Relativistic Invariance in Quantum Mechanics (*).

E. P. WIGNER
Palmer Physical Laboratory - Princeton, N.J.

(ricevuto il 30 Agosto 1955)

Summary. (*). — A detailed analysis is made of the theoretical possibilities of constructing a quantum mechanics and consequently of a description of the elementary particles, based on a definition of the postulates of relativistic invariance and then on the symmetry properties connected with the complete Lorentz group, including displacements in space and time and inversions of both the space and the time coordinates. It is then pointed out how the validity of every conclusion deriving from symmetry considerations depends essentially from the fundamental problem of the measurability of the field quantities.

(*) Editor's care.

1. — Introduction.

Once quantum mechanics has proved to be able to give, in principle, a satisfactory account of everyday phenomena (1), its reconciliation with, and adjustment to, the principle of relativity became one of the foremost objectives of theoretical physics. The period began around 1925; the articles of DIRAC on the relativistic equation of the electron (2) and of HEISENBERG and PAULI on quantum electrodynamics (3) were the first two milestones on the road toward it. The full aim of the reconciliation of the two most important physical theories of our century has not yet come to a conclusion although, more recently, the problem of elementary particles begins to compete for the first


place in the interest of the theoretical physicist. In the period, since 1925, a wealth of ideas has been formed and it would be vain for me to attempt a full review of them. I must ask forgiveness of those whose thinking I appreciated too little or failed to understand. All of us share the fault of appreciating ideas of others less than those of ourselves.

Because of the wealth of problems, points of view and results on the role of symmetry in quantum mechanics, I shall have to limit myself rigidly in the choice of the subjects to be discussed and even as far as this subject is concerned, I shall discuss principally only one point of view. The point of view which I shall adopt is that the problems of physics are still rather far from their solution and that the role of symmetry and invariance is that of a guide in the development of the proper physical concepts, rather than something that one simply reads off from the ready equations. To illustrate this, I shall not say that physical equations are invariant under rotations in space because the Hamiltonian contains only distances and the absolute values of the momenta. Rather, my point of view would be to search for those Hamiltonians which give a physical theory that is invariant under spatial rotations and other relativistic transformations. This point of view will be the one adopted throughout and my endeavour will be to be quite consistent in this.

Second, I shall restrict myself entirely to the symmetry demanded by the special theory of relativity; that is, the group which will underly my considerations will be the Lorentz group, including displacements in space and time and inversions of both the space and the time coordinates. It was after some hesitation that I decided to restrict myself in this way because the attempt to incorporate at least some of the ideas of general relativity into quantum mechanics is both a challenging and a fascinating one.

Third, I shall restrict myself to the consideration of the symmetry elements of space-time. This will exclude, in particular, the symmetry properties of interaction. This applies even to the symmetry between positive and negative charges—so very important a symmetry and so natural to consider for every physicist (4). I did this because it is hardly possible to know in this regard where to stop: from charge conjugation one is led to isotopic spin and to the generalizations of these concepts to explain properties of the semistable particles found in cosmic ray phenomena (5). It was reassuring, therefore, to read in our program that the theories of the semistable particles will be treated by other speakers.


Finally, even as far as the subject of space-time symmetry is concerned, I will try to present a point of view rather than detailed results.

2. – Relativistic Invariance.

Let me begin with the question of what we mean by relativistic invariance. In macroscopic, that is non-quantum, theory this concept is easily defined. To do it, the concept of the complete description of a physical system is useful. This shall consist of a full specification of the paths of all particles, together with a full description of all fields at all points of space-time. Given such a complete description of a physical system in a coordinate system, the equations of motion permit one to determine whether the complete description is compatible with them: the equations of motion give a criterion whether the physical system could have behaved in the way specified by the complete description. The principle of relativistic invariance, or of any invariance, then makes three postulates. These have been stated, with admirable clarity, a short time ago by R. Haag (6) in an article which I have not yet seen in print. They are

(a) It should be possible to translate a complete description of a physical system from one coordinate system into every equivalent coordinate system.

