Test of Time-Reversal Invariance in the Reactions Mg$^{24}$ + $d$ $\rightarrow$ Mg$^{25}$ + $p$

William G. Weitkamp,* Derek W. Storm, D. C. Shreve, Wilfred J. Braithwaite, and D. Bodansky

Department of Physics, University of Washington, Seattle, Washington

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No evidence for a violation of time-reversal ($T$) invariance has been found in a study of the inverse reactions Mg$^{24}$ + $d$ $\rightarrow$ Mg$^{25}$ + $p$ using approximately 10-MeV deuterons and 15-MeV protons. In order to reduce systematic uncertainties, the reactions were compared by measuring, as a function of incident energy, the ratio of differential cross sections at angles corresponding to maxima in the angular distributions. Particular attention was paid to accurately determining solid angles, peak areas, counter efficiencies, electronic efficiencies, contributions of target impurities, and beam energies. Ratios for the two reactions agree to within an over-all uncertainty (standard deviation) of 0.3%. Lack of an adequate model for the reaction makes quantitative interpretation of the result difficult. However, a tentative upper limit of 0.3% is set on the ratio of the $T$-noninvariant to $T$-invariant reaction amplitudes.

I. INTRODUCTION

The first experimental indication of a violation of invariance under time reversal ($T$) was found in the study of the decay of the $K^+$ meson carried out by Christenson, Cronin, Fitch, and Turley. Their discovery of a violation of CP invariance, which implies a violation of $T$ invariance if interpreted in accordance with the CPT theorem, has stimulated widespread interest in other studies of $T$ invariance.

The present paper is concerned with an experimental test of $T$ invariance in nuclear reactions. The desirability of such a test was suggested by the $K^+$ results, as well as by improvements in technology which permit a considerably higher experimental precision than was attained in earlier, similar, experiments. A direct connection between the $K^+$ violation and a similar violation in nuclear interactions is suggested by the hypothesis of Bernstein, Feinberg, and Lee, in which such violations are attributed to a large breakdown of $T$ invariance in the electromagnetic interaction of hadrons.

Experimental studies of $T$ invariance in nuclear interactions have fallen into several categories: comparisons of cross sections in inverse reactions, comparisons of polarization and asymmetry in proton elastic scattering, studies of electromagnetic transitions between oriented nuclear states, and studies of $\beta$-neutrino correlations in the $\beta$ decay of polarized nuclei. To date, no violation of $T$ invariance has been reported in such studies.

There has been considerable uncertainty in translating the experimental results, which consist of upper limits on observable quantities, into meaningful upper limits on the violation of $T$ invariance. The magnitude of the possible violation has been variously described in terms of the ratio of the $T$-odd (noninvariant) to the $T$-even (invariant) components of the force, of the interaction Hamiltonian, or of the reaction amplitude. In all such formulations the conclusions have depended upon assumptions made concerning unknown details of the processes. In view of the consequent large uncertainties in the analyses, no consistent attempt will be made in the summary below to distinguish between the alternative descriptions.

For the electromagnetic transition studies, the smallest upper limit has been reported by Kajfosz, Kopecký, and Honzátko, who conclude that the possible $T$ noninvariance is less than 2%. Experiments of still greater precision are being carried out by Kistner as well as by Frauenfelder and collaborators. Slightly higher limits have been reported for the polarization and asymmetry comparisons, although these too are in the region of a few percent. No explicit upper limit on $T$ invariance was extracted from the $\beta$-neutrino

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5 The relevance of such experiments for testing $T$ invariance has been discussed by E. M. Henley and B. A. Jacobsohn, Phys. Rev. 113, 225 (1959).


correlation measurements of Calaprice, Commins, Gibbs, Wick, and Dobson.\textsuperscript{16} Upper limits on $T$ non-invariance in inverse reactions were quoted as 2\% in a study\textsuperscript{a} of $p + p \rightarrow d + d$ and as 3\% in a study\textsuperscript{b} of $C^{12} + \alpha \rightarrow N^{14} + d$. More recently, von Witsch, Richter, and von Brentano\textsuperscript{a} have studied the excitation functions for the $Mg^{24} + \alpha \rightarrow \Lambda P^{2} + p$ reactions. Drawing upon theoretical work of Ericson\textsuperscript{18} and Mahaux and Weidenmüller,\textsuperscript{19} they set an upper limit of 0.4\% on the possible $T$ noninvariance.

The test of $T$ invariance described in the present paper is also based on a comparison of inverse nuclear reactions. In common with other such comparisons it is a test of the relation

$$\frac{(2s_A + 1)(2s_B + 1)}{(2s_B + 1)(2s_B + 1)} \sigma_T^A(\theta, E) = 1. \quad (1)$$

This expression is generally called the principle of detailed balance. In Eq. (1), $\sigma_T(\theta, E)$ and $\sigma_T(\theta, E)$ are the differential cross sections for the forward ($A + a \rightarrow B + b$) and reverse ($B + b \rightarrow A + a$) reactions at the same c.m. angle $\theta$ and channel energy $E$; and $s_i$ and $p_i$ are the spin and c.m. momentum of particle $i$. This relationship is obtained, for unpolarized projectiles and targets and a detection system insensitive to polarization, from the reciprocity theorem\textsuperscript{20} by summing the contributions of all initial and final spin states. The reciprocity theorem, in turn, is a direct consequence of $T$ invariance.\textsuperscript{20} Therefore, the fulfillment of the principle of detailed balance is a necessary condition, in the absence of polarization effects, for the validity of $T$ invariance.

The experiment is specifically concerned with the inverse reactions $Mg^{24} + d \rightarrow Mg^{25} + p$ at incident deuteron and proton energies of about 10 and 15 MeV, respectively. Rather than attempting to measure absolute cross sections to high precision, we have obtained, as a function of energy, the ratio of differential cross sections simultaneously measured at two angles. Detailed balance requires these ratios to be equal for the two reactions when compared at the same pair of c.m. angles and the same $\Lambda^{16}$ excitation energy.

The general experimental procedure which was followed and the reasons motivating it are discussed in Sec. II of this paper. Details of the experimental arrangement are described in Sec. III. Experimental corrections and sources of uncertainty are discussed in Sec. IV. The experimental results are described in Sec. V. In Sec. VI the interpretation of the experimental results is discussed, with emphasis on the problems encountered in attempting to establish a clear relationship between the upper limit on the experimental discrepancy and an upper limit on the $T$ noninvariance.

