The value of the energy corresponding to the $1S \rightarrow 2S$ switch in Li may also be calculated. If we add one more electron to the K-ionised Li atom, the work done will be approximately that of the first ionisation potential of Be, namely $9.5$ volts. The result is therefore $53.0$ volts.

The K-ionisation potential of the Li atom has been calculated by Braunbek (loc. cit.) as $64.6$ volts. Effectively, he subtracts a value equal to $3 \times 5.35$ or $16$ volts in place of our $18.1$ volts. The subtraction of this amount of energy can hardly be justified.

The Passage of $\alpha$- and $\beta$-Particles through Matter and Born’s Theory of Collisions.

By E. J. Williams, Manchester University.

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In a recent paper* the present writer considered the relation between the results of observations on various phenomena connected with the passage of $\beta$-particles through matter and the requirements of the quantum theory. A close quantitative comparison of the new quantum theory with experiment was, however, not made, because the theoretical requirements were not known with sufficient accuracy. The theoretical estimates were mainly based on calculations made by Gaunt† in 1927, and in his calculations the effect of close collisions with impact parameter less than atomic dimensions were not adequately dealt with. These close collisions contribute appreciably to such phenomena as the stopping-power and ionisation, and in order to allow for them in such cases somewhat arbitrary assumptions concerning their effect had to be made. The differences between the theoretical values arrived at and the experimental values were not so large that they could be definitely dissociated from these assumptions, and for this reason no fundamental significance was attached to them. A treatment of collisions more complete than that of Gaunt was given recently by Bethe‡ on the basis of Born’s theory of collisions. Bethe’s calculations deal with the effect of all collisions, and the

formulæ obtained by him for the stopping power, primary ionisation, etc., enable us to make a closer comparison of the new quantum theory with experiment than was possible before. This is done in the first part of the present paper. It is satisfactory that the new formulæ, whilst agreeing in a general way with those previously used as a representation of the quantum theory, are in better quantitative accord with experiment. The present position is, however, not completely satisfactory. In some cases there are still large discrepancies. These discrepancies, if real, are, of course, more serious than those found in previous discussions because there is much less room for ascribing them to incompleteness or approximation in the theoretical calculations.

The main assumptions made in Bethe’s calculations are that the velocity, \( v \), of the moving particle is large compared with the Bohr-orbit velocity, \( u \), of the atomic electrons traversed, and that it is small compared with the velocity, \( c \), of light; quantities of the order of \( u^2/v^2 \) and \( v^2/c^2 \), being neglected. A third simplification which we must not overlook, especially in dealing with many-electron atoms, is the representation, in Bethe’s calculations, of the atomic electrons by hydrogen-like wave-functions. As regards the first two assumptions, \( \frac{1}{2}mu^2 \) being roughly equal to the ionisation potential, \( J \), the corresponding conditions of applicability of Bethe’s results may be formally written

\[
\frac{u^2}{v^2} \simeq \frac{J}{\frac{1}{2}mv^2} \ll 1, \tag{1A}
\]

\[
\frac{v^2}{c^2} \ll 1. \tag{1B}
\]

These conditions are adequately satisfied in most of the cases considered in the first part of this paper. The test of Born’s theory of collisions under these simplifying conditions is very desirable, especially in view of the considerable contemporary work which is being done on slow electrons which do not satisfy (1A).

The second part of the present paper (§ 2) is devoted to a discussion of the relativity region. By comparing Bethe’s non-relativity formulæ with experimental results for particles with velocities comparable with \( c \) we can deduce the nature of the actual relativity effect. A direct application of quantum mechanics to the problem has not been made, and there are therefore no theoretical formulæ with the same generality as those derived by Bethe for the non-relativity case. A correction for the relativity effect in the case of “distant” collisions was, however, deduced recently by the writer.* This correction can

* W2, loc. cit.
be incorporated into Bethe's non-relativity formulae. The requirements of these corrected formulae are compared with experiment. The conclusions arrived at regarding the relativity effect are applied in § 2 (c) to the ionisation produced by high energy β-particles such as those associated with penetrating radiation.*

* It may be well to give an indication of the bases of the respective calculations of Bethe and of Gaunt. Bethe uses the theory of collisions developed by Born in 1927. Born's theory is essentially a statistical one, the progress of individual collisions not being dealt with. In the case of one electron atoms the problem is to find a solution of the wave equation for 2 particles (the "moving" particle (α or β) and the atomic electron) in each other's field and the field of the atomic nucleus. The wave-equation in such a case may be written

$$\nabla^2 \psi/M + \nabla^2 \psi/m + (8\pi^2/\hbar)(H - V)\psi = 0, \quad (A)$$

M and m are the respective masses of the moving particle and atomic electron; R and r their respective co-ordinates with respect to the atomic nucleus which is assumed fixed; H is the total energy of the system and is equal to \(\frac{1}{2}mv^2 - E_0\), where \(v\) is the "initial" velocity of the moving particle and \(E_1\) is the negative energy of the undisturbed atom; V is the potential energy of the system and is equal to

\(-E'/|r'| + (EE'/|R|) + (-Ee/|R - r'|),\)

where \(E', e\) and \(E\) are the nuclear charge, the electronic charge and charge of the moving particle respectively. The perturbation term is \((EE'/|R|) - (Ee/|R - r'|)\). Without this term the equation is, of course, satisfied by a wave-function, \(\psi_0\), representing the atom in its ground state and a free moving particle. In Born's theory the effect of the perturbation term is found by successive approximation. In his calculations Bethe goes no further than the first approximation, \(\psi_0\), which may be found in the usual way by substituting \(\psi_0 + \psi_0'\) for \(\psi\) in (A), and neglecting the term containing the product of \(\psi_0'\) and the perturbation potential. The error in the first approximation is of the order of \(u^2/v^2\) where \(u\) is the Bohr-orbit velocity of the atomic electron—this leads, together with certain approximations made by Bethe in integrating for the total loss of energy, etc. (pp. 357–359 of his paper), to the condition of applicability (1A) of Bethe’s formulæ. In the case of \(n\)-electron atoms the potential field of the nucleus is replaced by the field of the nucleus and \((n - 1)\) electrons, a procedure which, though not rigorous, is in general fairly accurate.

The method used in Gaunt’s calculations is radically different from the above. His calculations deal with distant collisions in which the moving particle may be assumed to move undisturbed in a straight line with a definite impact parameter relative to the atom encountered. Its passage by the atom gives rise to a perturbing potential varying in a known manner with the time. The result of this perturbation is found by solving a wave-equation (including the time) for a single particle, viz., the atomic electron.

Though Gaunt’s results are expressed in terms of impact parameter it is possible to compare them with those of Bethe at least in one respect, viz., the relative probability of excitation of different states, and of ionisation, in collisions for which the deflection of the particle is small. It is found that the two theories lead to identical results. There is little doubt that the appreciable difference between Gaunt’s final formula for the stopping power and that of Bethe, is due to the arbitrary assumption regarding close collisions made in Gaunt’s work.
§ 1. Comparison of Theory and Experiment for Slow Particles.

§ 1 (a). Stopping-power.—The formula obtained by Bethe for the average rate of loss of energy, \( \overline{dT}/dx \), by an electric particle with charge \( z \times e \), and velocity \( v \), is

\[
\frac{dT}{dx} = (4\pi z^2e^4 N/me^2) \sum_{r=1}^{Z} \log \left\{ \frac{(2) me^2}{E_r} \right\} \\
= (4\pi z^2e^4 NZ/me^2) \log \left\{ \frac{(2) me^2}{\overline{E}} \right\},
\]

(2)

\( N \) being the number of atoms per unit volume of the material traversed, and \( Z \) the number of electrons per atom. \( E_r \) is an energy relating to the \( r \)th electron in the atom, and is very nearly equal to the ionisation potential. \( \overline{E} \) is the geometric mean value of \( E_r \) for the whole atom.* It will be hereinafter referred to as the "average excitation potential" for the atom. The factor 2 inside brackets in the log. term is to be used for \( \alpha \)-particles but not for \( \beta \)-particles.