(b) That the translation of a dynamically possible description be again dynamically possible. Expressed in a somewhat more simple language: a succession of events which appears possible to one observer should appear possible also to any other observer.

(c) That the criteria for the dynamical possibility of complete descriptions be identical for equivalent observers.

It is not customary to formulate the principle of invariance as explicitly as I just did. In particular, the possibility of translating the description of the behavior of a physical system from one coordinate system into another is usually taken for granted. The same applies to postulate (b). The remaining postulate can then be stated as the invariance of the equations of motion. However, if one wishes a precise formulation of the principle of invariance in quantum theory, the translation of the languages of the different observers becomes a crucial point. It may be well, therefore, to emphasize the problem of translation already in macroscopic, that is, classical theory.

The problem of translation between different coordinate systems, or be-

between different observers, becomes so much more serious in quantum theory because the basic concept of quantum theory is that of observation by individual observers (7). If the observation of one observer can not be translated into the language of other observers, the principle of relativity becomes practically meaningless. This would be true, for instance, if the observation were an instantaneous process but extending over a finite part of space (or over all space), as would correspond to most measurements considered in non-relativistic quantum theory. Since the $t =$ constant planes of space-time are different for different observers, there is no obvious way to translate the observations of observers in relative motion with respect to each other. In order to make a translation possible, one usually introduces a wave function which summarizes the results of all previous measurements. One then can formulate the translation from one coordinate system into another in terms of the wave function or state vector.

The wave function or state vector also contains all possible information on the probabilities of the outcomes of future experiments. The translation therefore relates the expectations of different observers regarding the outcomes of experiments which they may carry out on a system. One sees that the translation has much more content in quantum theory than in classical theory; it answers a much more intricate question. One also sees that the fewer experiments are permissible «in principle», the easier it will be to satisfy the principle of invariance, the less meaningful such a principle will become.

All wave functions or state vectors which are constant multiples of each other are often said to form a ray. If all self adjoint operators are observable, it is possible to distinguish between any two rays in Hilbert space, that is between any two vectors which are not constant multiples of each other. If not all self adjoint operators are observable, as is undoubtedly the case (8), it may not be possible to distinguish every pair of rays in Hilbert space. This will make it easier to satisfy the principle of relativistic invariance but will make it also a bit less significant. I wish therefore to devote a few minutes to the discussion of observables, or to the equivalent question of the possibility of distinguishing between state vectors even though this discussion will prove somewhat futile.


3. – Heisenberg and Schrödinger Representations.

There are two ways in which quantum mechanics can be formulated: the Heisenberg picture and the Schrödinger picture. The principal concepts of the Heisenberg picture are certain observables and measurements. These are the ones which change in time and are subject to the laws of motion. The state vector, which lives a relatively withdrawn life, remains unchanged. In the Schrödinger picture, on the contrary, the wave function is the principal concept; it is the one which changes in time while the operator of a measurement remains the same, no matter at what time the measurement is carried out. In the Schrödinger picture the operators have rather little prominence, they can be replaced rather completely by transition probabilities.

The same two pictures are possible also in relativistic quantum theory but I wish to submit that one obtains a more simple and unified picture if one looks at the laws of motion as a particular relativistic transformation: as a time displacement. The Heisenberg’s equations of motion for operators are on a par with the equations for the other infinitesimal transformations of relativistic invariance. From this point of view, the equations

\[ -i \frac{\partial \psi}{\partial x^\mu} = [P_\mu, \psi] \]

are four equivalent equations giving the infinitesimal displacements in space and time. Similarly, Schrödinger’s equation for the time rate of change of the wave function is one of the transformation equations of the state function under the inhomogeneous Lorentz group; the hermitean nature of Hamilton’s operator guarantees the time displacement operators to be unitary. The point of view which I am advocating extends the concepts of Heisenberg and Schrödinger picture to all relativistic transformations—of which the equation of motion is only one—otherwise it does not introduce any new element into the theory.