II. GENERAL EXPERIMENTAL PROCEDURE

To enhance the prospect of observing an experimentally significant discrepancy, one clearly wishes a reaction in which the $T$-noninvariant force has a large effect, and in which the discrepancy itself can be determined with high precision. Unfortunately, with the present limited understanding of nuclear reactions and of possible $T$-noninvariant forces it is difficult to determine which reactions best meet these criteria. Targets and projectiles of low spin are preferable. It is also desirable to have a number of competing reaction channels open. At the time this experiment was begun, it was not clear whether a direct reaction or one that involved a compound nucleus was to be preferred, and, as discussed further in Sec. VI, the situation still remains unclear.

In the absence of compelling theoretical arguments, the choice of the experimental configuration was determined primarily by the desire for high experimental precision. In particular, the choice of the $(d, p)$ and $(p, d)$ reaction pair was dictated by the availability of intense deuteron and proton beams from the University of Washington tandem Van de Graaff accelerator. Beams of other particles, such as $\alpha$ particles, were only available at intensities less than a factor of 20 or more.

In order to reduce undesirable effects of energy loss in the targets and to facilitate the identification of the $(p, d)$ deuterons, the incident particle energies were made as high as stable operation of the accelerator permitted. The incident proton energy was about 15 MeV and, by the energetics of the reactions, the corresponding deuteron energy was about 10 MeV.

The selection of the targets, $Mg^{24}$ and $Mg^{25}$, was made primarily because: (a) the first excited states are well separated from the ground states; (b) isotopically and chemically pure targets were readily available; (c) the estimated width for Ericson fluctuations\textsuperscript{21} (100 keV) is considerably larger than the expected total spread in incident particle energy [about 10 keV (FWHM)]; and (d) the laboratory energies of the outgoing particles do not change rapidly with angle, making it possible to use very similar counting systems in the comparison of cross sections measured at different angles. Lighter targets would be less favorable from the kinematic standpoint and heavier targets would introduce complications from the smaller Ericson fluctuation widths. These considerations are partially balanced by the disadvantage of the relatively high spin $\frac{1}{2}$ of $Mg^{24}$.

In order to reduce systematic experimental errors, the study of the $Mg^{24}(d, p)Mg^{25}$ and $Mg^{25}(p, d)Mg^{24}$ reactions was carried out by using two counters simul-
taneously, one at the angle $\theta_1$ and the other at $\theta_2$, and by measuring the ratio

$$R(E, \beta_1, \beta_2) = \frac{\sigma(\theta_2, E)}{\sigma(\theta_1, E)},$$  \hspace{1cm} (2)

where $\sigma(\theta, E)$ is the differential cross section at the angle $\theta$ and the energy $E$. To facilitate comparison between the ratio for incident protons $R_p$ and the ratio for incident deuterons $R_d$, $\sigma$ and $\theta$ are expressed in the c.m. system and $E$ is taken as the excitation energy in the $\text{Al}^{26}$ system. The essence of the present experiment is then the comparison of $R_d$ and $R_p$ at the same angles and energy. In this method, accurate knowledge of the integrated beam current and the target thickness is not necessary. Furthermore, by using similar counting systems at the two angles, a partial cancelation of systematic counting errors is achieved.

A difference between $R_d$ and $R_p$ can exist only if there is a violation of $T$ invariance. In principle, the comparison could be restricted to just one pair of angles, $\theta_1$ and $\theta_2$, at a single energy. However, uncertainties in the determination of angle and energy made it necessary to explore $R$ both as a function of angle and of energy.

The comparison between $R_d$ and $R_p$ was made at extrema in the angular distributions and excitation functions (for $R$) in order to minimize errors due to small differences in the angles and energies used in the $(d, p)$ and $(p, d)$ measurements. A preference for maxima or minima in these distributions depends, in principle, upon the nature of the experimental uncertainties. Consider a point where the differential cross section is $\sigma$. Then, because the magnitude of the cross section is dominated by the $T$-even reaction amplitude, the even amplitude is approximately proportional to $\sqrt{\sigma}$. There is no reason to expect the odd and even amplitudes to be correlated and therefore, on the average, the ratio $F$ of odd to even amplitudes is proportional to $1/\sqrt{\sigma}$. This might suggest studying the reaction at minima in $\sigma$ so as to make $F$ as large as possible.\textsuperscript{10,18}

However, experimental uncertainties must also be considered. For fixed incident flux and fixed counting time, the number of observed counts is proportional to $\sigma$, giving a statistical uncertainty proportional to $\sqrt{\sigma}$. In addition, there are systematic uncertainties. For simplicity, let us consider two types: (1) uncertainties which are proportional to the cross section of interest (e.g., geometrical uncertainties); and (2) uncertainties which are independent of the cross section (e.g., background due to impurities or pileup of elastically scattered particles). Then $\delta$, the total fractional uncertainty in the cross section, is given by adding the statistical and individual systematic contributions in quadrature:

$$\delta \propto \frac{1}{\sigma} \left[ (\sqrt{\sigma})^2 + (K_{\sigma})^2 + (K_2)^2 \right]^{1/2},$$  \hspace{1cm} (3)

where $K_\sigma$ and $K_2$ are proportional to the type (1) and type (2) systematic uncertainties, respectively.

Optimum sensitivity is achieved by minimizing $\delta/F$, where

$$\frac{\delta}{F} = \frac{1}{(1 + K_1^2 + K_2^2/\sigma)^{1/2}}. \hspace{1cm} (4)$$

Thus for fixed incident flux and counting time, which correspond to the experimental realities, the sensitivity of the experiment is independent of whether the data are taken at maxima in $\sigma$ when there are no significant systematic uncertainties ($K_1 = K_2 = 0$). But if, for example, the uncertainties are predominantly geometrical, the data should be taken at minima in $\sigma$. In the present experiment, the systematic uncertainties were predominantly of type (2), arising from pileup and background. Therefore, it was preferable to take data at maxima.

