
discussed in detail in Section 4, $CP$ violation can be incorporated in the
standard electroweak theory if there exist six quarks. Extensions of the
standard electroweak model that provide mechanisms for milliweak $CP$
violation are discussed in Section 5.

3. Superweak. It is also possible that $CP$ violation is associated with a
class of interactions much weaker (an effective factor of the order of $10^{-9}$)
than the usual weak interactions provided these new interactions allow
$\Delta S = 2$ in lowest order. In the standard theory $m'$ and $\Delta M$, associated
with the $\Delta S = 2$ $K^0$-$\bar{K}^0$ mixing, occur only in second order. A new $\Delta S = 2$
interaction can contribute to $m'$ in first order and so be important for $K^0$-
$\bar{K}^0$ mixing even though it is too weak to have any significant effect directly
on decay amplitudes. Thus a major prediction of the superweak models
distinguishing them from most milliweak theories is that $\epsilon'$ is essentially
zero (of order $10^{-11}$). In addition, searches for $CP$ violation in other weak
interactions (with the possible exception of systems analogous to the $K^0$
like $D^0$ or $B^0$) will also give negative results. Superweak gauge models are
also discussed in Section 5.

It is possible that in particular milliweak models the major $CP$-violating
effect comes from $m'$ whereas for some dynamical reason $CP$-violating
effects in the decay amplitudes are small. As a result in such models $\epsilon'$ may
be very small. Such models are sometimes said to be superweak in char-
acter. We feel this is misleading since all such models contain some $CP$-
violating observables that are orders-of-magnitude larger than they would
be for a truly superweak model. There may, of course, be models in which
$\Delta S = 2$ lowest order effects and $\Delta S = 1$ $CP$ violation are both important;
while such models are milliweak in character, we refer to the $\Delta S = 2$ lowest
order term as a superweak mechanism.

4. KOBAYASHI-MASKAWA MODEL

4.1 The KM Matrix

In the standard electroweak model (see the accompanying review by Mar-
ciano & Parsa) the interactions of the quarks with the charged gauge
bosons $W$ are given by

$$g u_j U_{ji} \gamma_5 (1 - \gamma_5) d_i W^\pm + h.c.$$ 17.

Here $u_j = (u, c, t)$ are the up-type quarks and $d_j = (d, s, b)$ are the down-
type. $U$ is the unitary KM matrix, the $3 \times 3$ generalization of the Cabibbo
mixing matrix. A convenient parameterization of $U$ devised by Maiani
\( U = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} \)

\[
U = \begin{pmatrix} C_\theta C_\beta & C_\theta S_\beta & S_\theta e^{-i\delta} \\ -C_\gamma S_\theta - C_\theta S_\gamma S_\phi e^{i\delta} & C_\gamma C_\theta - S_\theta S_\phi S_\gamma e^{i\delta} & C_\beta S_\gamma \\ 0 & -C_\theta S_\gamma - C_\gamma S_\theta S_\phi e^{i\delta} & C_\gamma C_\beta \end{pmatrix},
\]

where \( C_\theta = \cos \theta \) and \( S_\theta = \sin \theta \). As originally noted by Kobayashi & Maskawa it is possible by defining the phase of the quark fields to eliminate all but one of the phases in \( U \). Thus all \( CP \) violation in this model depends on the phase \( \delta \). Experimental data on strange particle and \( B \) decay rates determine the values of \( U_{us} \) (18) and \( U_{cb} \) and set a limit on \( U_{ub} \) (19–21). Given these values we have made the empirical observation (22) that the mixing angles have a hierarchical structure such that we can expand in powers of \( \lambda = \sin \theta = 0.22 \) with

\[
\sin \gamma = A \lambda^2 \\
\sin \beta e^{-i\delta} = A \lambda^3 (\rho - i\eta).
\]

The experimental data are then summarized by

\[
A = 1 \pm 0.2 \quad 19a.
\]
\[
\rho^2 + \eta^2 \leq 0.3. \quad 19b.
\]

If \( U \) is expanded in powers of \( \lambda \) to order \( \lambda^3 \), the matrix has the simple form

\[
U = \begin{pmatrix} 1 & \frac{-\lambda^2}{2} & \lambda A \lambda^3 (\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A \lambda^2 \\ A \lambda^3 (1 - \rho - i\eta) & -A \lambda^2 & 1 \end{pmatrix}. \quad 22.
\]

We have chosen a phase convention (that is, a definition of the phases of quark fields) in Equations 18 and 22 such that \( U \) is manifestly \( CP \) invariant to order \( \lambda^2 \) and \( CP \) violation shows up first in order \( \lambda^3 \). Of course, the physics is independent of the phase convention.

It has recently been pointed out (23–25) that all \( CP \)-violating observables are proportional to a quantity \( J \) that is independent of phase con-
There are nine different ways of writing $J$ corresponding to crossing out one row and one column of $U$ and then multiplying together the diagonal elements of the resulting $2 \times 2$ matrix by the complex conjugates of the off-diagonal elements. From Equations 18 and 22, one obtains

$$J = C_\beta C_\gamma C_\delta \sin \theta \sin \gamma \sin \beta \sin \delta \approx A^2 \lambda^4 \eta.$$  

4.2 Calculation of $\epsilon$ and $\epsilon'$

To calculate the $\Delta S = 2$ quantities $\Delta m$ and $m'$ we need to go to second order in the weak interaction, that is, order $G^2$ where $G$ is the weak Fermi constant. The usual approach is to calculate an effective four-quark $\Delta S = 2$ operator by evaluating the box diagram of Figure 1 summing over intermediate $u$, $c$, and $t$ quarks. The result then depends on the matrix element of this operator between $K^0$ and $\bar{K}^0$, in particular on

$$\langle \bar{K}^0 | s \gamma_5 (1 - \gamma_5) d \bar{s} \gamma_5 (1 - \gamma_5) \bar{d} | K^0 \rangle = B [4 f_K^2 m_K / 3].$$

The last equation defines $B$, which is equal to unity when the matrix element is evaluated by inserting the vacuum in all possible ways (26). There exist some indications that $B$ is less than unity. Bag model calculations are very sensitive to the parameters but generally give values of the order 0.5 or less (27). Donoghue et al (28, 29) obtain the result $B = \frac{1}{3}$ by the use of SU(3) and PCAC, or, equivalently chiral SU(3) x SU(3). However, an analysis by Wise and collaborators (30) indicated that the corrections to this approximation are of the order of 100%. A completely different approach (31) using a QCD sum rule technique has also yielded the result $B = \frac{1}{3}$. In contrast, applications of dispersion relations (32) have

![Figure 1](https://example.com/figure1.png)

*Figure 1* Box diagram yielding effective $\Delta S = 2$ operator. A sum must be made over nine choices for the pair of quarks such as $(t, t)$ $(t, u)$, etc.
yielded values of $B$ of unity or greater. Initial attempts to calculate $B$ on the lattice have not yet yielded definite results (33, 34).

The original calculation of $\Delta m$ of Gaillard & Lee (26) using $B = 1$ gave the correct value for $\Delta m$; indeed it provided a prediction for the charm quark mass $m_c$ before it was discovered. Because of the small value of $U_{ud}U_{ts}$ it is possible to ignore completely the $t$ quark in calculating $\Delta m$ provided $m_t \leq m_W$. Thus it might seem possible to use this calculation to demonstrate that $B \approx 1$. However, the quark model calculation using the box diagram only makes sense for large virtual momenta, or short distances, for which the quarks might be treated as free with QCD corrections treated perturbatively. There are, however, long-distance contributions associated with intermediate low-mass states such as $\pi$, $\eta$, and $2\pi$. Thus we must write (35)

$$\Delta m = \Delta m_{\text{box}} + D\Delta m,$$

where $D\Delta m$ represents the long-distance or dispersive contributions. Direct calculation (36–39) of $D$ is very sensitive to strong interaction form factors and SU(3) breaking; the result is probably of order unity but even the sign is uncertain. As a result we cannot calculate $\Delta m$ from the box diagram and we cannot use the empirical value of $\Delta m$ to determine $B$. Thus the best we can say is that probably $B \leq 1$, but there is no reliable calculation.