In spite of the well known and oft discussed equivalence of the Heisenberg and Schrödinger pictures, the three postulates of relativistic invariance present themselves in rather different lights in the two pictures and focus the attention at different aspects of the theory. Let me begin with the Schrödinger picture. Postulate (b) when formulated in terms of transition probabilities simply means

---


(10) The concept of time dependent operators was introduced already by M. Born and P. Jordan: *Zeits. f. Phys.*, 34, 858 (1925).

(11) These equations are contained already in the articles of reference (9).
that the transition probability is independent of the frame of reference: if \( \psi \) and \( \psi' \) are two states and \( \varphi', \psi' \) their translations, the transition probabilities \( \varphi \rightarrow \psi \) and \( \varphi' \rightarrow \psi' \) are equal. One can deduce from this and the distinguishability of all rays in Hilbert space by a purely mathematical argument \(^{(12)}\) that the translation is effected by a unitary or anti-unitary operator

\[
\varphi' = O\varphi; \quad \psi' = O\psi;
\]

where \( O \) is unitary or antiunitary. The operator \( O \) depends, of course, on the two coordinate systems between which it effects the translation.

It then follows from postulate \((c)\) that the operation \( O \) depends only on the relation of the two frames of reference, not on their absolute position in space-time. From this, one can conclude again by a purely mathematical argument \(^{(12)}\) that the operators \( O \) form, up to a factor, a representation of the group connecting the equivalent frames of reference. This group is the inhomogeneous Lorentz group in ordinary special relativity theory. In the Schrödinger picture, the expression for the relativistic invariance is simple and concise and one is naturally led to consider the simplest sets of state functions which are already relativistically invariant. As is well known—and I shall review the underlying arguments—the set of all possible states of each elementary particle forms such a simplest relativistically invariant set and I shall also review the question which of the properties of these particles can be obtained from the postulates of relativistic invariance. While this part of the consideration of the simplest systems of the Schrödinger picture is quite satisfactory, it must be emphasized that the basic postulate that all self adjoint operators are observable is surely incorrect and it is equally incorrect to assume that all wave functions which differ more than by a multiplicative factor can be distinguished \(^{(8)}\). This fact casts a shadow on the whole theory as one does not know how far one will have to go in the restriction of the concept of the observable and which of the wave functions represent the same physical system. A second equally great, or perhaps even greater flaw of the Schrödinger picture as here defined is that it does not naturally suggest the consideration of local field observables or anything to substitute for these.

When turning now to the Heisenberg picture, it is well to reemphasize that the actual content of the Heisenberg and Schrödinger pictures in their original forms is the same and that the same physical theories can be formulated in either picture. However, in the Heisenberg picture it is more natural to speak about the translation of observables (rather than wave functions) and to consider such observables which have a meaning in all coordinate systems. Since

\(^{(12)}\) E. P. WIGNER: *Gruppentheorie und ihre Anwendungen etc.* (Braunschweig, 1931), Anhang to Chapter XX.

the usual non-relativistic observables refer to an instant of time but to at least a finite portion of space, their set is not translatable: most non-relativistic observables of a coordinate system at rest are not observables for a moving coordinate system. This difficulty is avoided in the Schrödinger picture by first extracting an essence from the observables, the state vector, and translating only the state vector. It is more natural, from the point of view of the Heisenberg representation, to translate the observables directly and this is more natural also physically. It implies the consideration of such sets of observables the members of which have a meaning for all frames of reference. Such sets of observables can be obtained from the non-relativistic set in two ways: either by extending that set or by restricting it. If one chooses the second way one is naturally led to observables which are defined at points of space-time, i.e. which are not only instantaneous but also refer to a single point of space. In this way one is led, rather naturally, to the operators of local fields. The other way to make the operators directly translatable between coordinate systems is to permit them to assume a finite extension not only in space but also in time. This possibility of non-local measurements has not been explored very fully. Both points of view are in a certain contrast with that of the Schrödinger picture as I presented it: this does not demand a direct translation of observables but only a translation of a concentrate of the results of observations, of the state vector. The fundamental equivalence of the Heisenberg and Schrödinger pictures manifests itself in the following way. If there are enough observables, the corresponding self adjoint operators will distinguish between any two rays of Hilbert space. Then, two sets of operators which satisfy our postulate (b) can be transformed into each other by a unitary similarity transformation \( Q \rightarrow UQU^{-1} \) or by such a transformation coupled with transition to the conjugate imaginary. This is again a purely mathematical result. If these transformations satisfy postulate (c) they again form a representation of the group connecting equivalent observers.