In consequence of these considerations, the final measurements took the form of the determination of the excitation function of $R(E) = R(E, \theta_{1m}, \theta_{2m})$, where $\theta_{1m}$ and $\theta_{2m}$ are angles at which maxima were found in the angular distribution. The most significant single result was the comparison of $R_p(E)$ and $R_p(E)$ at a local maximum in the excitation function.

### III. EXPERIMENTAL ARRANGEMENT

Details of the experimental arrangement are shown in Fig. 1.

**A. Incident Beam**

The collimated and magnetically analyzed beam from the accelerator was defined in energy to within about 4 keV (rms). It entered the scattering chamber through an aperture system in which the smallest aperture had a diameter of $x_B$ in. To minimize effects of slit scattering the beam was focused to a spot whose diameter was about $x_B$ in. The beam was stopped and the current measured in a Faraday cup at the far end of the scattering chamber.

No plausible polarization mechanisms exist in the accelerator and beam transport system, and the beam therefore was assumed to be unpolarized.

**B. Targets**

The targets were held in frames at the center of the scattering chamber and could be rotated about an axis
perpendicular to the scattering plane. The Mg targets were prepared by evaporating Mg metal (isotopically enriched to 99.9%) to an average thickness of 150 
\( \mu g/cm^2 \) on 75-\( \mu g/cm^2 \) carbon backings. The Mg was reduced from the oxide by aluminum at the time of evaporation and was evaporated onto a 75-\( \mu g/cm^2 \) aluminum backing. The Mg targets were about 300 \( \mu g/cm^2 \) thick. The Mg target thicknesses correspond in each case to energy losses of about 8 keV for the respective incident particles.

**C. Counters**

Each counter was mounted on an independently movable arm, with a solid-angle-defining aperture located 10 in. from the center of the scattering chamber. Counter 1, at the forward angle, had a 1/2-in.-diam aperture and counter 2, at the backward angle, had a 1/2-in.-diam aperture, the different areas being chosen to equalize roughly the counting rates.

The protons in the \((d,p)\) measurements had energies from about 13 to 15 MeV, depending upon angle, and their ranges substantially exceeded those of other charged reaction products. It was therefore possible to reject all other charged particles by placing poly-


ethylene degraders directly in front of the proton detectors. In particular, this eliminated the intense flux of scattered deuterons, reducing the effects of electronic pileup at the expense of increasing the width of the proton peaks due to energy straggling. The detectors themselves were Li-drifted silicon detectors.

The deuterons from the \((d,p)\) reaction ranged between 8 and 10 MeV. To distinguish the deuterons from other particles, particularly protons of higher range, a two-detector telescope was used in a conventional \(dE/dx-E\) arrangement. The detector thicknesses were tailored to exceed only slightly the deuteron ranges, thereby facilitating the separation of deuterons from protons. The detectors were transmission-mounted surface-barrier silicon detectors.

**D. Electronics**

The electronic arrangements used for the \((d,p)\) and \((p,d)\) studies were similar, differing only to the extent that no electronic particle identification was required in the \((d,p)\) case. The \((p,d)\) system will therefore be described in some detail, and the \((d,p)\) system may be inferred from it by omitting features required for particle identification.

A block diagram of the \((p,d)\) electronic arrangement is shown in Fig. 2. Detectors \(\Delta 1\) and \(E1\) comprise counter 1, and detectors \(\Delta 2\) and \(E2\) comprise counter 2. The signals from the two counters were treated as symmetrically as possible. The over-all system consists of the following main components, which will be described below: (1) a particle identification system for selecting the deuteron group of interest; (2) a pulse-height analysis system; (3) a pulser system for measuring the electronic efficiency; and (4) a system which routed the pulses to the proper quadrant of a multichannel pulse-height analyzer.

1. **Particle identification system.** The signals from the \(\Delta 1\) and \(E1\) preamplifiers were added. After amplification the added signal and an independently amplified \(\Delta 1\) signal were each fed into a differential pulse-height analyzer (DPHA). Because it was only necessary to identify deuterons in the neighborhood of the ground-state peak and because the deuteron energy loss exceeded that of any single proton in each detector, it was relatively easy to select the deuterons with appropriate DPHA windows. A coincidence between the DPHA outputs generated a signal \(C1\) for each deuteron in counter 1 whose energy was in the vicinity of the \((p,d)\) ground-state group. An analogous signal \(C2\) was similarly generated for deuterons in counter 2.

2. **Pulse-height analysis system.** The signal displayed in the pulse-height analyzer for counter 1 was proportional to the sum of the signals in \(\Delta 1\) and \(E1\). To develop this signal, a second output was taken from the \(\Delta 1\)-\(E1\) pulser; amplified, passed through a gate opened by the identification signal \(C1\), and further amplified in a biased amplifier. The outputs of the
biased amplifier for counter 1 and an identical biased amplifier for counter 2 were mixed, and the mixed output was fed into a 512-channel pulse-height analyzer.

3. **Pulser system.** Electronic pulzers were used in a system for determining the dead time and pileup losses during data collection. The pulser rates were kept continuously proportional to the detector counting rates by triggering the pulser from a converter whose output frequency was proportional to the beam current in the Faraday cup. Signals from the pulser were fed into the four preamplifiers in parallel with the detector pulses. Each pulser signal was separately adjusted in height to match roughly the energy loss of the particles of interest in the associated detector. A toggle circuit was used to drive the Δ1-E1 pair of pulser channels and the Δ2-E2 pair on alternate input triggers from the converter.

Because the electronic system treated the pulser and detector pulses in the same manner and because the variations of pulse rate with time were the same, the fractional losses suffered by the detector signals were equal to the fractional losses for the pulser signals. The pulser loss was measured separately for each counter by comparing the number of pulses appearing in the appropriate peak in the multichannel analyzer output with the total number of signals generated by the pulser.

4. **Routing system.** As indicated above, four different sets of signals were fed into the multichannel analyzer from the mixer, namely the detector signals and the pulser signals for both counters 1 and 2. A routing control unit was used to identify these signals and to route them to the proper quadrant of the multichannel analyzer. This was accomplished by appropriate coincidence and anticoincidence combinations. The routing unit was automatically shut off while the analyzer was occupied with a previous signal, preventing accidental mishandling of signals in the analyzer.