There are three $\Delta S = 1$ amplitudes that convert $s$ to $d$:

$$A(s \rightarrow u + u + d) \sim U_{us}U_{ud}^* \approx \lambda$$ 26a.

$$A(s \rightarrow c + c + d) \sim U_{us}U_{ud}^* \approx -i\eta \lambda^2 \lambda^5$$ 26b.

$$A(s \rightarrow t + t + d) \sim U_{us}U_{ud}^* \approx -\lambda^2 \lambda^5 (1 - \rho) - i\eta \lambda^2 \lambda^5,$$ 26c.

where we have kept the leading power of $\lambda$ for the real and imaginary terms. These are the terms that enter into K-decay amplitudes as well as into each leg of the box diagram. It follows by inspection that in this phase convention only Equations 26b and 26c violate $CP$ so that the $CP$-violating amplitudes satisfy the $\Delta I = \frac{1}{2}$ rule and $\text{Im} A_2 = 0$. As a result from Equation 11, we find

$$\sqrt{2}\varepsilon' = -0.045(\text{Im} A_0/\text{Re} A_0)e^{i\theta}.$$ 27.

Given the small experimental value of $\varepsilon'$ from Equation 14b, it then follows that we can ignore the second term in Equation 10 so that

$$\sqrt{2}\varepsilon = (m'/(\Delta m))e^{i\theta}.$$ 28.

In the box diagram for $\Delta m$ using either $c$ or $u$ in the legs, the result is
proportional to \( \lambda^2 \) from Equations 26. On the other hand, when we use the box diagram for \( m' \) we must pick up one \( CP \)-violating factor proportional to \( \eta \lambda^5 \) so that \( m' \sim \lambda^6 \eta \). It follows that

\[
m' / \Delta m \sim \lambda^4 \eta.
\]

Since \( \lambda^4 \approx 2.5 \times 10^{-3} \) we see that the small value of \( \epsilon \) in the KM model is explained by the hierarchy of mixing angles (Equation 19) without invoking a small value of the phase \( \delta \).

Substituting into Equation 28 the result of the box diagram (40) calculation of \( m' \) together with the experimental value of \( \epsilon_{\text{mix}} \), we find

\[
\epsilon e^{-i\theta} = 0.6 \lambda^4 A^2 \eta B [ -\eta_1 + \eta_2 \ln (m^2_3/m^2_1) + \eta_3 \lambda^4 A^2 (1 - \rho) (m^2_t/m^2_\tau)] \]

where we have used the value \( m_c = 1.5 \text{ GeV} \). The \( \eta_i \) are QCD correction factors (41) to the simple box diagram calculation given by \( \eta_1 = 0.7 \), \( \eta_2 = 0.4 \), \( \eta_3 = 0.6 \) for \( \Lambda^2_{\text{QCD}} = 0.01 \text{ GeV}^2 \). The last term in Equation 30 is an approximation for \( m_t \ll m_w \). The exact result (42) not using this approximation has been tabulated (43, 44); for values of \( m_t \) between 35 and 60 GeV we find a good approximation to the exact result is

\[
\epsilon e^{-i\theta} = 3.1 \times 10^{-3} A^2 \eta B [1 + \frac{1}{2} A^2 (1 - \rho) (m_t/42 \text{ GeV})^2].
\]

For the value \( B = \frac{1}{2} \) and \( m_t < 60 \text{ GeV} \) the experimental value of \( \epsilon \) cannot be fitted, given the experimental constraint of Equation 21. However, with \( B = 1 \) and \( m_t = 45 \text{ GeV} \), the KM matrix model can fit the value of \( \epsilon \) provided \( \eta \geq 0.4 \), that is, not too far from its present upper limit. From Equation 21 it then follows that \( |\rho| \leq |\eta| \) and so the phase \( \delta \) is large: \( |\tan \delta| \geq 1 \).

In obtaining Equation 30 we have assumed that \( m' \) can be calculated from the box diagram and so have neglected the long-range, or dispersive, contributions that we have claimed are so important for \( \Delta m \). Dispersive contributions to \( m' \) would correspond to \( CP \) violation in virtual decays such as \( K \rightarrow 2\pi \rightarrow \bar{K} \). However, given the small experimental value of \( \epsilon' \) we know that these virtual decay amplitudes have very little \( CP \) violation and so, although they are important for \( \Delta m \), one can show that they are unimportant for \( m' \) (45, 46).

To determine \( \epsilon' \) from Equation 27 we need to use the \( CP \)-violating amplitudes, Equations 26b and 26c. On the other hand, the natural way to obtain \( K \rightarrow \pi \pi \) in the quark model is through Equation 26a. If Equation 26a were the only diagram, one would obtain \( \epsilon' = 0 \). It was noted by Gilman & Wise (47) that Equations 26b and 26c may be significant even though they involve c and t quarks because they contribute to the transition \( s \rightarrow d + \text{gluons} \) and thence to the so-called penguin graphs of Figure 2.
These penguin graphs were first discussed (48) as a possible explanation of the $\Delta I = \frac{1}{2}$ rule since the transition $s \to d + \text{gluons}$ automatically has $\Delta I = \frac{1}{2}$. At present, most calculations (49–51) do not give a large enough magnitude for this contribution to explain the $\Delta I = \frac{1}{2}$ rule but it still seems likely that penguin graphs are the major source of $\text{Im} \ A_{0}$.

The calculation of the penguin graphs was recently summarized by Donoghue et al (52). The result can be written using Equation 27

$$\sqrt{2} \varepsilon' e^{-i\theta'} = -A^2 \lambda^4 \eta (0.045) [0.017 \ln (m_u^2/m_c^2)] P,$$

where the last factor $P$ is the matrix element of the four-quark operator derived from the penguin graph, and the $\ln$ factor is the coefficient calculated from the integral over the virtual $t$ and $c$ quarks. Again the main uncertainty is the value of the matrix element $P$, for which Donoghue et al give a range between 0.7 and 2.6, so that using the experimental value for $\varepsilon$,

$$|\varepsilon'/\varepsilon| = (1-4)A^2 \lambda^4 \eta = (3-10) \times 10^{-2} A^2 \eta.$$

An argument by Hagelin (53) summarized recently (45) gives a positive sign for $\varepsilon'/\varepsilon$ provided we accept the sign of $P$ given by the bag model. If we use the empirical constraints on $A$ and $\eta$, in particular the lower limit on $\eta$ needed to fit the value of $\varepsilon$, then Equation 33 yields

$$7.0 \times 10^{-3} > |\varepsilon'/\varepsilon| > 1.0 \times 10^{-3}.$$

The conclusion is that given the theoretical uncertainties in calculating the factors $B$ and $P$ the present value of $\varepsilon$ and limit on $\varepsilon'$ are both consistent with the KM model. In order to fit the value of $\varepsilon$ the $\text{CP}$-violating parameter $\eta$ must be close to the upper limit implied by Equation 21, which is derived from the limit on the decays due to $b \to u + e + \nu$. Thus a significant decrease on this limit could indicate trouble for the KM model. It follows from Equation 34 that the next set of experiments on $\varepsilon'$ (discussed in Section 2) have a good probability of finding a nonzero value if the KM model is correct.

Throughout we have discussed the KM model assuming only three

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Figure 2  Penguin diagram: $g$ is an intermediate gluon. A sum must be made over the three choices $u, c, t$. 