This argument is unquestionably correct but it can be replaced, in local field theories, by a more direct argument which is based on the fact that the field operators have a direct translation into every coordinate system—they transform like scalars, vectors, etc. The equations of motion are the same in each coordinate system and they are invariant if the field quantities are properly transformed. Thus the operators are directly translated and the translation satisfies postulates (b) and (c). The existence of a similarity transformation \( UQU^{-1} \) to transform field quantities from one to another coordinate system would not be necessary if one could really consider the field operators as the only observables. The existence of \( U \) is nevertheless usually inferred from the invariance of the commutation relations between fields \(^{(14)}\). This

\(^{(14)}\) Cg. e.g. J. M. JARCH and F. ROHRICH: The Theory of Photons and Electrons (Cambridge, Mass., 1955), p. 11.
inference is surely not valid; whether it is correct to the extent to which it is meant to be correct, is open to some doubt (15). Nevertheless, the success of the field theories and simplicity of the preceding argument suggest a more detailed discussion of the problems of these theories.

4. – The Field Theories.

The restricted set of operators which were defined as local field operators do not form a sufficiently large set to distinguish between all rays in Hilbert space. Therefore, if one adopts the restriction that only local fields are observable, one has significantly modified or rather restricted the original theory. In view of the history of theoretical physics in the last ten years, which is a rather unbroken line of successes of local field theories, one is very much tempted to limit the concept of observables to these local fields. As was mentioned before, they form a relativistically invariant set. Furthermore, the limitation of the concept of observables to these is rather natural and excludes in particular all observables which cause trouble with the relativistic invariance and some of which are, in fact, so far removed from ordinary observables that one would not know how to go about their measurement. The superselection rules about which I am going to speak are also an expression for the fact that not all self adjoint operators are observable. The limitation of the observables to fields includes all superselection rules which have been recognized so far (16).

The postulate that only field quantities are observable has not been formulated to my knowledge as explicitly as I just formulated it but must have been present in the minds of many field theoreticians. In particular, Feynman once made a statement which came very close to this. It is, of course, true that several difficulties have to be overcome before one will be fully satisfied that it is reasonable to consider only the field quantities as observables. First among these is the definition of the field after renormalization. Second, it will be necessary to modify the analysis of field measurements, as given by Bohr and Rosenfeld and extended by Corinaldesi (17). It seems to me


that in its present form this analysis shows rather the difficulty than the possibility of local field measurements. Third, if one looks at the set of local fields more closely, they form only a limiting case of a relativistically invariant set and there is no set of field quantities the measurement of which would be truly translatable. Thus even in a purely electromagnetic field, if one knows the magnetic field on a space-like surface, all one can say about the electric field, or about the magnetic field in another frame of reference, is that it is most probably infinite. The great successes of the field theories of which we are so proud are mostly in the line of calculations and not in the line of conceptual clarifications. For these reasons it appears doubtful to me that the solution of the problem of observables is as simple as restriction to local field quantities.

5. - Discussion of Elementary Particles by means of the Schrödinger Picture.

It was mentioned before that, in the Schrödinger picture, a unitary or antiunitary transformation corresponds to every relativistic transformation and also that the relativistic transformations include, as a particular case, the equation of motion. It would be erroneous to infer from this that the transformation properties of a system will define all its physical properties. The transformation properties and the equations of motion are only one aspect of the behavior of a physical system. There are many others, such as the configuration of the particles contained therein, which do not affect the transformation properties but remain physically significant. This comes back to the general question of observables and it is clear that not all observables can be uniquely determined on the basis of their transformation properties.