**IV. CORRECTIONS AND UNCERTAINTIES**

The nature of the present experiment made it crucial to minimize both the statistical and systematic uncertainties in the comparison of \( R_d \) and \( R_p \). The systematic uncertainties and the corrections applied to the data are discussed below. Unless otherwise specified, they are evaluated at the local maximum in the excitation function for \( R(E,\theta_{1m},\theta_{2m}) \), where the data were taken most carefully and where the statistical errors were smallest. The quoted uncertainties are to be interpreted as standard deviations.

**A. Solid Angle**

Uncertainties in relative solid angle in the \((d,p)\) and \((p,d)\) measurements were reduced by using the same counter holders and counter apertures for both reactions. The distances of the defining apertures from the center of the scattering chamber could be reproducibly set to within 0.001 in., corresponding to an uncertainty in distance of 0.01% and an over-all uncertainty in \( R_d \) or \( R_p \) of 0.03%. The actual ratio of aperture areas (counter 2 to counter 1) was 4.00±0.01, as determined by direct measurement of the apertures and by counting equal particle fluxes in the detectors. In calculating the values of \( R_d \) and \( R_p \), this ratio was taken to be exactly 4.00 because any errors cancel in the comparison of \( R_d \) and \( R_p \). Effects on the solid angle arising from scattering in the edges of the aperture were experimentally studied and were found to be negligible, partly because the aperture edges subtended a relatively small solid angle for scattering and partly because most of such scattered particles were so degraded in energy that they were unambiguously rejected.

To the extent that the plane of the target does not contain the axis of revolution of the counters, the distances can be incorrect. A small permanent misalignment in the position of the target introduces negligible error in the comparison of \( R_d \) and \( R_p \) because the laboratory angles used in measuring each ratio are virtually the same. Mechanical looseness in the target holder, however, permitted a small variable shift in position, introducing an uncertainty of 0.07% in \( R_d \) or \( R_p \). Bowing and wrinkling of the target can further shift its position. This caused uncertainties ranging from 0.03 to 0.11% in \( R_d \) or \( R_p \) in the earliest groups of runs. In later runs this uncertainty was eliminated by taking half of the data with the target rotated 180° from its normal position. (The beam energy was then adjusted slightly to compensate for energy loss in the target backing.)

If the position of the incident particle beam shifts over the face of the target, the relative solid angles for the two counters will, in general, be altered. This potentially major source of error was eliminated by orienting the target with its plane bisecting the angle between the two counters. In addition to affecting the solid angle, shifts in the beam position change the scattering angle. Errors of 0.1° were possible, but this led to no appreciable uncertainty in \( R_d \) or \( R_p \). For example, even a shift of 0.5° would not alter the cross section at the forward maximum (\( \theta_{1m} \)) by as much as 0.1%.

**B. Peak Areas**

The extraction of cross-section ratios from the pulse height analyzer spectra presented different problems for the \((d,p)\) and \((p,d)\) measurements, and the two cases will be discussed separately.

1. **\((d,p)\) spectra.** A typical \((d,p)\) spectrum is shown in Fig. 3. The resolution of this counter, 170 keV, full width at half-maximum, (accounted for mostly by the polyethylene foils) was adequate to separate almost completely the ground-state group from the first-excited-state group (0.582 MeV in Mg\(^{24}\)). The total number of counts in the ground-state peak was deter-
mined by summing the actual counts, except near the region of overlap with the first-excited-state peak. The low-energy side of an "ideal" spectral peak, determined by elastically scattering protons from gold or carbon targets at an appropriate energy, was used to determine the contribution to the ground-state group from counts in this region. The low-energy side of the ideal peak was terminated at the same relative location as the peak obtained in the counter efficiency measurement (described in Sec. IV C), thereby correcting for the small number of events in the extreme low-energy tail of the peak by including them in the efficiency correction.

A background, subtracted from the peaks, was estimated from the number of counts in the channels just above the ground-state peak, and from the residual counts in the valley which could not be accounted for in terms of the ideal peak shapes. Both counters showed evidence of small backgrounds, typically about 0.2% of the total counts in the peak. (Counts from target impurities will be dealt with separately in Sec. IV E). Different amounts of background were present in the two detectors so that $R_p$ had to be raised by $(0.1 \pm 0.1)\%$.

2. $(p,d)$ spectra. A typical $(p,d)$ spectrum is shown in Fig. 4. Here the ground-state group is well separated from the group to the first excited state of Mg$^{25}$ at 1.368 MeV, but the peak rides on a substantial background pedestal. Studies of the background indicate that it is primarily caused by pileup of pulses, presumably from the high proton flux in the deuteron detectors. This conclusion follows from findings that: (a) the ratio of background to peak counts was proportional to the beam intensity; (b) at equal background rates the spectrum from a natural Mg target (10% Mg$^{25}$) had only $\frac{1}{10}$ the number of peak counts as did the spectrum from a Mg$^{25}$ target; and (c) a similar background, with no peaks, was seen when a target of Al was bombarded with protons.

The background contribution to the counts under the ground-state peak was estimated by drawing a smooth curve through the data points on either side of the peak and interpolating at the peak, as shown in Fig. 4. For energies near the maximum of the excitation of $R_p$ where greatest precision was desired, data were taken at reduced beam intensity to reduce the fractional background contribution. At these points the background counts were in the neighborhood of 1.0% of the peak counts.

Nonstatistical uncertainties in the background subtraction are in part random, resulting from unintentional variations in the manner in which the background curve was drawn. It was estimated that this random error acted to raise the over-all random error in $R_p$ to 10% above the purely statistical random error. Beyond this there is a systematic uncertainty in the procedure which is not reduced by repetition of runs. However, such errors largely cancel in calculating $R_p$ because the appearance of the background is very similar in counters 1 and 2. The systematic background uncertainty in $R_p$ was estimated to be about 0.14%. This was the largest single source of systematic uncertainty in the experiment.

C. Counter Efficiency

Ideally, the counters should detect every particle passing through the defining aperture. However, there
may be losses from nuclear interactions in the detectors and from detector imperfections. To determine the losses from such causes, a detailed study was made of the counter efficiency using a $p-p$ (or $d-p$) coincidence method.