---
generations. The assumption of a fourth generation introduces three new mixing angles and two new phases. With the aid of these new parameters one may fit the experimental value of $\varepsilon$ without any significant constraint on the sign or magnitude of $\varepsilon'$. (54–57).

The conclusion that the phase $\delta$ in the KM matrix of Equation 18 must be large has led to speculation whether in some sense $CP$ violation is maximal (58–60). Definitions of maximal $CP$ violation in this context seem to be arbitrary; for example alternative definitions are $\rho = 0$ so that $U_{ab}$ has a phase $\delta$ of $90^\circ$ or $\rho = 1$ so that $U_{td}$ has the $90^\circ$ phase. Nevertheless the concept of maximal $CP$ violation may be useful in formulating ansatzes for the quark mass matrices (61–63).

### 4.3 $D$ and $B$ Systems

There exist three systems analogous to the $K^0$-$\bar{K}^0$ system made up of heavy quarks:

- $D^0$-$\bar{D}^0$  
  $D^0 = (c\bar{u})$

- $B^0$-$\bar{B}^0$  
  $B^0 = (b\bar{d})$

- $B^\prime_s$-$\bar{B}^\prime_s$  
  $B^\prime_s = (b\bar{s})$.

Systematic studies of $D^0$ and $B^0$ decays have been made in recent years (64, 65). At the time of this writing, there is no definitive observation of $B^\prime_s$ although its existence seems certain. For each of these systems we expect there will be mixing analogous to the $K^0$-$\bar{K}^0$ mixing so that there are two decaying states characterized by different lifetimes and by a mass difference $\Delta m$. No experimental evidence for such mixing has yet been found.

In the case of $K^0$ there is one predominant decay mode, $K^0 \rightarrow 2\pi$, for which the final state is $CP$ even so that $\Gamma_s \gg \Gamma_L$. On the other hand, there are many possible final states in $D$ and $B$ decays. As a result it is expected that the fractional difference in the lifetimes of the two eigenstates is very small in these systems. Possible experimental observations depend upon $\Delta m$, theoretical values of which are given in Table 2. For the $B^0$ system these are calculated using the box diagram, which compared to the box calculation for $\Delta m(K)$ yields

$$\Delta m(B^0) = \frac{\lambda^4 m_t^2}{\lambda^2 m_c^2} [((1 - \rho)^2 + \eta^2)] \frac{m_B}{m_K} \simeq 30 \left( \frac{m_t}{45 \text{ GeV}} \right)^2,$$

$$\Delta m(B^\prime_s)/\Delta m(B^0) = \lambda^{2/3}.$$  \hspace{1cm} 35a.

These results assume that in the B system analogue of Equation 24 $f_K = f_B$ and the B parameter has the same value. In fact different theoretical
estimates for $f_0^2$ and thus for $\Delta m$ in the B system differ by an order of magnitude (43, 66, 67). For the $D^0$ system the box diagram is inappropriate because the heaviest relevant virtual quark is the s quark so that dispersive terms (cf Equation 25) should dominate (68, 69). Only a very crude estimate of these is available; however, it is clear that $\Delta m(D^0)$ is very small. It is seen from Table 2 that mixing effects, which are proportional to a power of $(\Delta m/\Gamma)$, are expected to be negligible for $D^0$, may be significant for $B^0$, and are probably large for $B^0_s$.

The simplest way to search for mixing is to produce a particle-anti-particle pair, for example $B^0\bar{B}^0$, and then look for same-sign semileptonic decays. Since $b \rightarrow c l^-\bar{\nu}$ and $\bar{b} \rightarrow \bar{c} l^+\nu$ the observation of $l^- l^-$ or $l^+ l^+$ in the final state implies a flavor oscillation ($b \leftrightarrow \bar{b}$ or $\bar{b} \leftrightarrow b$) of one of the pair. For incoherent production of the pair, we obtain

$$R = \frac{N(l^+ l^+)+N(l^- l^-)}{N(l^+ l^-)} \approx \left(\frac{\Delta m}{\Gamma}\right)^2.$$ 

For the $D^0\bar{D}^0$ system experimental limits on $\Delta m/\Gamma$ are about 0.1 (70) whereas only crude limits exist for the $B^0$ system (71).

Analogous to Equations 26 we have

$$A(b \rightarrow u - \bar{u} + d) \sim U_{ub} U^*_{ud} \approx A \lambda^2 \rho - i \eta A \lambda^4,$$

$$A(b \rightarrow c + \bar{c} + d) \sim U_{ub} U^*_{cd} \approx - A \lambda^3,$$

$$A(b \rightarrow t + \bar{t} + d) \sim U_{tb} U^*_{td} \approx A \lambda^3(1 - \mu) + i \eta A \lambda^2,$$

$$A(b \rightarrow u - \bar{u} + s) \sim U_{ub} U^*_{us} \approx A \lambda^4 \rho - i \eta A \lambda^4,$$

$$A(b \rightarrow c + \bar{c} + s) \sim U_{cb} U^*_{cs} \approx A \lambda^2,$$

$$A(b \rightarrow t + \bar{t} + s) \sim U_{tb} U^*_{ts} \approx - A \lambda^2 + i \eta A \lambda^4.$$ 

Table 2 Estimates of $\Delta m$ and $\epsilon$ for D and B systems

| System | $\Gamma$ (sec$^{-1}$) | $\Delta m^b$ (sec$^{-1}$) | $\Delta m/\Gamma$ | $|\epsilon|$ |
|--------|----------------|-------------------|------------------|--------|
| $K^0$  | $10^{10}$      | $5 \times 10^9$   | 0.5              | $2 \times 10^{-3}$ |
| $D^0$  | $2 \times 10^{12}$ | $10^9\sim 10^{11}$ | $10^{-3}\sim 10^{-1}$ | $\sim 10^{-3}$ |
| $B^0$  | $10^{12}$      | $2 \times 10^{11}$ | $\sim 0.2$      | $\sim 0.3$ |
| $B^0_s$| $10^{12}$      | $3 \times 10^{12}$ | $\sim 3$        | $\sim 0.02$ |

*a For the $K^0$ this is $K^0$, we assume $\Gamma(B^0) = \Gamma(B^0)$.

*b Estimates of $\Delta m$ for the B systems differ.
These are all the amplitudes that enter into the legs of the box diagrams. [Note that the predominant amplitude contributing to $B$ decays is $A(b \rightarrow c + \bar{u} + d) = A\lambda^2$.] To find the $CP$ violation in the mass matrix we calculate $m'$ from the imaginary part of the box diagram. For $B^0$ since the virtual $t$ quark dominates one sees by inspection of Equation 36c that the box diagram contains as a factor $A\lambda^2(1 - \rho + i\eta)$ so that

$$m'/\Delta m = \frac{\eta(1 - \rho)}{(1 - \rho)^2 + \eta^2} = \frac{1}{2} \sin \Theta,$$

where $(\Theta/2)$ is the phase of $U_{td}^*$. One can show that if we consider decay amplitudes with little or no $CP$ violation (such as $b \rightarrow c + \bar{c} + s$)

$$\varepsilon(B^0) = \frac{A(B_L \rightarrow CP \text{ odd})}{A(B_H \rightarrow CP \text{ odd})} = i \tan \frac{\Theta}{2} = \frac{i\eta}{1 - \rho},$$

where $B_L$, $B_H$ are the $B^0$ eigenstates and $(CP \text{ odd})$ refers to a particular final state that is a $CP$ eigenstate such as $\psi + K_S$. In contrast to $K^0$ we find $\varepsilon(B^0)$ is not suppressed by powers of $\lambda$ but is expected to have a magnitude of 0.25 or more. Unfortunately, experiments to measure $\varepsilon(B^0)$ (72–74) are extremely difficult because they involve the observation of relatively rare exclusive decays. For the $B_s$ system inspection of Equation 37c shows $(m'/\Delta m) \approx i\varepsilon(B_s^0) = \eta \lambda^2$, so that the $CP$-violating effect is less than 3%. We note in passing that the product of the real times the imaginary piece of the amplitude is always proportional to $J$ (Equation 23) and so to $\lambda^6$. Thus only for $B^0$, for which $\Delta m$ is also of order $\lambda^6$, is the $CP$-violating parameter $\varepsilon$ not suppressed by powers of $\lambda$. However, values of $\varepsilon(B_s)$ unsuppressed by powers of $\lambda$ are found if one considers final states like $F^-K^+$ with branching ratios of order $\lambda^2$ (75).