The comparison of theory and experiment may be made either by calculating the theoretical range from (2) and comparing with the observed range, or by deducing, from the observations, the rate of loss of energy for a given velocity and comparing with (2) directly. A better idea of the validity of (1\(\alpha \)) may be formed if the latter method is adopted. This will be done in most cases, the velocity being chosen so as to satisfy the condition (1) as well as possible.

(2) is of the same type as previous theoretical formulae, and differs from them only in respect of the argument of the log. term. It will therefore be convenient to compare the stopping-power according to (2) with experiment, and with previous formulæ, in terms of the value of this log. term, or, more precisely, in terms of \( (\overline{dT}/dx) \div (2\pi z^2e^4 NZ/me^2) \). This quantity was denoted by \( P \) in a previous paper by the writer (W1).† Its value for \( \alpha \)-particles according to (2) is

\[
P_\alpha = 2 \log \left( \frac{2me^2}{\overline{E}} \right).
\]

(3)

In his calculations for \( \beta \)-particles Bethe does not take into account the fact that the electron with the greater energy after a collision is of necessity taken

* The value of \( \overline{E} \) for the hydrogen atom is 15 volts, and its values calculated by Bethe for air and copper are 35 and 80 volts respectively. The atomic numbers of all the elements considered in the present paper come within this range, and since \( \overline{E} \) occurs in a log term its value for any case may be determined with sufficient accuracy by interpolation from the above values. The interpolated values for oxygen, mica and argon are 37, 42 and 59 volts respectively. The value adopted here for molecular hydrogen is 1.1 \( \times \) 16 volts, in conformity with the value for atomic hydrogen, viz., 1.1 \( \times \) its ionisation potential.

† *Loc. cit.*
as the $\beta$-particle, nor that the frequency of violent collisions is, as shown by Mott,* appreciably affected by the application of Pauli’s exclusion principle. The correction of (2) for these effects can readily be made for the cases concerned here in virtue of the fact that the energy of the $\beta$-particles is large compared with the ionisation potentials of the atomic electrons traversed. In such cases we can, in making the corrections, use the formulæ derived by Mott for collisions of $\beta$-particles with free electrons. The corrected value of $P_\beta$ is

$$P_\beta = 2 \log \left( \frac{mv^2}{E} \right) - \log \left( \frac{8}{\varepsilon} \right).$$

(4)†

The new term represents a correction of about 10 per cent.

The observed and theoretical values of $P$, for $\alpha$- and $\beta$-particles traversing light elements are given in the following table. Only light elements and, in the case of $\alpha$-particles, the highest velocities, are included in order that the condition of applicability, (1a), of Bethe’s results may be best satisfied. The order of magnitude of the values of $u^2/v^2$ (which (1a) requires to be $\ll 1$) are given in the fourth column. Apart from the case of $\alpha$-particles traversing oxygen the condition is adequately satisfied, especially when we consider that the $k$-electrons, to which the highest value of $u^2/v^2$ refer, constitute only a small fraction of the total number of electrons. The observed values of $P_\beta$ and its values according to the Bohr-Gaunt formula are quoted from W1. The observed values of $P_\alpha$ have been deduced from the observations of Mrs. Harper and Mrs. Salaman‡ on the ranges of $\alpha$-particles, and of Gurney§ on the relative stopping-power of different gases for $\alpha$-particles.||

†  Treating the atomic electrons as free, the frequency of collisions in which energy $Q$ is lost, according to the results arrived at by Mott using the exclusion principle, is $\phi_Q (Q) = k/Q + k/(T - Q)^2 - k/Q (T - Q)$, where $k = 2\pi NZ e^4/mv^2$. This applies up to $Q = mv^2/4 = T/2$, above which $\phi_Q (Q) = 0$. The formula used by Bethe is $\phi_Q (Q) = k/Q^2$, which is applied up to $Q = T$. $\phi_Q (Q)$ and $\phi_Q (Q)$ agree for $Q \ll T$. The difference between the corresponding rates of loss of energy is therefore

$$\int_{T/2}^{T} \phi_Q (Q) Q dQ - \int_{T/2}^{T} \phi_Q (Q) Q dQ = k \log \left( \frac{8}{\varepsilon} \right) = (2\pi NZ e^4/mv^2) \log \left( \frac{8}{\varepsilon} \right).$$

|| The writer has made a fairly careful analysis of the results of observations on the rate of loss of energy by $\alpha$-particles. The results of Mrs. Harper and Mrs. Salaman for the ranges are given to three and four significant figures, and the agreement of their results for air with those of Geiger (‘Zeit. für Phys.,’ vol. 8, p. 45 (1921)) and of Henderson (‘Phil. Mag.,’ vol. 42, p. 538 (1921)) would seem to justify this. However, the values of $P$ deduced from their observations (calculated from the values of $\delta R/\delta v$ where $\delta R$ is the difference in
Table I.—Stopping-power.

<table>
<thead>
<tr>
<th>Moving particle</th>
<th>v/c</th>
<th>Gas</th>
<th>(u^2/u^2)</th>
<th>Observed</th>
<th>Theoretical*</th>
<th>(P_{\text{Bethe}})</th>
<th>(P_{\text{obs}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)-particle</td>
<td>0.136</td>
<td>H(_2)</td>
<td>0.001</td>
<td>11.7</td>
<td>17.1</td>
<td>11.5</td>
<td>0.98</td>
</tr>
<tr>
<td>(\alpha)-particle</td>
<td>0.230</td>
<td>O(_2)</td>
<td>0.001→0.07</td>
<td>10.6</td>
<td>18.0</td>
<td>12.2</td>
<td>1.14</td>
</tr>
<tr>
<td>(\alpha)-particle</td>
<td>0.230</td>
<td>A</td>
<td>0.001→0.2</td>
<td>10.0</td>
<td>16.3</td>
<td>11.2</td>
<td>1.12</td>
</tr>
<tr>
<td>(\alpha)-particle</td>
<td>0.064</td>
<td>H(_2)</td>
<td>0.01</td>
<td>11.1</td>
<td>12.9</td>
<td>10.9</td>
<td>0.98</td>
</tr>
<tr>
<td>(\alpha)-particle</td>
<td>0.064</td>
<td>He</td>
<td>0.02</td>
<td>9.3</td>
<td>12.1</td>
<td>9.9</td>
<td>1.07</td>
</tr>
<tr>
<td>(\alpha)-particle</td>
<td>0.064</td>
<td>O(_2)</td>
<td>0.01→0.5</td>
<td>7.5</td>
<td>10.6</td>
<td>9.4</td>
<td>1.25</td>
</tr>
</tbody>
</table>

range between \(\alpha\)-particles with initial velocities differing by \(\delta v\) show an irregular variation with velocity of the order of 5 per cent. This indicates either errors of the order of 1 per cent. in the observed ranges, or errors approaching \(\frac{1}{3}\) per cent. in the relative velocities of \(\alpha\)-particles from different sources, as recently determined by Briggs (Proc. Roy. Soc., A, vol. 118, p. 549 (1928)) and Laurence (Proc. Roy. Soc., A, vol. 132, p. 543 (1929)). The fact that the irregularities in the variation of \(P\) for different gases roughly correspond suggests the latter. Laurence claims an accuracy of about 0.1 per cent. for the velocities. His results for \(\alpha\)-particles from ThC, RaC, RaC' and RaF are 2.054, 1.923, 1.709 and 1.592 \(\times 10^9\) cm./sec. respectively. The closest values to these which give a uniform variation of \(P\) with velocity (for hydrogen) are 2.056, 1.927, 1.702 and 1.591 \(\times 10^9\) cm./sec. It might be pointed out that in dealing with the results for ranges a small correction is applied for the difference between the observed "extrapolated" range and the average range. This difference is greater, the greater the initial velocity. As it increases uniformly with velocity it cannot, of course, be a source of the above-mentioned irregular variation of \(P\) with velocity.