The opposite problem of finding the transformation properties, that is the representation, for a given system, is somewhat easier to solve. If we consider, in particular, an elementary particle, it is natural to postulate that it should not be possible to decompose its states into linear sets which are also relativistically invariant. If such a decomposition were possible, one would call each subset of invariant states the states of a different particle. The same assumption or convention, expressed in terms of the concepts of representation theory, states that the representation which gives the transformation of the states of an elementary particle is irreducible. Irreducible is the mathematical term for the absence of linear sets of states (except the set of all states) which are invariant under the group considered. One is thus led to the conclusion that an irreducible representation of the inhomogeneous Lorentz group—that is the group of Lorentz transformations and displacements in space and time—corresponds to every elementary particle. The representatives can be unitary or antiunitary operators.
The point of view which is outlined above (18) is not quite customary and has often been misunderstood. It is more customary (19) to start with a wave function which, similar to Dirac’s wave function (2), contains the variables of space-time and a spin coordinate and find equations which can be transformed relativistically. This more customary procedure has the advantage that it gives a description in which the coordinates are diagonal. It has the disadvantage of proceeding by trial and error. It has the further disadvantage that the same physical situation can be described by a variety of equations and it is not easy, within the framework of the theory, to recognize which equations are equivalent. If one is malicious, one can say that it has the disadvantage of its advantage being illusory because the variables \( x, y, z, t \) do not actually correspond to the position of the particle in space-time: actually in many cases, such as that of light quanta, it is not physically meaningful to define the position of the particle (20). This does not diminish, of course, the merit of the original discovery of invariant equations, such as Proca’s.

The standpoint which I am adopting conforms with the view that all relativistic transformations are on the same footing and that the equation of motion is a special case of these. The fact that all representations have infinitely many dimensions is only an expression for the fact that each particle is capable of assuming infinitely many linearly independent states. These are distinguished principally by their momenta, or positions, and considering the state of affairs as it really exists, it would be most embarrassing if any physical system could assume only a finite number of linearly independent states.

The first question which arises in connection with the investigation of the irreducible representations is: to which coordinate transformations correspond unitary and to which antiunitary operators. As far as the elements of the proper inhomogeneous Lorentz group is concerned, the answer is easy to give. All elements of the proper group can be obtained continuously from the unit element (this is what one means by the proper group) and the corresponding transformations must be obtainable continuously from the unit transformation.

\(^{(18)}\) It is contained in various publications such as reference (13) and V. Bargmann and E. P. Wigner: *Proc. Nat. Acad. Sc.*, 34, 211 (1948); E. Inoue and E. P. Wigner: *Nuovo Cimento*, 9, 719 (1952); R. Haag: reference (15).


This is not possible for antiunitary transformations and the operators which correspond to elements of the proper group, that is which do not involve either a space or a time inversion, must be unitary.

Since every transformation of the inhomogeneous Lorentz group is either a member of the proper group, or is a product of a member of the proper group and a space inversion, or a time inversion, or both, it suffices to determine the unitary or antiunitary character of the operations which correspond to inversions. It is important to note then that if there are two quantum mechanical systems in one of which a unitary operator corresponds to a symmetry transformation (such as an inversion), while in the other one the same symmetry transformation is represented by an antiunitary operator, the two systems can not be united to form a common system, even if one excludes interaction, unless all the observables which relate to the joint system are represented by sums of two operators, one of which refers entirely to the first, the second one entirely to the second system. This appears a too drastic restriction of the observables and one must conclude that every transformation is represented either in all physical systems by a unitary transformation, or in all physical systems by an antiunitary transformation. The first is the case for space inversions, the second for time inversion (21).

Once the unitary or antiunitary nature of the relativity transformations is known, the determination of the irreducible representations is again a purely mathematical problem (22). The results will not be given here in detail. Each representation is characterized by two parameters which will be called, in anticipation of considerations which follow, mass and spin. If the mass is positive, it can assume any value: relativistic invariance alone does not give any indication of possible mass values or their ratios. The spin determines the number of linearly independent states which have the same momentum four vector, except that this number is always 1 or 2 if the mass is zero, that is, if the momentum vectors are on the light cone. Second, contrary to common opinion, the parity of a particle is not determined by its relativistic transformations alone. The same applies for its transformation with respect to time inversion.