One may also consider $CP$-violating effects that do not involve $B^0$-$\bar{B}^0$ mixing. An example would be to look for a difference between $B^+ + B^-$ decays (76–79). In this case it is necessary to look for an interference between two quark decay amplitudes since any one amplitude can always be chosen as real by a suitable phase convention. In addition the effect depends on final-state interactions since without these $B^+$ and $B^-$ decay rates to any channel are equal by $CPT$ invariance. Thus the calculation has a dynamical uncertainty, unlike Equation 38, which depends only on KM matrix elements. An example of interfering amplitudes would be

$$A(b \rightarrow c + \bar{u} + s) = A\lambda^3,$$
$$A(b \rightarrow u + \bar{c} + s) = A\lambda^3(\rho - i\eta),$$

which could produce rate differences in the decays $B^- \rightarrow D_s^0 + K_s + X^-$ and $B^+ \rightarrow D_s^0 + K_s + X^+$, where $D_s^0$ subsequently decays to $K_S$. The $CP$-
violating effect here is not suppressed by powers of $\lambda$ because the decay rates themselves are of order $\lambda^5$ in contrast to the dominant $B$ decays, which are of order $\lambda^4$.

It is clear that the KM theory predicts interesting large $CP$-violating effects in $B$ decays. However, in practice these effects will be very difficult to detect.

5. ALTERNATIVE GAUGE MODELS

Electroweak gauge models have the following general form of Lagrangian

$$L = G(W, \psi, \phi) + H(\phi) + Y(\psi, \phi),$$

where $\psi$, $\phi$, and $W$ stand for fermion, scalar boson, and gauge vector boson fields, respectively. The term $G$ includes all the kinetic energy terms plus the interactions required by the gauge principles. $G$ is completely determined by the particle content and the gauge group and is necessarily $CP$ invariant. The only sources for $CP$ violation are (a) Complex Yukawa couplings in $Y$, (b) Complex coefficients in $H$, and (c) Complex values for the vacuum expectation values (VEV) of some of the scalar particles so that even though $L$ is $CP$ invariant the vacuum and therefore the physics is not (80). The source (c) is referred to as spontaneous breaking of $CP$.

In the standard electroweak $SU(2) \times U(1)$ model as originally discussed with two generations of quarks there was no possibility for $CP$ violation. With only one Higgs doublet $H$ is $CP$ invariant by hermiticity and the VEV of $\phi$ can be chosen real as a result of the gauge symmetry. Any complex couplings in $Y$ can only show up in the mass matrix and thence as complex phases in the unitary quark mixing matrix $U$. However, with only two generations phase transformation like $s \rightarrow se^{\alpha}$ (see Equations 15) can be used to remove these phases. Thus it was clear that some extension of the model was required.

It was first pointed out by Kobayashi & Maskawa (8) that a simple extension was to have three generations of quarks. In this case the complex couplings in $Y$ reveal themselves by the presence of a single nonvanishing phase in the mixing matrix $U$, as discussed in Section 4. With the discovery of the third generation $b$ quark this became the standard model of $CP$ violation. In this model the origin of $CP$ violation lies in the same mysterious Yukawa interactions that determine the quark mass spectrum and mixing angles.

From one point of view one may say that in the KM model $CP$ violation is not a fundamental symmetry of nature at all. As soon as the quark content allows $CP$ violation within the framework of the gauge theory, it occurs and it is not small. In contrast, theories with spontaneous $CP$
violation start out with CP invariance as a fundamental invariance of the Lagrangian. While the VEVs break this symmetry, one would expect at energies much larger than the magnitudes of the VEVs that the CP symmetry would be restored; in this sense the CP violation is soft. An argument against soft CP violation is that one may desire to use CP violation at very high energies in the early universe as a mechanism for establishing the baryon asymmetry of the universe (81). It has been pointed out that even with spontaneous CP violation it is possible that not all CP-violating effects disappear at high energies (82). In any case it is difficult to relate in any direct way the low-energy CP violation in the $K^0$ system to the baryon asymmetry.

In this section we consider alternatives to the Kobayashi-Maskawa model. These require an enlargement of the gauge group or an expansion of the Higgs sector. The emphasis is on spontaneous CP-violation models, in part because these are a more restrictive class, in part because they provide a contrast with the KM model. All the models provide new mechanisms for CP violation in $K^0$ decays; it should be noted that in many models these mechanisms supplement rather than replace the KM mechanism. Indeed, unless a model contains some symmetry forcing the KM phase $\delta$ to zero there may always be some contribution to CP-violating effects from the KM mechanism.

5.1 Superweak Two-Higgs Model

It was pointed out by T. D. Lee (80) that if there are two Higgs doublets $\phi_1, \phi_2$ then for a suitable range of parameters in $H(\phi_1, \phi_2)$ there will be a CP-violating phase in VEVs:

$$\langle \phi_1 \rangle = v_1 \exp(i\alpha_1), \quad \langle \phi_2 \rangle = v_2 \exp(i\alpha_2).$$

While either $\alpha_1$ or $\alpha_2$ could be set to zero, the relative phase $(\alpha_1-\alpha_2)$ is significant. A major problem in theories with two doublets is the presence of flavor-changing neutral Higgs couplings. (With a single Higgs the Higgs coupling is proportional to the mass matrix and thus diagonal with respect to mass eigenstates.) The resultant Higgs exchange produces $\Delta S = 2$ at tree level and thus tends to give too large a contribution to $\Delta m$ of the $K^0$ system. Two ways to avoid this are commonly discussed. The first is to give a large mass ($\sim 10$ TeV) to the flavor-changing neutral Higgs boson. However, there are several arguments that yield upper limits to Higgs boson masses in the $SU(2) \times U(1)$ model of the order of a few hundred GeV (83, 84); these limits hold for and may be even more restrictive (85, 86) for each of the bosons in a two-Higgs model. The second way is to adjoin a discrete symmetry to the theory such that for each type (weak
isospin projection) of fermion only $\phi_1$ or $\phi_2$ couples, but not both. However, this discrete symmetry when applied to the Higgs potential rules out terms like $\phi_1 \phi_1 \phi_1 \phi_2$, which are needed in order to get a significant nonzero value of $(\alpha_1-\alpha_2)$ when the potential is minimized. This relation between spontaneous $CP$ violation and flavor-changing neutral currents was recently reviewed by Branco et al (87).

The only possibility remaining is to fine-tune the parameters to give very small flavor-changing couplings so that $\Delta m$ is of the order of the experimental value even for a Higgs mass of a few hundred GeV. One also fine-tunes the Higgs potential to give a small value for the spontaneous $CP$ violation. It follows that the phase $\delta$ in the KM matrix is very small and essentially all the $CP$ violation comes from the $\Delta S = 2$ Higgs boson exchange. This then is the simplest realization of the superweak mechanism. It is not a true superweak theory in that the major contribution to $\varepsilon'$ still comes from the very small KM phase. It is also unappealing that the superweak character derives from fine tuning.