The experimental value of \(P\) for helium is deduced from the value of \(P\) for H\(_2\) and Gurney's determination of the relative stopping-power of H\(_2\) and He. The latter was obtained by observing the pressure of gas required to reduce the energy of \(\alpha\)-particles of given initial energy to a certain value, which was fixed by the ionising power of the \(\alpha\)-particles after traversing the gas. The relative stopping-powers obtained by Gurney for selected portions of the range differ, in some cases by as much as 5 per cent., from the values required by the work of Mrs. Harper and Mrs. Salaman. Even if we attribute all this discrepancy to the ranges, it does not however mean a serious error, since it concerns differences between ranges. In the case of hydrogen, e.g., an adjustment of the ranges of only 1 or 2 mm. (i.e., about 1 per cent.) brings agreement with Gurney's values. This is of importance in connection with the significance of Table (2) for ranges.

Taking into account all the evidence, we conclude that the observed values of \(P_{\alpha}\) in Table (1) are accurate within about 2 per cent. but not more so.

* In the case of the larger velocities involved, the relativity terms in the relation between energy and velocity are not appreciable. A complete relativity expression for \(dT/dx\) has not yet been deduced, but in all probability the correct procedure to a first approximation, and the procedure adopted here, is to use \(T\) as variable on the L.H.S. of (2) and \(v\) as variable outside the log. term on the R.H.S.
E. J. Williams.

The table shows that for both $\alpha$- and $\beta$-particles Bethe's formula is in much better agreement with experiment than the Bohr-Gaunt formula. In the case of $\beta$-particles the requirements of the latter exceed the experimental values by as much as about 60 per cent., whilst the average difference between the values given by Bethe's formula and those observed is only about 10 per cent. The latter is probably within the errors of experiment and the errors arising from known sources in the theoretical calculations. The case for which the discrepancy between Bethe's results and experiment is largest is that of $\alpha$-particles traversing oxygen, and this is the case for which the condition of applicability of the theoretical formula is least satisfied. The Bohr-orbit velocity of the $k$-electrons in this case is comparable with the velocity of the $\alpha$-particle, and this violates (1A). We must also remember that owing to Bethe's approximate method of dealing with many-electron atoms, viz., the use of hydrogen-like wave-functions, the error in the computed value of the average excitation potential $\overline{E}$ may be quite appreciable for oxygen.

We shall now consider the variation of the stopping-power with velocity, and in view of the nature of previous discussion on this subject, we shall do so in terms of the index $n$ of $v$ in the range-velocity relation

$$R = kv^n.$$  \hfill (5)

All theories require $P$ to increase with the velocity, and this means a departure from the Thomson-Whiddington range-velocity law according to which $n = 4.$

The theoretical value of $n$ for particles with initial velocity $v_0$ is

$$n_{\text{theor.}} = 4 - (\xi/P'),$$ \hfill (6)

where $\xi$ is the index of $v$ in the theoretical log. expression for $P$, and $P'$ is the absolute value of $P$ for a velocity of about $3v_0$. The interest lies in the departure of $n$ from 4, and since this is small it is necessary to make very accurate determinations of $n$ in order to give significant results. In the case of $\beta$-particles, existing data give only a rough indication of the value of $4 - n$. According to the results obtained by the writer for oxygen, the value of $n$ for $\beta$-particles with energy of about 10,000 volts traversing oxygen is $3.7 \pm 0.1$. The value required by Bethe's theory is, from (4) and (6), $4 - 4/10 = 3.6$. The results obtained by Nuttall and the writer for somewhat slower $\beta$-particles traversing hydrogen give $n = 3.3$. The theoretical value in this case is also

* By definition of $P$, $(dT/dx) \propto P/v^3$, i.e., $dv/dx \propto P/v^3$, so that if $P$ is constant $R \propto v^4$.

† The theoretical range-velocity law for all velocities cannot, of course, be represented by a single term as in (5), and the theoretical value of $n$ refers only to a small region of velocities.
3·6. In neither case is the difference greater than the possible experimental error. The experimental values of \( n \) for \( \alpha \)-particles are known more accurately. In the case of moderately fast \( \alpha \)-particles \((1·8 - 2·0 \times 10^9 \text{ cm./sec.)}, \) traversing \( H_2 \), the value of \( P' \) in (6) is 9·7, so that according to Bethe's formula
\[
n = 4 - \frac{4}{9·7} = 3·59.
\]
The observed value of \( n \) for this case according to the experiments of Mrs. Harper and Mrs. Salaman is 3·36. For air the theoretical and observed values are 3·3 and 3·1 respectively. The observed values appear to be systematically greater than the theoretical. These differences cannot be removed by adjusting the value of \( P' \) in (6) because this is fixed within a few per cent. by the absolute stopping-power at high velocities. The differences rather indicate the necessity for a higher power of \( v \) in the log. expression for \( P \) than that given by Bethe's formula—a serious modification. A closer consideration of the position shows, however, that there are no grounds here for questioning the validity of Bethe's theory. We must remember that in dealing with \( n \) the whole range of the \( \alpha \)-particle is involved, and towards the end of the range, in the first place, the condition (1A) of applicability of Bethe's formula ceases to be satisfied, and secondly, the capture and loss of electrons by the \( \alpha \)-particles—ignored in Bethe's calculations—must appreciably affect the rate of loss of energy. The position can be best considered in terms of the ranges, the values of which for hydrogen are given in the following table. The observed range for the lowest velocity has been obtained from Briggs' result for the velocity that gives a range of 1·4 cm. in air, and Gurney's determination of the relative stopping-power of hydrogen and air for this range. To make the table more complete the results obtained by Nuttall and the writer for \( \beta \)-particles are also included. In calculating the theoretical ranges the value of the theoretical excitation energy \( \overline{E} \) used here (as in Table I) is
\[
1·1J = 1·1 \times 16 \text{ volts.}
\]

Table II.—Ranges in Hydrogen (at 15° C., 76 cm. pressure).

<table>
<thead>
<tr>
<th>Particle</th>
<th>Initial velocity ( \times 10^{-9} )</th>
<th>Theoretical average range (cm.)</th>
<th>Observed range (cm.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha ) particle</td>
<td>2·054</td>
<td>39·2</td>
<td>40·9</td>
</tr>
<tr>
<td>( \mu )</td>
<td>1·923</td>
<td>31·0</td>
<td>32·7</td>
</tr>
<tr>
<td>( \mu )</td>
<td>1·709</td>
<td>20·3</td>
<td>21·6</td>
</tr>
<tr>
<td>( \mu )</td>
<td>1·592</td>
<td>15·8</td>
<td>17·3</td>
</tr>
<tr>
<td>( \mu )</td>
<td>1·082</td>
<td>4·1</td>
<td>5·7</td>
</tr>
<tr>
<td>( \beta )-particle</td>
<td>5·11</td>
<td>0·77</td>
<td>0·76</td>
</tr>
<tr>
<td>( \mu )</td>
<td>4·08</td>
<td>0·34</td>
<td>0·37</td>
</tr>
</tbody>
</table>
It will be seen that the theoretical distance travelled by an α-particle in losing velocity from \(2 \cdot 054 \times 10^9\) to \(1 \cdot 592 \times 10^9\) is \(23 \cdot 4\) cm. The observed distance travelled in this interval is \(23 \cdot 6\) cm., or \(23 \cdot 2\) after correcting for straggling. The agreement is very good, and is, of course, to be identified with the agreement between the observed and theoretical values of \(P\) already given in Table I. Whilst the difference between the ranges is thus well represented by the theoretical formula, the ranges themselves differ sensibly—the theoretical ranges are \(1 \cdot 7, 1 \cdot 7, 1 \cdot 3, 1 \cdot 5\) and \(1 \cdot 6\) cm. respectively less than the observed ranges. These differences correspond to the difference found between the observed and theoretical values of the exponent \(n\) in the range-velocity law. The significant result is that these differences are nearly the same for the different velocities*. This means that the source of the discrepancy is to be found towards the end of the range in each case. The agreement elsewhere is excellent, and in terms of the excitation energy \(E\) in the log. term it shows that within about 10 per cent. the theoretically expected value, viz., \(1 \cdot 1 \times 16\) volts, is the best value of \(E\) which represents the experimental results. The discrepancy towards the end of the range can be explained if the effects mentioned above (capture and loss of electrons, and \(a^2/v^2\) terms) can cause the α-particle to travel about \(1 \cdot 4\) cm. more than Bethe’s formula requires. The distance an α-particle travels as \(\text{He}^+\) in \(\text{H}_2\) is about \(2 \cdot 0\) cm. The phenomenon of capture and loss of electrons will therefore give the α-particle an extra range of the order of \(1\) cm. provided the stopping-power for \(\text{He}^+\) is about one-half that for an α-particle, which is not unreasonable.