Separate measurements were made for the proton and deuteron counters. For example, to measure the efficiency of the proton counter 1 (see Fig. 5), one of the protons from elastic $p-p$ scattering was detected in counter A, and the other, after passing through a specially inserted thin transmission counter B, was stopped in counter 1. The degrader is taken to be part of counter 1. The energy of protons entering counter 1 was the same as the energy of normal $(d,p)$ protons. Coincidences between pulses of appropriate heights in counters A and B signaled the entry of a proton of correct energy into counter 1, and were used to open the input gate of the pulse-height analyzer. The efficiency was determined from the ratio of the number of counter-1 events appearing in the spectrum peak to the number of gating pulses. To eliminate effects of purely electronic losses, the pulser method described above (see Sec. III D) was employed in this determination. Apparent losses due to accidental coincidences were negligible.

It was found that the efficiencies of the proton counters, including losses in the polyethylene degraders, averaged about 99.6%. The efficiency was $(0.12 \pm 0.06)\%$ less in counter 1 than in counter 2. These losses can be understood in terms of nuclear interactions in the detectors, the efficiency being less in counter 1 because of the greater range of the higher-energy protons. There was no reason to attribute any of the observed losses to detector defects, especially as the losses did not change as the detectors aged or when different detectors were used.

The same method was used to explore the detector efficiencies for the deuteron counters, using a deuterated polyethylene target to obtain $d-d$ elastic scattering coincidences. The losses due to nuclear interactions are expected to be less in this case because the deuteron ranges are substantially less than the proton ranges. The final experimental correction for the deuteron detectors decreased $R_p$ by $(0.04 \pm 0.04)\%$.

### D. Electronic Efficiency

Corrections for dead time and pileup losses in the electronic system were obtained using the pulser method discussed in Sec. III D. Typical losses were in the neighborhood of $1\%$, varying proportionally with the beam intensity and differing at the two angles and in the two reactions. The pulser peaks were considerably sharper than the detector peaks, and therefore there were fewer problems in determining the area under the pulser peaks than were encountered for the detector peaks. On the other hand, the difference in peak shapes created minor difficulties in determining the areas for pulser and detectors in exactly the same way, with a resultant uncertainty of about 0.05% in $R_d$ and in $R_p$.

### E. Impurities

The targets used in these measurements contained small amounts of chemical and isotopic impurities, and it was necessary to correct the data for their contributions. The $\mathrm{Mg}^{25}(p,d)\mathrm{Mg}^{24}$ case was the easier to study because $(p,d)$ reactions are much less endoergic on $\mathrm{Mg}^{25}$ than on most common light nuclei. In fact, a study of $(p,d)$ $Q$ values and reaction kinematics showed that no plausible impurity with $A < 43$ could contribute a significant number of spurious counts to the $\mathrm{Mg}^{25}(p,d)\mathrm{Mg}^{24}$ peaks. Deuteron elastic scattering was used to investigate heavier impurities in the $\mathrm{Mg}^{25}$ targets. Small amounts of impurities were evident in the vicinity of $A = 56$ and 200, comprising by weight approximately 0.1 and 0.0006% of the target, respectively. The supplier’s spectroscopic analysis of the target material and an examination of $Q$ values showed that only Fe$^{57}$ and Pt might be troublesome. Measurement of the yield of $(p,d)$ reactions from targets of these elements showed that they could make no significant contributions to the data. It was concluded that $R_p$ could not be in error by more than 0.01% because of impurities.

The situation was kinematically less favorable in the $\mathrm{Mg}^{24}(d,p)\mathrm{Mg}^{25}$ reaction. Furthermore, assays made by deuteron or $\alpha$-particle elastic scattering gave evidence for the presence of a number of troublesome nuclei, especially $\mathrm{Mg}^{25}$, $\mathrm{Si}^{28}$, $\mathrm{P}^{31}$, $\mathrm{S}^{32}$, and $\mathrm{Cl}^{35}$. In the case of $\mathrm{Mg}^{25}$, $\mathrm{Si}^{28}$, and $\mathrm{Cl}^{35}$, measurable yields of protons coinciding in energy with those in the $\mathrm{Mg}^{24}(d,p)\mathrm{Mg}^{25}$ peak came from reactions to excited states of the residual nucleus. More energetic groups of protons from these impurity $(d,p)$ reactions showed up as small peaks on the high side of the $\mathrm{Mg}^{24}(d,p)\mathrm{Mg}^{25}$ ground-state group. An example is the peak labeled Cl$^{35}$ in Fig. 3. By taking $(d,p)$ spectra with targets of the impurity isotopes, or by using known cross sections, it was possible to use these peaks to infer how much a given impurity contributed to the yield under the $\mathrm{Mg}^{24}(d,p)\mathrm{Mg}^{25}$ peak or to set upper limits. Upper limits on the effects of $\mathrm{P}^{31}$ and $\mathrm{S}^{32}$ were calculated from an esti-
mate of the amount of impurity in the target and known \((d, p)\) cross sections.

The net effect of all impurities in the Mg\(^{24}\) targets was to require an increase in \(R_d\) of between 0.02 and 0.06% depending upon the individual Mg\(^{24}\) target used, with an uncertainty of 0.06%.

### F. Beam Energy

It is convenient to compare the energies for the Mg\(^{24}\)\(+d\)\(\rightarrow\)Mg\(^{24}\)+\(p\) reactions in terms of the equivalent excitation energy in Al\(^{28}\). The accuracy with which the energy scales for the two reactions match depends upon the accuracy with which the nuclear masses are known and the accuracy of the accelerator calibration. The masses used in the (relativistic) calculation of the energy scales were taken from the tabulation of Mattauch et al.\(^{23}\) with quoted uncertainties of 2 or 3 keV.

The accelerator was calibrated by relating the magnetic field in the analyzing magnet to the energy of the \(AP\(\(\phi, n\)\)S\(\(\phi\)\) reaction threshold. It was assumed that the momentum of the particles was directly proportional to the magnetic field at the location of the magnetometer probe. The threshold energy was taken to be the weighted mean of all measurements listed by Freeman et al.\(^{24}\) 5800±3 keV. The combination of uncertainties in the threshold energy and possible deviations from proportionality between magnetic field and momentum lead to an absolute uncertainty of about 20 keV in the energy scales. However, because the magnetic field in the analyzing magnet is nearly the same for 10-MeV deuterons and 15-MeV protons, the relative uncertainty in the \((d, p)\) and \((p, d)\) energy scales was only about 5 keV. It will be seen below (Sec. V) that the observed excitation functions match in energy to within somewhat better than 5 keV. Because \(R(E)\) is at a local maximum at those energies at which the most precise data were taken, it is relatively insensitive to small energy uncertainties, falling, for example, by about 0.1% at 5 keV away from the maximum. The resulting uncertainties in the comparison of \(R_d\) and \(R_p\) depend upon the method of comparison, as seen in Sec. V. In the most favorable method the uncertainty is only 0.03%.