5.2 Other Superweak Models

Most superweak models are associated with a new mass scale $M$. This requires extending the group $SU(2) \times U(1)$ to $SU(2) \times U(1) \times G$ where $M$ is related to the breaking of $G$. In such models $CP$ violation may arise from the exchange of heavy neutral gauge bosons that change flavor and thus allow $\Delta S = 2$ at tree level. A particularly popular idea (88–92) is that the group $G$ is a horizontal symmetry relating different flavors. Some of the general features of such a model are discussed by Decker et al (93). Such models, however, have a complicated Higgs structure with $CP$ violation associated with Higgs exchange as well as gauge boson exchange; thus there can be many variations on this theme.

Some exotic possibilities arise if the group $G$ includes the SU(3) of color as a subgroup, as occurs in grand unified theories. Then one finds among the Higgs bosons [for example, in the $126$ of SO(10)] some that transform like diquarks. Calling these $H_6$ one may have the tree-level graph $s + s \rightarrow H_6 \rightarrow d + d$ providing a superweak $\Delta S = 2$ transition (94, 95). The boson $H_6$ may also be involved in neutron-antineutron oscillations (96).

While some superweak models may have interesting implications for new physics at high energies or for rare processes, in general they all yield the same negative results as far as prospective searches for further $CP$ violation are concerned.

5.3 Weinberg Three-Higgs Model

Weinberg (97) suggested a model of $CP$ violation in which flavor-changing neutral Higgs couplings were forbidden by a discrete symmetry, but $CP$
violation still occurred as a result of the couplings among three-Higgs boson fields. While this model can be formulated by setting coefficients in $H$ complex, we assume here that the $CP$ violation arises spontaneously. In this case Branco (98) has shown that the KM matrix $U$ is real so that $CP$ violation arises only as a result of the exchange of physical Higgs particles. Of the original three charged fields two emerge as physical particles; the major $CP$-violating effect can be identified as a phase in the mixing matrix that diagonalizes the charged Higgs mass matrix (99, 100).

Like the $W^+$ the charged Higgs boson exchanges cause a flavor change $\Delta S = 1$. The calculation of $\varepsilon$ in this model involves calculating the imaginary part of box diagrams like Figure 1 with one or both $W$s replaced by charged Higgs bosons to obtain the parameter $m'$. In addition one must calculate $\text{Im} A_0$, which is dominated by the penguin graph of Figure 2 with the $W$ replaced by charged Higgs bosons. One can then use Equation 10 with the empirical values of $\Delta m$ and $\text{Re} A_0$ to obtain an equation for $\varepsilon$. While the model contains a number of parameters one finds even if one tries to maximize the $CP$-violating phase that the charged Higgs boson mass must be considerably smaller than $m(W)$ because of the weakness of Higgs couplings. Thus if the model were correct, charged Higgs bosons should be discovered in future high-energy experiments.

When this calculation was first carried out (101, 102) it was found that the $\text{Im} A_0$ term in Equation 10 dominated

$$\frac{(\text{Im} A_0/\text{Re} A_0)}{(m'/\Delta m)} = 5.2 \left( \ln \frac{m_H^2}{m_e^2} - \frac{3}{2} \right) > 15,$$

where $m_H$ is the charged Higgs mass and $m_H \geq 15$ GeV from experiments. As a result from Equation 27 $\varepsilon'/\varepsilon \approx -w = -0.045$, which is much larger in magnitude than the experimental result (Equation 14b). Thus the model appeared to be ruled out. Factors as large as 10 in Equation 41 that might arise by changing approximations or varying parameters would not change this conclusion. However, it was pointed out recently (103–105) that the original calculations are probably incorrect. On the one hand, the main contribution to $m'$ is probably not the short-distance contribution from the box diagram but a long-distance term associated with the virtual transition $K^0 \rightarrow \eta' \rightarrow \bar{K}^0$. On the other hand, the evaluation of the matrix element needed to obtain $\text{Im} A_0$ incorrectly used a naive version of PCAC. The unfortunate conclusion is that both $m'$ and $\text{Im} A_0$ are hard to calculate and very sensitive to SU(3) breaking. Two estimates are (104, 105) $\varepsilon'/\varepsilon = -0.006$ (Donoghue, Holstein) and $\varepsilon'/\varepsilon = -0.016$ (Sanda), but both have large uncertainties. Thus this model remains a viable alternative.
5.4 $SU(2)_L \times SU(2)_R \times U(1)$ Model

One of the simplest ways to enlarge the gauge group is to add an $SU(2)_R$ gauge interaction mediated by bosons $W_R$ that couple to right-handed currents. The resultant group is labeled $SU(2)_L \times SU(2)_R \times U(1)$ where $SU(2)_L$ is the $SU(2)$ associated with the usual bosons $W^\pm$ and $Z^0$. Naturally the mass of $W_R$, which is associated with the breaking scale of $SU(2)_R$, is much larger than the mass of $W^\pm$, although an interesting case is that in which $m(W_R) \approx 20-200 \ m(W)$. It is generally assumed that parity is a spontaneously broken symmetry in this model and is restored for energies well above $m(W_R)$. This model has been extensively reviewed by Mohapatra and others (106, 107).

Mohapatra & Pati (108) pointed out that with two generations this model could allow $CP$ violation. The mass matrix for quarks is in general diagonalized by a biunitary transformation

$$U^d L M_d U^d_R = M^d_{\text{diag}},$$
$$U^u L M_u U^u_R = M^u_{\text{diag}},$$

where $M_u$ ($M_d$) are the up (down) mass matrices. The KM matrix determining the couplings to $W$ is

$$U = (U^u_L)^+ U^d_L.$$  \hspace{1cm} (42b)

There is a similar matrix $U_R$ for the coupling to $W_R$. With two generations a phase convention can be chosen so as to make $U$ real but then $U_R$ will contain complex elements. As a result $CP$ violation will occur once $W_R$ exchanges are included.

The box diagram, Figure 1, in which one $W_L$ is replaced by a $W_R$, is of particular importance. In the first place, it contributes to $\Delta m$ for the $K^0$ system. As a result of enhancement factors that arise when $W_L$ is replaced by $W_R$, the box calculation yields the result (109) $m(W_R) > 20 m(W_L) \sim 2 \ \text{TeV}$ in order that $\Delta m$ not be too large. Because $U_R$ contains complex phase factors the box diagram also contributes to $m'$ and thus to $\epsilon$. If we assume $m(W_R)$ is approximately equal to the lower limit of 2 TeV so that the $CP$-conserving part of the left-right box is approximately equal to $\Delta m$, the relevant phase factors in $U_R$ must be of the order $10^{-3}$ in order to give $(m'/\Delta m) \sim \epsilon = 2 \times 10^{-3}$. This is in contrast to the KM model where a large phase factor is needed in $U$ in order to get a large enough value of $\epsilon$. The reason, of course, is that $CP$ violation in this model occurs with only two generations whereas in the KM model $CP$ violation is suppressed because mixing with the third generation is essential.
Quantitative conclusions about this model are difficult for several reasons. In general the model has many CP-violating phases: for three generations there are six phases in $U_R$ as well as a phase describing the mixing of $W_L$ and $W_R$. In addition with three generations there is the standard KM $CP$ violation in addition to that due to $W_R$ exchange. There is also $CP$ violation of a superweak variety because the model necessarily contains flavor-changing neutral Higgs bosons. Thus it is necessary to consider specific forms of the model.