We conclude from the evidence put forward that, as regards rate of loss of energy, Bethe’s results agree satisfactorily with experiment, the case of hydrogen, for which the uncertainties in the calculated and experimental values are least, being very well represented by the theoretical formulæ.

\(\S\) 1 (b). Primary Ionisation.—Bethe’s formula for the number of primary ions, \(I\), produced per centimetre of its path by a β-particle traversing hydrogen-like atoms is

\[
I = (2\pi Ne^4/mv^3) \log (42 mv^2/J), \tag{7}
\]

\(J\) representing the ionisation potential. The classical value of \(I\) is

\[
I_{ct} = 2\pi Ne^4/mv^3J. \tag{8}
\]

We shall assume that (7) is applicable to molecular hydrogen provided we

* Corrected for straggling, the differences are about \(1 \cdot 3, 1 \cdot 4, 1 \cdot 2, 1 \cdot 4, \) and \(1 \cdot 6\) cm., respectively.
substitute for $J$ the appropriate value, viz., 16 volts.* Observations on the ionisation in hydrogen were recently made by Williams and Terroux.† These apply to velocities ranging from $0.5c$ upwards. The discussion of relativity effect in § 3 (b) shows that for $\beta = 0.5$ the relativity correction can be neglected, so that for this velocity we can legitimately use the non-relativity formula (7). According to this the number of primary ions produced per centimetre by a $\beta$-particle with velocity $0.5c$, traversing hydrogen (at N.T.P.), is $12.6$. The observed number is $14.7$, and the agreement is satisfactory. It is interesting to note that the number according to the classical formula is only $3.5$. The numerical failure of the classical theory here is therefore much greater than in the case of stopping-power, a state of affairs emphasised in W2.

§ 1 (c). Total Ionisation. (Primary + Secondary)—The outstanding features of the experimental results are:—(a) the ratio, $dT/dN$, of the rate of loss of energy, $dT/dx$, to the rate of production of ions, $dN/dx$, is little dependent on the velocity of the ionising particle; (b) the total ionisation in the inert gases, in relation to their ionisation potentials, is abnormally high compared with that in other gases such as hydrogen, nitrogen and oxygen; as an example of this the observed average energy spent per ion-pair in helium is actually less than that in hydrogen though the ionisation potential of helium is more than $1\frac{1}{2}$ times that of hydrogen. The new quantum theory adequately accounts for (a). In so far as it has been applied up to the present it gives, however, no indication of (b), and we find here the most serious discrepancy between theory and experiment. (b) is discussed in the recent book on “Radiations from Radioactive Substances,” by Rutherford, Chadwick and Ellis. An explanation of it on the new quantum theory which is there anticipated is not forthcoming. However, as we shall see later, at least part of the discrepancy is due to systematic error in the experiments.

A general explanation of (a) follows from the theoretical result that the relative numbers of excitation collisions and of light ionisation collisions (in which the energy losses are of a certain order of the ionisation potential) are practically independent of the nature and velocity of the moving particle.‡ The relative

* Since in (7) $J$ occurs outside as well as inside the log. term this assumption may involve more numerical error than the corresponding assumption in the case of stopping-power. In the latter case a variation of 50 per cent. in $J$, for example, affects the stopping-power by only about 5 per cent.


‡ From Bethe's paper the probability that a collision, in which the momentum transfer is $\hbar$, raises an atomic electron from its undisturbed state $\psi_0$ to a state represented by a wave-function $\psi_n$ is proportional to the matrix element $\int e^{i(\sigma r)} \psi_0 \psi_n \, d\tau$, $r$ being the
number of violent collisions, resulting in comparatively high-speed secondary electrons, does, of course, depend on the velocity of the primary particle. The frequency of these violent collisions is, however, not of much consequence to the ultimate total ionisation. The reason for this is that nearly all the dissipation of energy takes place ultimately in the form of light collisions, and since the partition of energy between different excitations and ionisations in such collisions is independent of the nature and velocity of the moving particle, it little matters whether it is the primary particle that is concerned in them, or fast secondary electrons which have derived their energy from the primary particle. It follows that for a given energy loss by the primary particle the total ionisation is nearly independent of its nature and velocity.

We shall now consider the absolute value of the energy expended per ion pair, viz., $dT/dN$, in the case of $\alpha$- and $\beta$-particles traversing hydrogen and helium. These gases afford a good example of the experimental result (b), whilst they are also the ones for which the theoretical requirements can be most accurately estimated. Even in these cases the total ionisation can only be roughly evaluated. The difficulty is that an appreciable fraction of the ionisation is produced by slow secondary electrons the behaviour of which as regards rate of loss of energy and primary ionisation may not be even approximately given by Bethe’s formulæ for these effects, on account of the condition of applicability (1A).* It is, however, possible to make estimates of the total ionisation which are sufficiently significant without having to deal with the detailed behaviour of the slower secondary electrons. It will be safe to assume that if $J/\frac{1}{2}mv^2$ is less than $1/10$, the condition (1A) is adequately satisfied. In that case we can deal with all secondary electrons with energy greater than $10J$, and the dissipation of energy can be traced up to the point when all further ionisation that is produced is due to secondary electrons with energy less than

co-ordinate of the atomic electron. For the light collisions considered, $q$ is nearly parallel to the velocity of the moving particle ($x$ axis, say), and $qr$ is also small. $e^{iqr}$ is therefore approximately equal to $1 + i|q|x$, and the above matrix element reduces to $i|q| \int x\psi_0\psi_n d\tau$. This is independent of the nature and velocity of the moving particle, and the result mentioned follows.

* In his paper Bethe makes a rough estimate of the total ionisation by assuming the applicability of his formulæ for primary ionisation and stopping-power to all secondary electrons, and also making one or two other approximations. He does not give an estimate of the possible error due to these approximations, so that his result has no definite significance. Bethe compares with experiment in one case only, viz., $30,000$ volt $\beta$-particles traversing nitrogen. The good agreement found gives a false indication of the position regarding total ionisation.
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10J. This further ionisation cannot be definitely evaluated, but we can fix limits to its value corresponding to zero and 100 per cent. efficiency respectively.* It is found in this way that for $\alpha$- and $\beta$-particles with velocities of the order of 1000 electron-volts, traversing hydrogen or helium, the theoretical energy expended per ion pair produced lies between 2·2J and 3·2J.† For hydrogen $J = 16$ volts, so that the limits are about 35 and 50 volts; for helium $J = 25·4$ volts and the limits are about 55 and 80 volts. The cases dealt with have been investigated experimentally by Gurney ($\alpha$-particles)‡ and Lehman.§ ($\beta$-particles). The experimental values for hydrogen are 35 and 37 volts for $\alpha$- and $\beta$-particles respectively, and for helium 29 and 31 volts respectively.|| We notice that there is little dependence on the nature of the primary particle, in accordance with theory. The actual observed values for hydrogen are just within the theoretical limits. For helium, however, even the lower limit to the theoretical energy per ion pair is nearly twice the observed value. If we accept the experimental results we must simply conclude that the distribution between different kinds of collisions, of the loss of energy by $\alpha$- and $\beta$-particles traversing helium, is very different from that required by Bethe’s theory — so much so that the success of the theory in accounting for other phenomena must to a large extent be of the nature of an accident. It is difficult to accept this position. The alternative is to question the low experimental values of the energy per ion pair, especially in helium. There are, in fact, several reasons

* By 100 per cent. efficiency is meant that a secondary electron with energy between $nJ$ and $(n + 1)J$ produces $n$ ions.

† Estimates were made for particles with different velocities, viz., 400 and 1000 electron-volts. The differences of 2 or 3 per cent. between the limits in the two cases was within the error of their evaluation.