It was found that the analyzing magnet changed calibration slightly during the several months that elapsed between groups of runs. The magnet was recalibrated immediately prior to each of the last groups of \((d, p)\) and \((p, d)\) runs. The excitation functions \(R(E)\) from previous groups of runs were adjusted in energy, with shifts of 9 to 16 keV, to match those of the last groups. The energy shifts were determined by comparing the excitation functions as well as by comparing ratios of the first-excited-state and ground-state yields in the two counters. Energy shifts found by these different matching procedures were consistent, and no additional uncertainty was introduced by this adjustment.

### G. Summary of Uncertainties

The data of the present experiment were taken in a number of extended groups of up to 50 runs, each run lasting about an hour. When repeated data points are combined the statistical uncertainties reduce in a simple manner. The reduction of systematic uncertainties by repetition is less straightforward. In fact, in the present experiment the only nonstatistical error which was so random as to be reduced by each repetition was a relatively small part of the \((p, d)\) background uncertainty. Other uncertainties, although not reduced by the mere repetition of points, were reduced if some change in the apparatus was made between runs. These included geometric uncertainties and target impurity uncertainties (when the target was changed). Finally, several uncertainties were not at all reduced by repetition, the same error being repeated each time. Uncertainties in electronic efficiency, in detector efficiency, and in extracting data from pulse-height spectra fell into this category. In combining the data from

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various runs, these distinctions in the reduction of systematic errors were applied.

A summary of corrections and uncertainties is presented in Table I. It is seen that the statistical uncertainties are comparable to the resultant of all the systematic uncertainties at the peak of the excitation functions for \( R \), where the statistical uncertainties are least. Elsewhere, the over-all uncertainties in the data points are almost entirely determined by the statistical uncertainties.

V. RESULTS

An exploratory series of runs showed that simultaneous maxima occurred in the angular distribution and in the energy dependence of \( R(E, \theta_1, \theta_2) \) at c.m. angles of \( \theta_1 = 29.7^\circ \) and \( \theta_2 = 119.2^\circ \) and at an excitation energy in Al\(^{40} \) of 20.68 MeV. The positions of these maxima determined the angles and energies for the final data of the experiment.

Excitation functions for the \((d, p)\) differential cross sections (in individual counters) were taken to help in understanding the reaction mechanism, as discussed in Sec. VI, and are not related to the experimental determination of \( R(E, \theta_1, \theta_2) \). Results at 29.7°, 119.2°, 148.3°, and 170.5° are shown in Fig. 6. These data have an uncertainty of about 10% in absolute value due primarily to uncertainties in target thickness. In addition there is a relative uncertainty of about 3% in the points due primarily to beam wander over the target nonuniformities. The target used for these measurements was about 40 keV thick.

A comparison of angular distributions for the \((d, p)\) and \((p, d)\) reactions at \( E = 20.68 \) MeV is shown in Fig. 7. They were obtained by measuring the ratio of cross sections at two angles, one of which was \( \theta_1 \) or \( \theta_m \). They are each normalized to unity at \( \theta_1 \). Typical uncertainties in the angular distributions are 0.7% in the \((d, p)\) data and 1.6% in the \((p, d)\) data. Within these uncertainties the angular distributions for the two reactions are identical.

The excitation functions for \( R_d(E) \) and \( R_p(E) \) are shown in Fig. 8. The corrections and uncertainties discussed in Sec. IV are included in the data of this figure. Typical uncertainties away from the peak are about 0.5% for \( R_d \) and 1.0% for \( R_p \) while at the peak the uncertainties are 0.22% for the \( R_d \) point at 20.679 MeV (deuteron laboratory energy of 10.046 MeV) and 0.31% for the \( R_p \) point at 20.678 MeV (proton laboratory energy of 14.953 MeV). Within these experimental

![Fig. 7. Angular distributions for the Mg\(^{28}(d, p)\)Mg\(^{27} \) and Mg\(^{28}(p, d)\)Mg\(^{27} \) reactions at an excitation energy of 20.68 MeV in Al\(^{40} \). Maxima in the angular distributions are labeled \( \theta_m \) and \( \theta_0 \). The normalized differential cross section \( \sigma'(\theta) \) is the ratio of the differential cross section at \( \theta \) to the differential cross section at \( \theta_0 \). The differential cross sections at \( \theta_m \) are about 6 mb/sr for the \((d, p)\) reaction and about 2 mb/sr for the \((p, d)\) reaction. The solid curve is drawn arbitrarily through the two sets of data to guide the eye. The lower part of the figure shows the fractional differences between \( \sigma'(\theta) \), the normalized differential cross section measured in the \((d, p)\) reaction and \( \sigma'(\theta) \), the normalized differential cross section measured in the \((p, d)\) reaction. The errors include both statistical and systematic uncertainties and are to be interpreted as standard deviations.](image)

| TABLE I. Corrections and uncertainties in the measurements of \( R_d \) and \( R_p \). All numerical values refer to the single points at the peak of the excitation curve of Fig. 9. Small additional uncertainties which arise from possible energy errors are omitted here. They are discussed in Sec. V. |
|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | Uncertainties (%) |
|                 | \((\text{standard deviations})\) |
|                 | \( R_d \) | \( R_p \) | \( \sigma' \) | \( \sigma'' \) |
| Solid angles    | 0.00 | 0.00 | 0.00 | 0.00 |
| Peak areas      | 0.10 | 0.00 | 0.10 | 0.14 |
| Counter efficiencies | -0.12 | -0.04 | 0.06 | 0.04 |
| Electronic efficiencies | -0.20° | -0.50° | 0.05 | 0.05 |
| Target impurities | 0.04 | 0.00 | 0.06 | 0.01 |
| Total systematic uncertainty | 0.16 | 0.17 | 0.15 | 0.26 |
| Random uncertainty | 0.22 | 0.31 |

* These are typical corrections. The actual corrections varied with each run.
uncertainties the shapes of the excitation functions and the magnitudes of \( R(E) \) are identical for the two reactions. Furthermore, they match in energy within the over-all relative uncertainty of 5 keV.