Chang (110) analyzed a model assuming spontaneous $CP$ violation so that the only phases were those occurring in Higgs boson vacuum expectation values. The $SU(2)_L \times SU(2)_R \times U(1)$ model necessarily has a Higgs boson representation $\Phi$ transforming as $(2,2,0)$ corresponding to two $SU(2)_L$ doublets $\phi_1$ and $\phi_2$. Assuming $\langle \phi_1 \rangle = K$ and $\langle \phi_2 \rangle = K'e^\alpha$, we find the phase $\alpha$ showing up both in $U_R$ and in the $W_L-W_R$ mixing and it also determines the phase in the KM matrix $U$. If $m(W_R) \approx 2$ TeV then the KM phase turns out to be very small so that the value of $\epsilon$ can be calculated from the left-right box if we neglect superweak Higgs exchange. The calculation of $\epsilon'/\epsilon$ in this model depends once again on very uncertain matrix elements. An analysis by Ecker & Grimus (111) yields the order-of-magnitude result $\epsilon'/\epsilon \approx \pm 5 \times 10^{-3}$, where the sign depends on the details of the model. However, if the flavor-changing neutral Higgs boson in this model is not chosen to be very massive, it provides a superweak contribution to $\epsilon$ and no contribution to $\epsilon'$ and so the value of $|\epsilon'/\epsilon|$ is expected to be lower than the estimate above. It is not possible to obtain spontaneous $CP$ violation in the $SU(2)_L \times SU(2)_R \times U(1)$ model with the minimum Higgs sector but it can be done with additional Higgs bosons (112, 113).

5.5 Supersymmetric Models

Supersymmetric models contain a set of new fermions (such as gluinos and photinos) for every standard boson and new bosons (such as squarks) for every standard fermion. In a “minimal” supersymmetric $SU(2) \times U(1)$ model the only $CP$ violation in $K^0$ decay comes from the same complex couplings in $Y$ that produce the KM $U$ matrix. However, because of supersymmetry breaking this $CP$ violation shows up in new ways. In particular it has been shown that there exists a flavor-changing quark-squark-gluino coupling (114) described by the standard KM $U$ matrix. (This is in contrast to normal QCD gluon coupling, which is diagonal.) Thus $\epsilon$ gets additional contributions from box diagrams in which the Ws are replaced by gluinos and the intermediate quarks by squarks. It is then possible to fit the data with a lower value of $\eta$ and hence from Equation 33 with a lower prediction for $\epsilon'$ (115, 116).
In general supersymmetric models contain additional CP-violating phases beyond that in the KM matrix. While these may not significantly affect the analysis of \( \epsilon \) and \( \epsilon' \) they may be very important for other observables such as dipole moments and \( B^0 \) mixing (117, 118).

6. OTHER OBSERVABLES

In order to distinguish between models of CP violation it is necessary to have additional experimental information. As emphasized in Section 2, a nonzero value of the parameter \( \epsilon' \) in \( K^0 \to \pi \pi \) decay could rule out the superweak model. The possibility of detecting CP violation in the heavy quark systems was discussed in Section 4. Here we discuss additional observables of current theoretical and experimental interest.

6.1 Neutron Electric Dipole Moment

A system with an electric dipole moment \( d \) has an interaction energy \( ds \cdot E/s \) where \( E \) is the external electric field and \( s \) is the spin. Such an interaction violates both parity \( P \) and time reversal \( T \) if the system is an elementary particle, or, more generally, a nondegenerate eigenstate. Since we know weak interactions violate parity, the search for nonzero electric dipole moments is essentially a search for a \( T \)-violating weak effect. Since it is easiest to study a neutral system the greatest effort has been devoted to the neutron. The experiments were recently reviewed by Ramsey (119); the present experimental limit is (120)

\[
\frac{d_n}{e} < 6 \times 10^{-25} \text{ cm.}
\]

The significance may be evaluated by comparing this to \( \mu_n \), the neutron magnetic moment

\[
d_n/\mu_n < 2 \times 10^{-11}.
\]

Given that we expect a factor of about \( 10^{-6} \) in order to have weak parity violation, this represents a significant limit on \( T \) violation.

Many milliweak models predict values (see Table 3) of \( d_n \) in the neighborhood of the present limit. The "predictions" are at best order-of-magnitude values because of both calculational uncertainties and the dependence on model parameters that are not constrained from the \( K^0 \) data. The KM model, however, predicts a much smaller value because the electric dipole moment vanishes in the lowest (one-loop) order; in fact the quark moments vanish even in a two-loop calculation (121). As a result in the KM model (122) as in a generic superweak model (123) the value of \( d_n \) is governed by \( \Delta S = 0 \) graphs analogous to the \( \Delta S = 2 \) graphs.
contributing to $m'$ for the $K^0$ system. This suggests a rough order-of-magnitude value of $10^{-30}$ to $10^{-32}$, but in specific superweak models the value can be much lower (88).

It must be emphasized, however, that a nonzero value of $d_n$ could always be blamed on "strong CP violation". This is the $P$-odd $T$-odd $\Delta S = 0$ interaction expected in QCD if CP is violated anywhere in the Lagrangian. Unfortunately, its magnitude (measured by the parameter $\Theta_{\text{QCD}}$) is fundamentally incalculable in most models, including the KM model (124). Since the neutron electric dipole moment is in practice the most sensitive probe of $\Theta_{\text{QCD}}$, the present limit of Equation 43 already tells us that strong CP violation will not be seen in any other foreseeable experiment. Thus a nonzero value of $d_n$ cannot be unambiguously related to weak CP violation and so would not rule out either the KM or superweak models. In fact there exist superweak models in which $\Theta_{\text{QCD}}$ is calculable and that predict values of $d_n$ as large as $10^{-25}$e·cm (95). However, if prospective experiments should lower the limit on $d_n$ by an order of magnitude or more, some alternative milliweak models may appear to be ruled out barring an accidental cancellation between the strong and weak contributions.

### 6.2 Electric Dipole Moments of Atoms

Atomic physics experiments can place extremely good limits on the electric dipole moments of neutral atoms. A recent experiment (125) on the ground state of xenon gives $d_{^{129}\text{Xe}}/e \leq 10^{-26}$ cm, and much more sensitive experiments are underway. There are several possible contributions to $d$ for the atom: (a) an intrinsic electric dipole moment or magnetic quadrupole moment of the nucleus, (b) a $T$-violating electron-quark interaction, or (c) an intrinsic electric dipole moment $d_e$ of the electron. A detailed atomic physics calculation is necessary to extract these. Because of enhancement factors (126–128) it is possible that these experiments can eventually rival the measurement of $d_n$ in sensitivity. In principle the measurement of both $d_n$ and nuclear $T$-odd moments might allow for a discrimination between strong and weak $CP$ violation.