In the calculations the number, $dN_3$, of primary ions is first obtained using Bethe’s formula. The energy, $dT_3$, expended by the secondary electrons with initial energy greater than 10J, before their energy is reduced to 10J, is then calculated. $dT_3$ is divided by the average energy per primary ion for the velocities concerned. This gives a number $dN_3$. Some of these tertiary electrons have energy greater than 10J and the energy they lose in being reduced to 10J is dealt with in the same way as $dT_3$, leading to a number $dN_4$ of quaternary ions. This number is, however, almost negligible and it is unnecessary to follow the process further. $dN_3 + dN_4 + \ldots$ gives the lower limit to the total ionisation. Of the number $dN_3 + dN_4 + \ldots$ a number $dN'$ of the electrons have energy greater than $J$. The energy distribution of these is known, and the upper limit to the ionisation they produce is obtained by assuming 100 per cent. efficiency as mentioned.


|| The values for $\alpha$-particles also involve Geiger’s absolute determination of the energy per ion pair for air, viz., 35 volts (see ‘Radiations from Radioactive Substances,’ by Rutherford, Chadwick and Ellis, p. 81).
for doing so, though it remains to be seen whether they are sufficient to bring about agreement with theory.

A spuriously low value of the energy per ion pair means that the ionisation current measured in the experiments exceeds that which corresponds to the rate of production of ions by the moving particle. This cannot be due to ionisation of the gas by collision because an ionisation current independent of the applied voltage is realised in all the experiments. The recent experiments of M. L. E. Oliphant* and others on the interaction between positive ions of He and a metal electrode show, however, that there are effects at the electrodes of a first order of magnitude, and which would in all probability escape detection in experiments of the kind which have been carried out on the ionisation produced by $\alpha$- and $\beta$-particles.† In the experiments referred to it is found that positive helium ions, attracted to an electrode, liberate electrons from it; the number, $\xi$, liberated per incident ion being, within experimental error, independent of the incident kinetic energy of the ion, provided this energy is less than about 200 volts. In all experiments on the ionisation by $\alpha$- and $\beta$-particles the ions reaching the electrodes never have energy approaching this magnitude, so that the electrode effect does not prevent a saturation current being reached. On account of this effect, however, the actual saturation current overrates the rate of production of ions in the gas by a factor $1 + \xi$, and the true energy, $S$, expended per ion pair is equal to the apparent value, $S_a$, $\times (1 + \xi)$. The effect occurs for positive ions of neon as well as of helium, and this fits in with the fact that an abnormally low value of $S$ is also found for neon. The actual value of $\xi$, though of a first order of magnitude, is however, not large enough to bring the corrected value of $S$ for helium within the theoretical limits. The value of $\xi$ for helium according to Oliphant is 0.25, which gives a corrected value of $S$ of about 30 $\times$ 1.25 = 37 volts per ion pair. In a letter to the writer Oliphant, however, points out that in all the experiments which have been carried out on the ionisation by $\alpha$- and $\beta$-particles, there are sufficient impurities in the gas owing to the presence of unclean surfaces, grease, etc., to give rise to another effect similar to that which occurs at the electrodes. The atoms, excited and ionised by the moving particles, undergo a process of exchange of energy with the impurities, resulting in the latter being ionised. As a consequence more ions arrive at the electrodes

† The suggestion of this "electrode" effect in ionisation measurements is due to Mr. R. W. Gurney, and I am indebted to Mr. Gurney for drawing my attention to the results obtained by Oliphant and others. I am also grateful to Mr. Oliphant for a letter in which he discusses the effect and suggests estimates of its magnitude.
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than are genuinely produced. In so far as excited as well as ionised atoms are able to take part in this effect it is more powerful than the electrode effect. The exchange of energy involved in the process has been shown by Kallman and Rosen* to go on with great efficiency in a large number of cases. Though the process is pre-eminently effective in the monatomic gases, it also takes place to a smaller extent in all the ordinary gases. In this connection we might mention the remarkable discrepancies between the results of different observers for the energy expended per ion pair by $\beta$-particles in air—the case most extensively investigated. Apart from any theory of spurious effect these discrepancies show that there must be a source of considerable error somewhere in the measurement of the total ionisation by electric particles. Osgood and Lehmann,† using $\beta$-particles with energy between 200 and 1000 volts, find a value of $S$ independent of the voltage and equal to 45 volts. W. Shmitz‡ finds practically the same value for the range 1000–9000 volts. E. Buchmann§ and H. Eisler|| on the other hand, find constant values close to 32 volts, the ranges investigated being 4000–13,000 and 9000–59,000 volts respectively. If these discrepancies are due to the ionisation of impurities in the gas by excited and ionised atoms, then the value of 45 volts is the one nearer the true value, the effect of impurities in the other experiments increasing the ionisation by nearly 50 per cent. This is more than we would expect in the case of air, and there are probably other sources of error. We shall not consider the matter further. Sufficient evidence has been put forward to show that the hitherto accepted values of the total ionisation by $\alpha$- and $\beta$-particles may be considerably in error. The validity of the theoretical requirements regarding this phenomenon must therefore be left open until the experimental position is more definitely established.

§ 1 (d). Straggling.—The phenomenon of straggling arises from the statistical fluctuations in the energy losses suffered by a moving particle in travelling a given distance. The straggling is contributed to mostly by the violent collisions, and in the case of $\alpha$-particles the extent of the fluctuations as calculated by Bohr¶ is measured by the integral

$$\sigma = \int_{Q_l}^{Q_m} Q^2 \phi(Q) dQ,$$

(9)

* 'Z. Physik,' vol. 64, p. 806 (1930).
‡ 'Phys. Z.,' vol. 29, p. 846 (1928).
¶ 'Phil. Mag.,' vol. 30, p. 531 (1915).
\( \phi(Q) \) being the probability of a collision in which energy \( Q \) is lost. On the new quantum theory, as on the classical theory, the value of \( \phi(Q) \) for \( \alpha \)-particles traversing free electrons (NZ per unit volume) is

\[
\phi^\alpha_0 \ (Q) = 8\pi e^4 NZ/mv^2Q^2
\]

the limits \( Q_l \) and \( Q_m \) being 0 and \( 2mv^2 \) respectively. The corresponding value of \( \sigma \) is

\[
\sigma_0 = 16\pi e^4 NZ.
\]

For \( \alpha \)-particles traversing free electrons the problem is, of course, the same as the scattering of electrons by \( \alpha \)-particles, and there is enough circumstantial evidence that such scattering is correctly given by theory. An explanation of any departure from (11) must therefore be sought for in the effect of atomic binding forces.

Many investigations have been made on the straggling of \( \alpha \)-particles, the most exhaustive being due to G. H. Briggs.* Briggs finds that for fast \( \alpha \)-particles \( (v = 1.9 \times 10^8 \text{ cm. sec.}) \) traversing mica the actual value of \( \sigma \) is about 1.9 \( \sigma_0 \). On the classical theory the direct effect of binding forces is to reduce \( \sigma \) but, indirectly, in giving rise to motion of the atomic electrons, the effect is to increase \( \sigma \) above \( \sigma_0 \). This increase is, however, small compared with the factor of 1.9 required to give agreement with the observed value. On the new quantum theory the effect of binding forces on \( \sigma \) is certainly greater than on the classical theory. As described in W2, the average value of \( \phi (Q) \) on the quantum theory greatly exceeds the classical value for values of \( Q \) of the order of the ionisation potential, \( J \). The excess of \( \sigma \) over \( \sigma_0 \) required by Bethe's theory, due to this concentration of energy losses near the ionisation potential, is approximately represented by a term of the form

\[
A \log \left( \frac{mv^2}{J} \right) \times \left( \frac{J}{mv^2} \right) \times \sigma_0.
\]

\( A \) is a numerical constant of the order of unity, its actual value depending upon the level of the electron in the atom. For the actual values of \( v \) and \( J \) concerned the log. term is also of the order of unity. The magnitude of the contribution to \( \sigma \) represented by (12) is therefore of the order of \( (J/mv^2) \times \sigma_0 \). Now fractional errors of the order of \( J/mv^2 \) are involved in all of Bethe's results, because of the application of Born's theory of collisions only as far as a first approximation (see 1A). It follows that the exact coefficient of \( (J/mv^2) \times \sigma_0 \), representing the full contribution of binding forces to the straggling, cannot