Several methods for a quantitative comparison of the \( R(E) \) excitation function data are discussed below:

1. **Single point at the peak.** The simplest quantitative way of comparing the values of \( R_d \) and \( R_p \) at the peak is to use the accurate single points nearest the peak at 20.679 MeV for \( R_d \) and at 20.678 MeV for \( R_p \). Neglecting the small displacement of these energies from the center of the peak (see Fig. 9 below), these points give for the ratios at the peak:

\[
R_d = 0.1902 \pm 0.0004, \quad R_p = 0.1901 \pm 0.0006.
\]

Examination of the observed excitation functions indicates that it is unlikely that either of these points is more than 4 keV from the true peak. The worst case, where one point is at the peak and the other is 4 keV away, corresponds to an error of less than 0.1% in the comparison of \( R_d \) and \( R_p \). Including all uncertainties, it is concluded that \( R_d \) and \( R_p \) agree within an over-all uncertainty of 0.4% (standard deviation).

2. **Consideration of all data.** The method described above does not take advantage of all the available information because only two data points are used. One way of comparing all the data is to fit a function \( R_c(E) \) to all the data points of \( R_d(E) \) and \( R_p(E) \) taken together. Then the difference between \( R_d(E) \) and \( R_p(E) \) is characterized by \( \epsilon \), the difference between the average deviations of \( R_d(E) \) and \( R_p(E) \):

\[
\epsilon = \frac{1}{\sum_i w_i} \frac{\sum_i w_i [R_d(E_i) - R_c(E_i)]}{\sum_j w_j [R_p(E_j) - R_c(E_j)]}.
\]

where the summations are over data points at \((d,p)\) and \((p,d)\) energies \( E_i \). The weighting factors for the individual points, \( w_i \) and \( w_j \), are inversely proportional to the squares of the random errors.

The introduction of the function \( R_c(E) \) is necessitated in this method by the fact that the sets of energies \( E_i \) and \( E_j \) are not identical. Thus \( R_c(E) \) provides a common reference. \( R_c(E) \) was chosen to be an eight-term Fourier cosine series fitted to all the data points. This function is shown in Fig. 8, where it can be seen to give a reasonable fit to the data. (Note, however, that in this method of comparison even a rough fit would suffice.) The resulting difference was found to be \( \epsilon = 0.0002 \pm 0.0005 \).
When expressed as a fraction of the mean value of \( R \) over the energy interval considered, this is a difference of \((0.1\pm0.3)\%\). The uncertainty was calculated treating random and systematic contributions separately, and is less than the uncertainty in (1) because the random uncertainties are reduced on including more points.

This method is not fully satisfactory because it only measures an average shift of \( R_d(E) \) from \( R_p(E) \); an upward shift in one energy region could be offset by a downward shift in another region, giving a zero value for \( \epsilon \) even though the two functions are not identical. Furthermore, the mean difference was calculated without allowance for the possibility of small systematic errors in the energy calibration. The value of \( \epsilon \) can be sensitive to such errors because most of the data of Fig. 8 are in regions where \( R(E) \) has a relatively large negative slope.

3. Data near the peak. A third method of comparing \( R_d \) and \( R_p \) is a compromise between the two methods discussed above and meets in part the objections to each. The comparison is made over a restricted energy interval centered about the peak in \( R(E) \) at 20.68 MeV. Figure 9 gives an enlarged view of the data in the vicinity of the peak. A value of \( \epsilon \) was calculated for the seven \((d,p)\) and five \((p,d)\) data points in this interval, using the procedure of method (2) with the same function \( R_p(E) \). It was found that systematic errors of up to 5 keV in the relative energy calibrations would not change \( \epsilon \) by more than 0.03\%. The final result, including all uncertainties, is \( \epsilon = -0.0001 \pm 0.0006 \), or \((-0.1\pm0.3)\%\).

Because the data near the peak were taken with the smallest statistical uncertainties and the greatest care in studying systematic uncertainties, it is reasonable to depend on these data alone, using the data at other energies primarily as evidence that the energies in the \((d,p)\) and \((p,d)\) reactions are properly matched. Thus the result of method (3) is taken to be the most appropriate measure of the agreement found in the present experiment. Both the degree of agreement and the uncertainties in (3) are close to those found in the alternative analyses (1) and (2). Thus it is concluded that, in the most precise comparison, the \((d,p)\) and \((p,d)\) results agree within an experimental uncertainty (standard deviation) of 0.3\%.

VI. DISCUSSION

The early measurements on inverse nuclear reactions\(^6\) were interpreted using an approximate prescription\(^9\) that the fractional violation of \( T \) invariance might be expected to be about one-half of the fractional discrepancy \( \Delta \) between the experimental cross sections. Were this prescription applied to the present experiment, one would conclude that an upper limit of 0.2\% has been established for the \( T \) noninvariance. However, it is unlikely that so simple a relation can be appropriately applied here, or, in fact, that a reliable quantitative limit can be established in the absence of a detailed theoretical model for the reaction in both its \( T \)-invariant and \( T \)-noninvariant parts.

In the context of certain restricted models, tests of detailed balance in direct reactions are insensitive tests of \( T \) invariance. Detailed balance occurs independently of \( T \) invariance in the plane-wave Born approximation\(^5\) and, as recently shown by Robson,\(^6\) in the distorted-wave Born approximation when a lower cutoff on the radial integral is taken outside the range of the nuclear potential. However, the contribution to the DWBA radial integral from the region within the nuclear potential, including the nuclear surface, is significant and can be sensitive to \( T \) noninvariance.\(^6\) Furthermore, it should be recognized that models such as the DWBA may not provide an adequate description of the direct-reaction mechanism. Therefore it appears premature to reach quantitative conclusions as to the sensitivity of the direct component of \((d,p)\) reactions to \( T \) noninvariance.