### Table 3 Theoretical estimates for CP-violating observables

<table>
<thead>
<tr>
<th>Model</th>
<th>$\epsilon'/\epsilon$</th>
<th>$d_n$</th>
<th>$d_e$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superweak</td>
<td>$\sim 10^{-11}$</td>
<td>$&lt; 10^{-30}$</td>
<td>$&lt; 10^{-30}$</td>
<td>$\sim 10^{-14}$</td>
</tr>
<tr>
<td>KM</td>
<td>$(1–7) \times 10^{-3}$</td>
<td>$10^{-31}–10^{-32}$</td>
<td>$\sim 0$</td>
<td>$\sim \epsilon'$</td>
</tr>
<tr>
<td>Weinberg Higgs</td>
<td>$\sim 10^{-2}$</td>
<td>$10^{-24}–10^{-25}$</td>
<td>$10^{-32}$</td>
<td>$\sim \epsilon'$</td>
</tr>
<tr>
<td>SU(2)$_L \times$ SU(2)$_R \times$ U(1)$_A$</td>
<td>$10^{-2}–10^{-3}$</td>
<td>$10^{-25}–10^{-26}$</td>
<td>$10^{-29}$</td>
<td>$&lt; \epsilon'$</td>
</tr>
<tr>
<td>Experimental value</td>
<td>$(-3 \pm 6) \times 10^{-3}$</td>
<td>$&lt; 6 \times 10^{-25}$</td>
<td>$&lt; 2 \times 10^{-24}$</td>
<td>$\leq 0.1$</td>
</tr>
</tbody>
</table>

*Our estimates refer to the model of (110). The values of $\epsilon'/\epsilon$ and $d_n$ are from (111).
The electric dipole moment of the electron $d_e$ could be as large as $d_n$ in models in which, in contrast to the KM model, $CP$ violation is allowed in the lepton sector. Furthermore $d_e$ has no strong $CP$ contribution to confuse the interpretation. H. Y. Cheng (129) has calculated $d_e$ in various models. In particular, in the $SU(2)_L \times SU(2)_R \times U(1)$ model an electron dipole moment is expected as a result of $W_L-W_R$ mixing together with the existence of a neutrino mass term. The maximum possible value is of the order $10^{-26}e\text{cm}$; however, in the simplest version with the $CP$ violation fitted to the $\epsilon$ parameter in $K^0$ decay the value is $10^{-29}$. The best published experimental limit is

$$\frac{d_e}{e} < 2 \times 10^{-24} \text{ cm},$$

deduced from older measurements of the electric dipole moment of the metastable state of the xenon atom (130). Planned experiments aim to reduce the limit of Equation 44 by several orders of magnitude. Unfortunately, even with such improvement most models do not predict that a nonzero value will be detected (see Table 3).

6.3 Time-Reversal Violation in Semileptonic Decays

Time reversal in weak decay processes can show up in $T$-odd correlations. In nuclear beta decay the simplest is a dependence of the rate on $\sigma_N \cdot \hat{p}_e \times \hat{p}_\nu$, where $\sigma_N$ is the nuclear spin, $\hat{p}_e$ the electron direction and $\hat{p}_\nu$ the neutrino direction. While such a dependence can occur as a result of the final-state electromagnetic interaction in the absence of $T$ violation, this dependence is very small and can be predicted. The experimental value of this correlation can be expressed as a relative phase $\phi_{AV}$ between $g_V$ and $(-g_A)$; for the neutron (9) $\phi_{AV} = (2 \pm 3) \times 10^{-3}$ radians, consistent with zero. A nonzero phase of this sort is expected in the $SU(2)_L \times SU(2)_R \times U(1)$ model as a result of $W_L-W_R$ mixing and could be as large as $10^{-3}$ (131). In the minimal model constrained to fit the $K^0 CP$ violation (110), however, the predicted value is only $10^{-5}$ to $10^{-6}$.

A similar correlation $\sigma_\mu \cdot \hat{p}_\mu \times \hat{p}_\nu$ has been searched for in the decay $K \to \pi\mu\nu$ where $\sigma_\mu$ is the muon polarization and $\hat{p}_\mu$ the muon direction. This result can be expressed in terms of $\text{Im} \xi$, where $\xi = f_-/f_+$ is the ratio of the two vector form factors. Combining the results from $K_L$ and $K^+$ decay (including the small theoretical correction for the electromagnetic interaction in the case of $K_L^0$ decay): $\text{Im} \xi = -0.01 \pm 0.02$. Since this is a purely vector decay it does not have a contribution from $W_L-W_R$ mixing as does $\phi_{AV}$. On the other hand, there could be a significant effect in the Weinberg Higgs model because there can be interference between scalar Higgs exchange (proportional to $m_\nu$) and the usual $W$ exchange. The
numerical result depends on parameters of the model and is probably of or less than $10^{-3}$ (132).

6.4 Nonleptonic Decays

It is natural to look for $CP$ violation in strange particle nonleptonic decays other than $K^0 \to 2\pi$. The goal is to find a clear signal of $CP$ violation in a decay amplitude. However, there exists already a severe limit on the $CP$-violating decay amplitude for $K^0 \to 2\pi$ given by $|\varepsilon'| \leq 2 \times 10^{-5}$. Future experiments discussed in Section 2 aim to bring this down to two parts per million. No other prospective experiment can possibly do nearly as well. In most models, including the KM model, $\varepsilon'$ is naturally suppressed by the factor $w (=0.045)$ as a result of the $\Delta I = \frac{1}{2}$ rule. Thus it is reasonable to hope that some $CP$-violating amplitudes may be 20 to 25 times as large as $\varepsilon'$, but even then it is hard to find an experiment as sensitive as the measurement of $\varepsilon'$.

One possibility is to look at other final states in $K^0$ decay. Thus in analogy with $\eta_{++}$ and $\eta_{00}$ one defines parameters for the $3\pi$ final states:

$$\eta_{000} = \frac{A(K_S \to 3\pi^0)}{A(K_L \to 3\pi^0)}$$
$$\eta_{+-0} = \frac{A(K_S \to \pi^+\pi^0\pi^-)}{A(K_L \to \pi^+\pi^-\pi^0)}$$

While the decay $K_S \to 3\pi^0$ is direct evidence for $CP$ violation (because all $3\pi^0$ spin-zero states are $CP$-odd), there is a $CP$-invariant decay $K_S \to \pi^+\pi^-\pi^0$ but it is inhibited by angular-momentum barriers and the $\Delta I = \frac{1}{2}$ rule. In any case the parameters $\eta_{000}$ and $\eta_{+-0}$ are measured in interference experiments ensuring that $K_L$ and $K_S$ go to the same final state, presumed to be the $I=1$ $CP$-odd state. Such an interference effect is a direct sign of $CP$ violation (133). From Equations 7 and 9 and $CPT$ we can write for either $\eta_{+-}$ or $\eta_{000}$

$$\eta_{3\pi} = \varepsilon + i \frac{\text{Im} \ A_{3\pi}}{\text{Re} \ A_{3\pi}} = \varepsilon + i \left( \frac{\text{Im} \ A_{3\pi}}{\text{Re} \ A_{3\pi}} - \frac{\text{Im} \ A_0}{\text{Re} \ A_0} \right) \equiv \varepsilon + \varepsilon'_{3\pi},$$

where we assume only a single final $3\pi$ state. The parameter $\varepsilon'_{3\pi}$, analogous to $\varepsilon'$ of Equation 11, provides unambiguous evidence for $CP$ violation in the decay amplitude. Using soft-pion arguments one can show in the KM model (134) and also in the Weinberg model (132) that $\varepsilon'_{3\pi}$ is of the same order of magnitude as $\varepsilon'$. In the $SU(2)_L \times SU(2)_R \times U(1)$ model in general $\varepsilon'_{3\pi}$ might be much larger than $\varepsilon'$ although a specific calculation by Chang (110) gives a very small value. Unfortunately it is difficult experimentally
to measure $\eta_{+0}$ or $\eta_{000}$ precisely enough to see an effect of order $\epsilon$, much less to see a small difference of the order $\epsilon'$. At present there are only poor limits of the order 0.1 on either (9), although an ongoing experiment on $\eta_{+0}$ is planned to reach a level of 0.003 (135).