The representation is something very abstract and before it can be considered to give a theory of the elementary particle, one must find the observables for the most common physical quantities. This can be done by accepted methods which have, however, little to do with the considerations which led

(21) This point is indicated also by H. Umezawa, S. Kamefuchi and S. Tanaka: Prog. Theor. Phys., 12, 383 (1955). The same result was obtained by the present writer on the basis of the consideration of time displacements and the positive nature of the energy (Göttinger Nachr., 1932, p. 546).

(22) See reference (13). It should be noted, however, that the time inversion considered there is Pauli's unitary operator (cf. Rev. Mod. Phys., 13, 203 (1941)).
to the representation. It may be of some interest, therefore, to point out that the operators for the components of the momentum can be determined, except for a factor, if it is demanded that they be invariant with respect to displacement and transform, under homogeneous Lorentz transformations, as a four vector. The only operators which satisfy this requirement are the infinitesimal operators of displacement (or constant multiples of these)—there is no other quartet of operators in the Hilbert space of the representation which is translation-invariant and a four-vector. This also provides a justification for our having called mass the length of the four vector formed by these infinitesimal operators. Newton and I tried to obtain the position operators in a similar way (22) and were led to one of the possibilities envisaged already by Papapetrou and by Maurice Pryce (25).

A somewhat more intricate and perhaps also more interesting question is that of the formation and disintegration of semistable compounds. A full answer to this question would be actually much more informative than might appear offhand: all unstable particles, such as mesons, can be considered to be semistable compounds and most reactions between particles can be thought of as proceeding via the intermediate formation of an unstable compound. Similar to the situation with respect to the masses of elementary particles, symmetry considerations permit one to draw no conclusion as to the actual energy of the semistable compounds, they only permit the definition and general characterization of these compounds. The corresponding rules were largely determined already by Michel (34), they give the characteristics of the compound states which can be formed by the collision of two particles with given spin and mass. The most important ones of these rules we owe to Landau and to Yang (25), the oldest one is that which states that a \(0 \rightarrow 0\) transition with the emission of a light quantum is absolutely forbidden. A 0 spin semistable particle cannot be formed in the collision between a 0 spin particle and a spin 1 particle with zero restmass (light quantum). The results for finite restmass are known from collision theory; if only one of the restmasses is zero, they show a great resemblance to the states of molecules, the spin of the zero restmass particle assuming the role of the electronic angular momentum along the internuclear axis. It should be pointed out that the aforementioned selection rules do not tell the whole story which can be obtained from symmetry arguments: inferences can be drawn concerning the state of

---


(25) Cf. L. Michel: Report on the 1953 meeting of the IUPAP, p. 272. This article contains also information on the rules resulting from the symmetries mentioned in reference (4).

polarization of the disintegration products of the semistable particle which are equally interesting (26).

I like to illustrate this with an example: the measurement of the parity ratio of two particles with zero spin. At the same time, this will throw some light on the question of the parity i though this question will be discussed, I presume, more in detail in later reports. In order to measure the parity difference between two particles, which we assume for the sake of simplicity to have zero spin, we can let a beam of polarized slow neutrons impinge on the particle with higher restmass. The neutrons can be replaced, of course, by other particles with spin \( \frac{1}{2} \). If the incoming particle is sufficiently slow only its \( s \) part, i.e. the \( l = 0 \) part, will react with the spin zero particle. Because of the axial nature of the spin, the collision system will have a plane of symmetry in the direction perpendicular to the direction of polarization of the incoming neutron. The neutrons will then react with the spin zero particle and will transform it, with a certain probability, into the spin zero particle with the smaller restmass. The mass difference between the two spin zero particles will increase the kinetic energy of the neutrons so that a certain fraction of the outgoing neutron beam will have a higher kinetic energy than the ingoing beam and will be distinguishable therefrom. If there is no parity difference between the two spin zero particles, the outgoing wave will also be an \( s \) wave, i.e. will have \( l = 0 \), and the direction of the spin of the neutrons will not have changed. The two spin components of the outgoing wave will be

\[
\psi_{\text{spin as originally}} = \frac{1}{r} \exp \left[ ikr \right] \quad \text{and} \quad \psi_{\text{spin flipped}} = 0.
\]