The sensitivity of statistical compound nuclear reactions has been discussed both by Ericson\(^18\) and by Mahaux and Weidenmüller\(^19\) in the analysis of the Heidelberg results\(^8\) for the reactions \( \text{Mg}^{24}\alpha \rightarrow \text{Al}^{27}\alpha + p \). Separate consideration was given to the limits which could be placed on the \( T \)-odd fraction \( F \) of the reaction amplitude and on the \( T \)-odd fraction \( \bar{F} \) of the Hamiltonian. There was agreement on the value of \( F/\bar{F} \) for the specific circumstances of that experiment, but the estimates of the ratio \( F/\bar{F} \) differed by a factor of 6. Thus uncertainties remain, although the understanding of the sensitivity may be better for statistical reactions than for direct reactions.\(^27\)

In the expectation that the sensitivity of direct and compound nuclear reactions to \( T \) noninvariance may prove to be different, a brief experimental study was undertaken to provide some measure of the relative sizes of these contributions in the \( \text{Mg}^{24}(d,p)\text{Mg}^{25} \) reaction. The energy dependence of the \( \text{Mg}^{24}(d,p)\text{Mg}^{25} \) differential cross sections, presented in Fig. 7, was studied for this purpose. The fraction \( Y \) of the average cross section which proceeds by direct reactions can be related to the excitation function correlation coefficient \( C \) through the approximate relation\(^21\) 

\[ Y = (1 - N_e C)^{1/2} \]

where \( C = (\sigma_d/\sigma_p)^{1/2} - 1 \) and \( N_e \) is the effective number of magnetic substates contributing to the reaction.

\( C \) was found to be 0.024 at \( \theta = \theta_{2m} = 119.2^\circ \) and 0.002 at \( \theta = \theta_{1m} = 29.7^\circ \). For \( N_e \) at its maximum possible value of 18 (see below), this corresponds to average compound nuclear fractions of \( 1 - Y = 1/2 \) at \( \theta_{2m} \) and \( 1 - Y \)

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18 D. Robson (private communication).
20 See P. A. Moldauer [this issue, Phys. Rev. 165, 1136 (1968)] for a more recent consideration of the sensitivity of compound nuclear and direct reactions as tests of \( T \) invariance.
= 1/50 at θ_{1m}. Were N_{e} less than 18, the compound nuclear fractions would be correspondingly less.

In view of the problems discussed above, it is not yet possible to place a reliable quantitative upper limit on the T noninvariance from the present experimental results. However, it is possible to obtain a rough estimate of the ratio F of the T-odd to T-even reaction amplitudes using a procedure which ignores any specific information concerning reaction mechanisms. The measured differential cross section is the sum of the partial cross sections of the contributing magnetic substates. On separating the reaction amplitude for the 4th substate into a T-even amplitude f_{2}(θ) and a T-odd amplitude f'_{2}(θ), the differential cross sections can be expressed as

$$
σ_{±}(θ) = A_{±} \sum |f_{2}(θ)\pm f'_{2}(θ)|^2.
$$

The + sign is arbitrarily assigned to the (d,p) reaction, and the − sign then refers to the (p,d) reaction. A_{±} and A_{−} are statistical factors depending upon the particles’ spins and (c.m.) momenta, but not upon angle. Introducing the ratio of the T-odd to the T-even reaction amplitude

$$
F_{1}(θ) = |f'_{2}(θ)|/|f_{2}(θ)|
$$

and assuming F\ll1, it follows that

$$
R_{±}(θ') = \frac{σ_{±}(θ')}{σ_{−}(θ')} = \sum |f_{2}(θ')|^{2}[1 \pm 2F_{1}(θ') \cos[φ_{2}(θ')]]^{2}/\sum |f_{2}(θ')|^{2}[1 \pm 2F_{1}(θ') \cos[φ_{2}(θ')]],
$$

where φ_{2}(θ) is a phase angle relating f'_{2}(θ) and f_{2}(θ).

For the extreme case where only a single magnetic substate contributes to the reaction, the fractional discrepancy in cross sections Δ is given by

$$
Δ = R_{+}/R_{−} - 1 \approx 4[F_{2}(θ) \cos[φ_{2}(θ)]]

-F_{1}(θ) \cos[φ_{2}(θ)].
$$

In the absence of detailed information on the reaction mechanism, an approximate relationship may be obtained by replacing F_{2}(θ) and F_{1}(θ) by a mean F and introducing the crucial assumption that the phase angles φ are random. Then one finds F ≈ Δ/3. It must be recognized that this relationship has, at best, only a statistical validity, because a larger value of F would correspond to the same value of Δ were, for example, φ_{2}(θ') and φ_{2}(θ) close to 90°, either due to chance or due to inherent aspects of the reaction mechanism.

In the present experiment there are 36 possible combinations of magnetic substates, corresponding, because of symmetry under reflection through the reaction plane, to a maximum of 18 independent, incoherent transition amplitudes. In the extreme case that all 18 have uncorrelated phases and are of equal magnitude, one would expect a reduction in Δ/F by about a factor of (18)^{−1/2}. Then F ≈ \sqrt{2Δ}, where F now corresponds to an average of F_{1}(θ) over the magnetic substates as well as over angle.

From this examination of extreme cases, it is concluded that F probably lies between Δ/3 and \sqrt{2Δ}. We therefore assign an approximate upper limit for F equal to the experimental upper limit for Δ, namely F < 0.3%. However, as suggested by the predictions of the restricted direct-reaction models discussed above, we cannot exclude the possibility that F might be substantially larger than this limit.

The method for determining Δ, the T-noninvariant fraction of the Hamiltonian, is even less clear than the method for F. The discussion of Mahaux and Weidenmüller suggests that Δ is of the same order of magnitude as F for the compound nuclear component of the present reaction. The relationship between Δ and F is not known for the direct component. One might guess that Δ is also roughly equal to F for the noncompound nuclear component of the reaction. Then the upper limit on Δ would be 0.3%. However, it must be emphasized that this is, at best, only an order of magnitude estimate.

In summary, no evidence has been found in the present study for a violation of time-reversal invariance. Predictions of T invariance for ratios of reaction cross sections have been verified with an experimental uncertainty of 0.3%. Although some attempts to interpret these results in terms of limits on the possible magnitude of the T-odd part of the nuclear interaction have been described above, the unambiguous establishment of reliable limits will require substantial improvements in the understanding of the reaction mechanism as well as specific models for the manner in which a T-odd component might enter into the reaction.

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