Another possibility that has been discussed is the decay $K^0 \rightarrow \gamma\gamma$ (133, 136). The one decay that has been observed is $K_L \rightarrow \gamma\gamma$ with a branching ratio of $5 \times 10^{-4}$. Presumably this goes primarily to the $CP$-odd state ($\gamma\gamma-$), the same final state occurring in $\pi^0$ decay. It is expected, but not yet observed, that $K_s$ decays to the $CP$-even state ($\gamma\gamma+$) with a comparable but somewhat larger rate. A study of interference effects in $K^0(\bar{K}^0) \rightarrow \gamma\gamma$ could then measure

$$\eta_- = \frac{A(K_s \rightarrow \gamma\gamma-)}{A(K_L \rightarrow \gamma\gamma-)} \equiv \epsilon + \epsilon'_\gamma.$$

Once again, $\epsilon'_\gamma$ is an unambiguous measure of a $CP$-violating effect. (There is also an analogous parameter $\eta_+$ corresponding to the $\gamma\gamma+$ final state, but theoretical arguments suggest this is almost exactly equal to $\epsilon$.) A rough estimate (136) gives $\epsilon'_\gamma \approx 30\epsilon'$ for the KM model. The possibility of measuring $\eta_-$ using tagged $K^0$ from $\bar{p}p$ collisions at the CERN facility LEAR has recently been discussed (138).

Leaving $K^0$ decay there are two possible types of observables: (a) a difference in decay rates or spectra between particle and antiparticle such as $\Lambda$ and $\bar{\Lambda}$, (b) a $T$-odd decay correlation. The first of these is dependent upon and proportional to final-state interactions, since in their absence particle and antiparticle decay identically from $CPT$ invariance. In contrast a "$T$-odd correlation" can be produced by final-state interactions even if $T$ invariance is good so that $T$-odd correlations are useful in nonleptonic decays only to the extent that the final-state phase shifts are well measured.

Comparing $K^+$ and $K^-$ one may look for a difference in the branching ratios for the $\tau$-decay mode (9)

$$\frac{\Gamma(K^+ \rightarrow \pi^+\pi^+\pi^-) - \Gamma(K^- \rightarrow \pi^-\pi^-\pi^+)}{average} = (0.7 \pm 1.2) \times 10^{-3}.$$  

In the absence of $\Delta I = \frac{1}{2}$ transitions and neglecting quadratic energy dependence over the Dalitz plot, one expects in general that this difference vanishes (139). In principle it is better to search for a difference in the slope parameters $a$ that characterize the energy variations over the Dalitz plot. Present data (9) give

$$\frac{a(\pi^+\pi^+\pi^-) - a(\pi^-\pi^-\pi^+)}{average} = (-7 \pm 5) \times 10^{-3}.$$
We turn now to hyperon decays, using the $\Lambda$ for illustrative purposes
(140, 141), although similar considerations hold for $\Sigma$ and $\Xi$ decays. The
decays $\Lambda \to p\pi^-$ and $\Lambda \to nn^0$ are normally analyzed in terms of the
amplitudes $A_s(1) \exp (i\delta_s)$, $A_p(1) \exp (i\delta_{1p})$, $A_s(3) \exp (i\delta_3)$, and $A_p(3) \exp (i\delta_{3p})$, where $(s, p)$ indicate the final orbital angular momentum and $(1, 3)$
the isospin $I = \frac{1}{2}, \frac{3}{2}$. The final-state pion-nucleon phase shifts in these
states are indicated by $\delta$. In the absence of $CP$ violation the $A$'s are all
real; possible $CP$-violating phases $\theta_1, \theta_3, \phi$ are defined by

$$
A_s(1)/A_p(1) = |A_s(1)/A_p(1)| \exp (i\theta_1)
$$

$$
A_s(3)/A_p(3) = |A_s(3)/A_p(3)| \exp (i\theta_3)
$$

$$
A_s(3)/A_s(1) = |A_s(3)/A_s(1)| \exp (-i\phi).
$$

The simplest measure of $CP$ violation is the rate difference

$$
\frac{\Gamma (\Lambda \to p\pi^-)}{\text{sum}} \times \sin \phi \sin (\delta_1 - \delta_3) \approx 7 \times 10^{-3} \sin \phi,
$$

where the last equality uses experimental values. This rate difference is
suppressed both by the $A \Delta = \frac{1}{2}$ rule and the smallness of the phase shifts.
As a result in the KM model the magnitude is expected to be less than $|\epsilon'|$.

An alternative is to compare the decay asymmetry parameters $\alpha$ for the
two decays; in the absence of $CP$ violation $\alpha(\Lambda)$ and $\alpha(\bar{\Lambda})$ have opposite
signs. Neglecting the $I = \frac{1}{2}$ final state the $CP$-violating effect is

$$
\frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}} \approx -\tan \theta_1 \tan (\delta_1 - \delta_{1p}) \approx -0.1 \tan \theta_1.
$$

A still more difficult parameter to measure is $\beta$, which is the coefficient of
the $T$-odd correlation $\sigma_\Lambda \cdot \sigma_p \times k$ where $\sigma_\Lambda, \sigma_p$ are the $\Lambda$ $(p)$ polarizations
and $k$ is the relative momentum in the final state. A nonzero value of $\beta$
can arise from both $CP$ violation or final-state interactions:

$$
\beta \approx \alpha \tan (\delta_{1p} - \delta_s - \theta_1) \approx 0.08,
$$

where the last equality neglects the $CP$ violation. Even if $\theta_1$ were as large
as $\epsilon$ it would only change $\beta$ by 1%. To get rid of the final-state interaction
effects one may consider comparing $\beta$ for $\Lambda$ and $\bar{\Lambda}$ decays (142) yielding

$$
(\beta + \bar{\beta}) = 2 \alpha \tan \theta_1 \approx \tan \theta_1.
$$

Estimates for $\theta_1$ are $2 \times 10^{-4}$ in the KM model and $10^{-3}$ in the Weinberg
Higgs model (142). This provides an example of an effect that could be
much larger than $\epsilon'$ but obviously still extremely hard to measure.
7. CONCLUSION

The complete Hamiltonian describing elementary particle interactions violates $CP$ and $T$ invariance. So far the only measure of this violation is the parameter $\varepsilon$ in $K^0$ decay with a magnitude of $2 \times 10^{-3}$. It is not surprising that there exist many viable models of $CP$ violation, all of which contain one or more free parameters fitted to the value of $\varepsilon$.

It is possible within the minimal version of the standard $SU(2) \times U(1)$ theory to explain the observed $CP$ violation in terms of the phase $\delta$ (or the parameter $\eta$) in the $3 \times 3$ KM matrix $U$. In this model the origin of $CP$ violation resides in the arbitrary Yukawa coupling of the quarks to the Higgs boson and so is intimately related to the general problem of the mass matrix.

In order to fit the value of $\varepsilon$ it is necessary that the phase $\delta$ be of the order $45^\circ$ or more; the smallness of the value of $\varepsilon$ results from the hierarchy in magnitudes of the mixing angles in $U$. The fit to the value of $\varepsilon$ leads to the predictions that $\varepsilon'/\varepsilon$ should be observable in prospective experiments on $K^0$ decay and that the rate of decay $b \to u + e + \nu$ should be within a factor of two of its present upper limit. Both these predictions are quantitatively uncertain because of the difficulty of determining the hadronic matrix elements parameterized by $B$ and $P$ (Equations 24 and 32). The large value of the phase $\delta$ implies that $CP$ violation should be large for $B^0 - \bar{B}^0$ mixing; unfortunately, observation of this $CP$ violation requires a very difficult study of exclusive decays.

Many searches for additional evidence for $CP$ violation are being performed or considered. Unfortunately, most of these cannot achieve the sensitivity to distinguish among a variety of models. The two most sensitive experiments are the search for a nonzero value of the parameter $\varepsilon'$ and for a neutron electric dipole moment $d_n$. A nonzero of $\varepsilon'$ would rule out the superweak model. A nonzero value of $d_n$ would not rule out any model as long as it could be blamed on "strong $CP$ violation," but a very low value would be difficult to reconcile with some models.

While this review has been devoted primarily to a number of popular models of $CP$ violation, it should be emphasized that no model seems particularly compelling. Progress in this field may very likely come from unexpected experimental or theoretical developments.

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