If the parities of the two spin zero particles are different, the outgoing neutron wave with the higher kinetic energy will be a \( p \)-wave, i.e. will have \( l = 1 \), and its two spin components will be, in the plane perpendicular to the direction of polarization of the incoming neutron, asymptotically

\[
0 \quad \text{and} \quad \exp \left[ i\varphi \right] \frac{1}{r} \exp \left[ ikr \right].
\]

It will be possible to observe the parity difference by observing the direction of the spin of the outgoing neutron in the plane perpendicular to the original direction of polarization: if there is no flip in the spin direction, the two parities

are the same, if there is a flip, the parities are opposite. This is a quantum
mechanical measurement in the best orthodox sense (7): corresponding to the
two possible values of the quantity to be measured, there are two possible
outcomes of the measurement. I brought this example up to illustrate further
relations based solely on symmetry considerations which go beyond the re-
lations of Table I.

\[ \begin{align*}
\text{Table I.} & \quad \text{Semistable Particles resulting from the Collision of two Particles.} \\
\text{Two identical particles (*)}. & \\
& \text{s}_1 = s_2 = 0 \rightarrow J = 0, 2, 4, \ldots \text{ with even parity. No states with odd parity.} \\
& \text{s}_1 = s_2 = \frac{1}{2} \rightarrow J = 0, 2, 4, \ldots \text{ with even parity. All } J \text{ with odd parity.} \\
& \text{s}_1 > s_2 ; m > 0 \rightarrow \text{All } J \text{ with both even and odd parities.} \\
& \text{s}_1 > s_2 ; m = 0 \rightarrow \begin{cases} J = 0, 2, 4, \ldots \text{ with both even and odd parities.} \\
J = 2s, 2s+1, \ldots \text{ with parity } (-)^s. \end{cases} \\
\text{Two different particles with finite restmass.} & \\
& \text{s}_1 = s_2 = 0 \rightarrow \begin{cases} J = 0, 2, 4, \ldots \text{ with even parity.} \\
J = 1, 3, 5, \ldots \text{ with odd parity.} \end{cases} \\
& \text{s}_1 = 0; s_2 > 1 \rightarrow \text{All } J \text{ with both parities, except } J = 0 \text{ only with parity } \omega = (-)^s \omega_1 \omega_2. \\
& \text{s}_1, s_2 > \frac{1}{2} \rightarrow \text{All } J \text{ with both parities.} \\
\text{First particle with finite, second with zero restmass.} & \\
& \text{s}_2 < \frac{1}{2} \rightarrow \text{same as two different particles with finite restmass.} \\
& \text{s}_1 = 0, s_2 > 1 \rightarrow J = s_2, s_2 + 1, s_2 + 2, \ldots \text{ with both parities.} \\
& \text{s}_1 > s_2 > \frac{1}{2} \rightarrow \text{All } J \text{ with both parities.} \\
& \text{s}_2 > s_1 > \frac{1}{2} \rightarrow J = s_2 - s_1, s_2 - s_1 + 1, s_2 - s_1 + 2, \ldots \text{ with both parities.} \\
\text{Two different particles with zero restmass.} & \\
& \text{otherwise } \rightarrow J = |s_1 - s_2|, \ |s_1 - s_2| + 1, \ |s_1 - s_2| + 2, \ldots \text{ with both parities.} \\
\end{align*} \]

(*) The particles with integer and half integer spin were assumed to have symmetric and anti-
symmetric wave functions, respectively, and the parities were assumed to be \( \pm 1 \) and comparable
throughout; \( s_1 \) and \( s_2 \) are the spins of the colliding particles. The case that the mass \( m \) of the
identical particles is 0 is included under this heading.