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be obtained from Bethe's calculations. We are therefore, at present, only able to express the order of magnitude of the effect of binding forces. This is

$$\sigma_i = \sigma_0 \{1 + 0 (J/mv^2)\}.$$  (12A)

The average value of $J$ for mica is about 250 volts, so that for fast $\alpha$-particles traversing this substance, the average value of $J/mv^2$ is 0.12. It follows that the coefficient of $J/mv^2$ in (12A) must be very high in order to account for the observed value of $\sigma$, viz., $1.9 \sigma_0$. The value of this coefficient deduced from Bethe's first approximation formulæ (see (12)) is certainly much too small.* The position regarding straggling is thus not very satisfactory. The experimental results may, of course, be questioned on the grounds that nearly all errors enhance the observed straggling. However, if the errors are small, as appears to be the case in the work of Briggs, it little matters whether they are systematic or not. Accepting the experimental results, we must conclude either that the correction to Bethe's formulæ to allow for the assumption (1A), made in his calculations, is exceedingly large, or that there are other appreciable sources of straggling. The capture and loss of electrons by $\alpha$-particles is certainly one other source of straggling, but unless there is a systematic difference between the frequency of capture and loss by individual $\alpha$-particles,† the extra straggling this phenomenon gives rise to is negligible for fast $\alpha$-particles. For such particles the independent straggling due to 'capture and loss' is only about one-twentieth of the straggling according to (11).‡ The fractional increase in the straggling produced by 'capture and loss' is therefore only about $\sqrt{1 + 1/20^2} \sim 1.001$.

* Added in proof.—In a recent letter to the writer, Bethe agrees with the form of (12) for the extra straggling due to binding forces, and he also calculates the exact value of $A$, which was not done by the writer. For hydrogen-like atoms in the ground state Bethe gives $A = 4/3$. Assuming this coefficient for all the electrons in mica, we find $\sigma = 1.35 \sigma_0$, which is appreciably less than the observed value of $1.9 \sigma_0$.


‡ A rough estimate of the straggling due to "capture and loss" can readily be made. Let $\lambda_1$ and $\lambda_2$ be the respective mean free paths for capture and loss. Consider $\alpha$-particles after traversing a distance $x$. Since $\lambda_2 \ll \lambda_1$ the average number of captures is $x/\lambda_1$ and the distance, $y$, travelled as $\text{He}^+$ is, on the average, $(x/\lambda_1) \times \lambda_2$. The probable variation in the individual free paths, as $\text{He}^+$, is of the order of $\lambda_2$. Assuming no departure from the average number of existences as $\text{He}^+$, this leads to a probable variation in $y$ of

$$\sqrt{\Sigma \lambda_2^2} = \sqrt{(x/\lambda_1) \times \lambda_2} = \sqrt{x/\lambda_1} \times \lambda_2.$$  Assuming all free paths as $\text{He}^+$ to be the same ($= \lambda_2$), the probable variation in $y$ due to fluctuations in the number of existences as $\text{He}^+$ is $\sqrt{x/\lambda_1} \times \lambda_2$; $\sqrt{x/\lambda_1}$ being roughly the probable variation in the number $x/\lambda_1$. The order of magnitude of the probable variation in the distance travelled as $\text{He}^+$ is therefore
In concluding this section, it might be pointed out that there is no evidence against the form of the general expression (12a) for \( \sigma \), though it would seem at first sight to be refuted by the results obtained by Briggs for different velocities. According to these the ratio of the straggling at different distances from the beginning of the range of the \( \alpha \)-particles, to the calculated straggling corresponding to \( \sigma = \sigma_0 \), is nearly constant. Actual calculation shows, however, that though \( v \) occurs to a second power in (12a), the systematic increase in \( \sigma_0/\sigma_0 \) with decreasing velocity which it requires is too small to make itself evident above the experimental irregularities.

§ 1 (c). Production of Branches.—The production of branches by slow \( \beta \)-particles and its satisfactory relation to the new quantum theory have been discussed in another paper.* In the experiments described there the energy of the branches is large compared with the ionisation potentials of the electrons traversed, so that the results are a test of the theory of collisions between free electrons.

§ 2. Relativity Effects.

In Bethe's treatment of collisions no account is taken of relativity effects, so that the results do not necessarily apply to particles with velocities comparable with that of light. In this section we shall consider in the first place the relation between the non-relativity formulae of Bethe and the experimental results for such fast particles. Assuming that Bethe's formule give the correct velocity variation apart from relativity effect, we can in this way deduce the nature of the actual relativity effect. Experimentally, let \( \sqrt{\sigma/\lambda_2} \times \lambda_2 \) be the rate of loss of energy by the \( \alpha \)-particle, and \( r (dT/ dx) \) the difference between the rate of loss of energy by an \( \alpha \)-particle and He\( ^+ \). Then the fractional probable variation, \( S' \), in the energy lost in travelling a distance \( x \), due to "capture and loss" is \( r \lambda_2/\sqrt{\sigma/\lambda_2} \). In close collisions with impact parameters much less than the electron-nucleus distance in He\( ^+ \) \( (dT/dx) \) He\( ^+ \) \( \sim (5/4) (dT/dx) \) He\( ^+ \). On the other hand, for distant collisions, the perturbation produced by He\( ^+ \) is nearly the same as that due to a particle with a single electronic charge, so that for such collisions \( (dT/dx) \) He\( ^+ \) \( \sim (1/4) (dT/dx) \) He\( ^++ \), (see also end of § 1A). Evidently the order of magnitude of \( r \) is not \( > 1 \) so that \( S' \lambda_2/\sqrt{\sigma/\lambda_2} \). For \( \alpha \)-particles with velocity \( \sim 1 \times 10^8 \) cm./sec., traversing air, Rutherford ('Phil. Mag.', vol. 47, p. 277, 1924) finds \( \lambda_2 = 0 \cdot 0011 \) cm., \( \lambda_1 = 0 \cdot 22 \) cm., so that for \( x = 1 \) cm., \( S' \approx 0 \cdot 002 \). The Bohr straggling, \( S \), under these conditions is \( 0 \cdot 04 \), and is therefore about 20 times the independent straggling produced by "capture and loss." As for variations with velocity \( \lambda_1 \propto v^6 \) and \( \lambda_2 \propto v \) so that \( S' \propto v^2 \), \( S \propto \sqrt{\sigma_0/(dT/dx)^2} \propto v \). \( S' \) is therefore \( \propto 1/v^2 \propto 1/\text{range}, \text{R} \). It follows that at 1 cm. from the end of its track the rate of straggling due to capture and loss becomes comparable with the Bohr straggling. Towards the end of the range the capture of 2 electrons to form neutral helium also becomes important.

mental results for high velocities are only available for β-particles, and among the phenomena which have been investigated are the rate of loss of energy and primary ionisation. We shall proceed to consider these phenomena.

§ 2 (a). Stopping-power.—In W1 the writer deduced from the observations of White and Millington* on the passage of fast β-particles through thin foils, values of the rate of loss of energy, \( d_w T/\delta x \), suffered by a β-particle due to collisions for which Q is less than a certain value, W. If \( \phi (Q) \) is the frequency of collisions in which energy Q is lost, then

\[
d_w T/\delta x = \int_0^W Q \phi (Q) \, dQ.
\]  

(13)

If we represent by \( dT/\delta x \) the rate of loss of energy due to all collisions, then

\[
d_w T/\delta x = dT/\delta x - \int_W^{\infty} Q \phi (Q) \, dQ. \tag{13A}
\]

The actual value of W for which the experimental results are given is 1500 volts, the substance traversed being mica. In mica the ionisation potentials of the great majority of the electrons are much less than 1500 volts, and for the purpose of deriving the theoretical expression for \( d_w T/\delta x \) we may assume that \( W/J \gg 1 \). In that case \( \phi (Q) \) for \( Q \gg W \) is negligibly affected by binding forces, and to evaluate the integral in (13A) we can use the value of \( \phi (Q) \) for free electrons. We find

\[
d_w T/\delta x = \left( 2\pi NZe^4/mv^2 \right) \left\{ \left[ 2 \log (mv^2/E) \right] - \int_W^{\infty} dQ/Q \right\}
\]

\[
= (2\pi NZe^4/mv^2) \log (2mv^2W/E^2). \tag{14}
\]

The experimental values of \( d_w T/\delta x \) (quoted from W1), and the values required by this non-relativity formula of Bethe are given in the second and fourth columns of the following table.† In this table all the velocities for which there are data are included in order to show the extent of the experimental irregularities.