The experiment which I mentioned also illustrates the point that if the
parity relation is measurable it can have only two values: equal and opposite.
In particular, any linear combination of the wave functions which correspond
to the two cases, such as one with the two components.

\[ \alpha \frac{1}{r} \exp [ikr] \quad \text{and} \quad \beta \exp [ip] \frac{1}{r} \exp [ikr], \]

\[ \begin{align*}
& x \frac{1}{r} \exp [ikr] \quad \text{and} \quad \beta \exp [ip] \frac{1}{r} \exp [ikr], \\
\end{align*} \]
would yield a spin direction of the outgoing neutron in the plane perpendicular to the polarization of the incoming neutron as illustrated. Such a distribution of the directions of polarization will not permit a symmetry plane perpendicular to the original direction of polarization and would be in direct conflict with the symmetry principle which establishes the concept of parity.

The preceding consideration can be generalized to show that if the parity difference between two particles can be measured, it can yield only the two results «equal» or «opposite». Can we conclude from this that only two parities exist? In our opinion (37), the answer to this question is «no», because it is quite possible that none of the measurements will yield any result. It would mean, for the preceding example, that the neutrons simply do not transform the spin zero particles into each other so that there is no outgoing beam on which to measure the spin. If there is such a block against measuring the spin we say that a superselection rule separates the two types of particles. The qualification «super» is added, not only for nationalistic reasons, but also to indicate that not only is a spontaneous transition impossible but that a transition cannot be induced even by measurements.

The preceding consideration, and the generalization to which I alluded, shows that all measurements of parity changes can give either one of the results: same parity, opposite parity, or give no result. It is possible that the matter simply ends here and that it is not reasonable at all to define a parity difference between two particles. In my opinion this is the case, for instance, as far as protons and electrons (or positrons) are concerned. However, it should be noted that the preceding consideration refers only to the parity change between one particle of each kind. Even if it should be, in principle, impossible to define such a parity difference, it is quite conceivable that the parity of a pair of particles A can be compared with the parity of particle B. In this case—and this is a certain change of point of view on my part—it might be desirable to define a parity $i$ or $-i$, or similar directly not measurable parities.

6. — Conclusion.

The question of symmetry, in particular, relativistic symmetry in quantum theory, is one in which not only I but several of my colleagues are deeply interested (37). The point of view which I adopted was that our knowledge

\[37\] My indebtedness is particularly great to Drs. V. Bargmann and A. S. Wightman.
of the interaction of particles is not yet adequate to read off the symmetry properties from the Lagrangians which we use. Rather, the symmetry properties should be a guide in establishing the proper physical picture. The element which was emphasized particularly on the basis of this point of view is that the conclusions at which one can arrive on the basis of symmetry considerations depend very much on the type of measurements which are possible «in principle». The situation is most similar, perhaps, to that in thermodynamics in which, also, many results depend on the type of equipment that one can use «in principle» to construct a perpetuum mobile. The more liberal one is in permitting in practice unrealizable equipment to establish a perpetuum mobile, the more the scope of thermodynamics is increased, the greater the number of phenomena which we can treat with it. Similarly, the symmetry considerations will have greatest scope if we permit most «measurements» as possible in principle, if we consider it permissible to assume any type of interaction with the measuring equipment. However, we know that there is a limitation in this regard, that it is, for instance, not permissible to assume that every self adjoint operator in the Hilbert space of field theories is measurable. The exact limitation is not known at present and it will be an intriguing problem to explore it.

---

RIASSUNTO (*)

Viene eseguita una particolareggiata analisi sulle possibilità teoriche per la costruzione di una meccanica quantistica e quindi di una descrizione delle particelle elementari, fondata su una definizione dei postulati di invarianza relativistica e sulle proprietà di simmetria connesse col gruppo completo di Lorentz, comprese traslazioni nello spazio-tempo e inversioni sia delle coordinate spaziali che della coordinata temporale. È poi messo in luce come la validità di ogni conclusione tratta da considerazioni di simmetria sia condizionata dal problema fondamentale della misurabilità delle grandezze di campo.

(*) A cura della Redazione.