The consideration of the variation with velocity, which follows, shows that for the smallest velocity represented in the table there is little relativity effect, and the absolute value may be legitimately compared with Bethe’s non-relativity formula. We see that for this velocity the difference between the requirements of this formula and the experimental value is about 20 per cent. The agreement is satisfactory. (14) is certainly a vast improvement on the

† The value of \( \bar{E} \) for mica is 42 volts (see footnote*, p. 111).
Table III.—Rate of Loss of Energy, $d_wT/dx$, at High Velocities.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Experimental.</th>
<th>Non-relativity.</th>
<th>Relativity-corrected.</th>
<th>Pw (Expt.)</th>
<th>Pw (Non-rel.)</th>
<th>Pw (Rel-corr.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d_wT/dx$</td>
<td>$(d_wT/dx)/1980$</td>
<td>$d_wT/dx$</td>
<td>$(d_wT/dx)/2400$</td>
<td>$d_wT/dx$</td>
<td>$(d_wT/dx)/2500$</td>
</tr>
<tr>
<td>0.64</td>
<td>1980</td>
<td>1.00</td>
<td>2400</td>
<td>1.00</td>
<td>2500</td>
<td>1.00</td>
</tr>
<tr>
<td>0.70</td>
<td>1680</td>
<td>0.85</td>
<td>2040</td>
<td>0.85</td>
<td>2120</td>
<td>0.85</td>
</tr>
<tr>
<td>0.75</td>
<td>1600</td>
<td>0.81</td>
<td>1790</td>
<td>0.75</td>
<td>1920</td>
<td>0.77</td>
</tr>
<tr>
<td>0.80</td>
<td>1440</td>
<td>0.73</td>
<td>1590</td>
<td>0.61</td>
<td>1720</td>
<td>0.71</td>
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<tr>
<td>0.87</td>
<td>1360</td>
<td>0.64</td>
<td>1360</td>
<td>0.57</td>
<td>1500</td>
<td>0.60</td>
</tr>
<tr>
<td>0.94</td>
<td>1110</td>
<td>0.56</td>
<td>1180</td>
<td>0.49</td>
<td>1380</td>
<td>0.55</td>
</tr>
<tr>
<td>0.96</td>
<td>1130</td>
<td>0.57</td>
<td>1130</td>
<td>0.47</td>
<td>1350</td>
<td>0.54</td>
</tr>
<tr>
<td>0.99</td>
<td>—</td>
<td>—</td>
<td>1075</td>
<td>0.45</td>
<td>1380</td>
<td>0.55</td>
</tr>
<tr>
<td>0.97</td>
<td>—</td>
<td>—</td>
<td>1050</td>
<td>0.44</td>
<td>2240</td>
<td>0.90</td>
</tr>
</tbody>
</table>
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The corresponding Bohr-Gaunt formula, which differs from the experimental value, for the above velocity, by nearly a factor of 2. (This corroborates the results for slow $\beta$-particles in § 2A.) The difference of 20 per cent. between the new formula (14), and the observed value, may partly be accounted for by the existence of a few electrons in mica with ionisation potentials of the order of the value of $W$, i.e., 1500 volts. In deducing (14) from Bethe's formula for $dT/dx$ we assumed that $J \ll W$ for all electrons.

It will be noticed that (14) is expressed in terms of the velocity of the $\beta$-particle, and not its energy, or momentum ($H_\beta$). Expressed in this way we see from the table that this non-relativity formula gives with remarkable accuracy the observed variation of $d_wT/dx$ with velocity, up to $\beta = 0.96$. The formula requires a decrease of 53 per cent. in $d_wT/dx$ for the velocity-range concerned, whilst the observed decrease is 43 per cent. This result shows conclusively that there cannot be a relativity factor such as $\sqrt{1 - \beta^2}$ outside the log. term in the formula. For the range of velocity concerned $\sqrt{1 - \beta^2}$ varies by a factor of nearly 3, whilst the results do not permit a correction greater than about 20 per cent.

The absence of a correcting factor outside the log. term is in accordance with the theory discussed in W2. In that paper it was shown that we must, however, introduce a correction inside the log. term in the quantum theory formula, in order to allow for the increase in the radius of action of the $\beta$-particle due to the Fitzgerald contraction of its field. This increased radius of action gives an extra loss of energy

$$(dT/dx)_R = (2\pi NZe^4/me^2) \log (1 - \beta^2)^{-1}.$$  \hfill \ (15)

This is practically all spent in energy losses less than $W = 1500$ volts, so that the new value of $d_wT/dx$ is

$$(d_wT/dx)_R = (2\pi NZe^4/me^2) \log \{2me^2W/\sqrt{E^2 (1 - \beta^2)}\}.$$ \hfill \ (16)

It is not proposed here to discuss the theory of this correction. We shall proceed to consider its relation to experiment.

The values of $d_wT/dx$ according to (16) are given in the sixth column of the above table. In order to show more clearly the variation with velocity the values of the ratio, $r$, of $d_wT/dx$ to the value of $d_wT/dx$ for the smallest velocity, are given in the table. It will be seen that the relativity correction we have introduced into Bethe's formula definitely improves the agreement with the experimental variation with velocity. In view of the importance of this
question of relativity correction, we shall also consider the results in terms of
\( \langle d_w T/dx \rangle = (2\pi N \zeta c^4/mu^2) \), a quantity denoted by \( P_w \) in W1. In view of the
slight discrepancies in the absolute values, it is the variation of this quantity
with velocity that is the most significant. Its non-relativity value, and
relativity-corrected value according to (16), are respectively given by

\[
P_{w, \text{N.R.}} = \log (2mc^2/E^2) + \log \beta^2 \tag{14A}
\]

\[
P_{w, \text{R.}} = \log (2mc^2/E^2) + \log \beta^2 + \log (1 - \beta^2)^{-1}. \tag{16A}
\]

The experimental and theoretical values of \( P_w \) are given in the last three columns
of the table. The increase in the experimental value of \( P_w \) as \( \beta \) increases
from 0·64 to 0·96 is 3·2 \( \pm \) 0·6. The increase according to (23) is 0·8, and
according to the corrected formula it is 2·9. Though the irregularities in the
experimental values are large, we may at least conclude that the correction
we have introduced is of the right order of magnitude.

\( \S 2 \text{ (b). Primary Ionisation.} \) The existing data for the primary ionisation
due to fast \( \beta \)-particles refer to the gases oxygen and hydrogen. The requirements of theory can be most accurately ascertained for the latter so that we
shall not consider the case of oxygen. The formula obtained by Bethe for the
primary ionisation in atomic hydrogen is

\[
I = (2\pi e^4 N/mu^2) 0.285 \log (42mu^2/J) = I_{\text{el.}} 0.285 \log (42mu^2/J). \tag{7}
\]

The value of \( I \) corrected for the extra relativity energy loss (15) is

\[
I_R = (2\pi N \zeta c^4/mu^2) 0.285 \log \{42mu^2/(1 - \beta^2) J\}. \tag{17}
\]

The theoretical values of \( I \) according to these formulæ, and the observed values
of \( I \), are given in the following table. The observed values are taken from the
smooth " \( I - H_\beta \) " curve through the experimental results obtained by
Terroux and the writer.* They refer to \( H_\alpha \) at N.T.P.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>Observed.</th>
<th>Non-relativity.</th>
<th>Relativity-corrected.</th>
<th>( L_{\text{obs}} )</th>
<th>( L_{N,R} )</th>
<th>( L_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( I )</td>
<td>( I/14.7 )</td>
<td>( I )</td>
<td>( I/12.6 )</td>
<td>( I )</td>
<td>( I/12.9 )</td>
</tr>
<tr>
<td>0·50</td>
<td>14·7</td>
<td>1·00</td>
<td>12·6</td>
<td>1·00</td>
<td>12·8</td>
<td>1·00</td>
</tr>
<tr>
<td>0·75</td>
<td>8·0</td>
<td>0·54</td>
<td>5·9</td>
<td>0·47</td>
<td>6·3</td>
<td>0·50</td>
</tr>
<tr>
<td>0·96</td>
<td>5·5</td>
<td>0·38</td>
<td>3·8</td>
<td>0·30</td>
<td>4·5</td>
<td>0·35</td>
</tr>
</tbody>
</table>

* *loc. cit.*
Passage of $\alpha$- and $\beta$-Particles through Matter.

The agreement between the absolute values for the smallest velocity is satisfactory and has already been referred to in § 1 (b).

The relation of the observed values of I for the highest velocities to the requirements of the non-relativity formula shows, as in the case of $d_{\nu}T/dx$, that there is no possibility of a relativity correction of the type $(1 - \beta^2)$ outside the log. term in the formula. At the same time the necessity for some correction is indicated, and since this must be put inside the log. term we shall now consider the values of

$$L = I / 0.285 (2\pi Ne^4/mv^2) = I/0.285 I_{cl}. \quad \text{(18)}$$

The theoretical values of $L$ corresponding to (7) and (17) are

$$L_{NR} = \log (42mc^2/J) + \log \beta^2 \quad \text{(7A)}$$

$$L_{R} = \log (42mc^2/J) + \log \beta^2 + \log (1 - \beta^2). \quad \text{(17A)}$$

The observed values of $L$ and those required by these formulae are given in the last three columns of the table. We see that the relativity correction introduced into the formula accounts for a large fraction of the increase in $L$ as $\beta$ approaches unity. The whole effect concerned is rather small, and it requires greater accuracy in the values of the primary ionisation than was aimed at in the experiments of Terroux and the writer, to decide whether the correction is adequate.* We might incidentally point out that the classical value of $L$ is a constant equal to 3.5.

* In addition to the stopping-power and ionisation, observations have also been made on the number of branches produced by fast $\beta$-particles (Williams and Terroux, loc. cit.). A reference to the results obtained may be made. They show that the frequency of branch collisions is about 1.6 times the non-relativity value required by the quantum theory for $0.6 < \beta < 0.8$, and about 2.4 times the non-relativity value for $0.8 < \beta < 0.96$. The total number of branches involved in the results is only 44 and the statistical errors are, therefore, fairly large. Nevertheless, it is fairly certain that a relativity correction of a first order of magnitude is required. This is in contrast with the state of affairs for $d_{\nu}T/dx$ and primary ionisation, and two important differences between the cases may be pointed out. In the first place the maximum deflection of the $\beta$-particles in the case of $d_{\nu}T/dx$ and primary ionisation is only 1 or 2 degrees, as compared with deflection of the order of 10° produced by branches. Secondly, binding forces play an important part in $d_{\nu}T/dx$ and primary ionisation, but have negligible effect in branch collisions. The writer is inclined to believe that it is the greater angle of scattering that brings about the greater relativity effect. If that is the case, we may infer that the relativity expression for $\phi(Q)$ for $\beta$-particles traversing free electrons is of the form

$$\phi'(Q) = \phi_0(Q) \left\{ 1 + A \times F(Q) \times f(\beta) \right\}, \quad \text{(i)}$$

$f(x)$ and $F(x)$ both increasing with $x$. $\phi_0(Q)$ is the non-relativity value of $\phi(Q)$. A correction of this form might arise from the spin interaction between a $\beta$-particle and a "knocked" electron.

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§ 2 (c). Application to Very High Energy $\beta$-particles.—The $\beta$-particles associated with penetrating radiation, and the "runaway" electrons from thunderstorms, have energy of the order of $10^9$ volts. There is no experimental data at the present time for the behaviour of such $\beta$-particles, and it may be of interest to apply to them the relativity formulæ put forward in the preceding sections. As we have seen, these formulæ appear to give satisfactorily the relativity effect at $10^6$ volts. We shall only consider the ionisation.

The theoretical primary ionisation according to (17A) is proportional to $\beta^{-2}\{k + \log \beta^2 \log (1 - \beta^2)^{-1}\}$. For hydrogen this quantity passes through a minimum value of $4.5$ primary ions per centimetre at $\beta = 0.97$ ($1.5 \times 10^9$ volts). It then increases with increasing velocity and reaches a value of $8$ ions per centimetre at $\beta = 0.970^* (10^9$ volts). The value of $k'$ in the above expression is nearly constant for light elements, so that the percentage changes in the ionisation are nearly the same for oxygen and air as for hydrogen. For oxygen the observed number of primary ions per centimetre produced by $\beta$-particles with velocity of $0.97c.$ is 22. The theoretical number produced by $\beta$-particles with energy of $10^9$ volts is therefore about 40. There is little difference between the primary ionisation in air and in oxygen, so that about the same number of primary ions would be produced in air. The numbers given refer to the gases at N.T.P.

The theoretical total ionisation produced by the high energy $\beta$-particles under consideration will similarly be about 70 per cent. greater than that due to $1\frac{1}{2}$ million volt $\beta$-particle. The observed probable total ionisation per centimetre in air due to the latter is about 40, so that the probable number of ions produced by $10^9$ volt electron will be about 70.†

It is interesting to notice that when $\beta$ is close to unity, the formulæ (16) and (17) depend only on $(1 - \beta^2)$, so that a $\beta$-particle with T volts produces the same ionisation as a proton with $1840$ T volts. A proton with $3 \times 10^8$ volts energy therefore produces the same ionisation as a $\beta$-particle with about $1\frac{1}{2}$ million volts.

§ 4. Summary and Conclusion.

The non-relativity theory of the passage of electric particles through matter developed by Bethe on the basis of Born's theory of collisions is compared with experimental results for the stopping-power, primary ionisation, total

* On p. 126, Table III, last line, this is inadvertently shown as $0.97\gamma$.
† The most probable ionisation is referred to because it is the most likely quantity to be measured if individual particles are dealt with as in the Wilson cloud method. In that case the abnormally high ionisation when a "branch" is produced is likely to be left out.
ionisation, straggling, and production of branches, by $\alpha$- and $\beta$-particles. Some of these phenomena, which have long resisted a quantitative theoretical explanation, are found to be accounted for by the new theory. For instance, in the simple case of the stopping-power of hydrogen for slow $\beta$-particles—simple because there is only one energy level and because the velocity of the $\beta$-particles concerned is large compared with that of the hydrogen electrons traversed and small compared with that of light—neither the calculations of Bohr on the classical theory, nor of Henderson on the old quantum theory, nor of Gaunt on the new quantum theory, give results within 40 per cent. of the experimental value. Bethe's formula (corrected here for exclusion principle effect) gives the observed value within the experimental error of a few per cent. Again, in the case of the primary ionisation produced in hydrogen by moderately fast $\beta$-particles, the value given by the classical theory is four times less than the observed value. The value deduced from Gaunt's calculations on the quantum theory is nearly twice the observed value. Bethe's formula for primary ionisation gives the observed value within experimental error ($\sim 10$ per cent.). It would seem from these cases that the passage through matter of electric particles, with moderate velocity, was at last solved—to establish further the non-relativity quantum theory. That state is, however, not reached. The total ionisation in the monatomic gases, and the straggling of $\alpha$-particles in light elements, present difficulties to the theory of Bethe as they do to previous theories. The positions regarding these effects are closely considered and, though they are disquieting, it is found that the results do not necessarily mean a refutation of the theory. Further experiment and calculation is necessary to make the situation clear.

In the second part of the paper the experimental results for fast $\beta$-particles are discussed. The non-relativity formulae of Bethe for stopping-power and primary ionisation, when expressed in terms of velocity, represent to within 20 per cent. the variation with velocity up to $\beta = 0.96$. At the same time the experimental results indicate fairly definitely the necessity for a small relativity correction. The introduction into Bethe's non-relativity formulae of a correction deduced in a previous paper by the writer is in the right direction and of the right order of magnitude. The paper concludes with an extension of the results obtained for the relativity effect to high energy $\beta$-particles, such as those associated with penetrating radiation.

In conclusion, I wish to thank Professor W. L. Bragg for his continued interest in this work, and also express my appreciation of discussions with Dr. Bethe on the subject.