The Electroresistive Effect and a Rectifying Property of Carborundum Crystals

The generalized Ohm's law

\[ E_i = \rho \sum_{k=1}^{3} R_{ik} J_k, \quad i = 1, 2, 3 \]  

(1)
does not adequately predict the actual current-voltage equilibrium states, particularly in the group of crystals which rectify alternating currents. Experiments performed by the writer show that the electrical resistances of single, homogeneous crystals of carborundum and zincite vary rapidly with the applied voltage even under isothermal conditions and when the voltage gradients are uniform. The isothermal variation of the electrical resistance with the applied voltage will be called the "electroresistive effect."

When the effects of the applied magnetic fields can be neglected, the current-voltage equilibrium may be approximated by

\[ R_i = \sum_{k=1}^{3} R_{ik} J_k + \sum_{k=1}^{3} \sum_{i=1}^{3} \sum_{n=1}^{3} R_{ikn} J_k E_i E_n + \cdots \quad (i = 1, 2, 3), \]

(2)

where \( J_k \) and \( E_i \) are the cartesian components of the current and electric field intensity and \( R_{ik}, R_{ikn}, \) etc., are components of second, third, fourth, etc. order tensors. These tensor components are in general functions of temperature and pressure. The terms containing \( R_{ikn} \) and \( R_{ikn} \), which are, respectively, linear and quadratic in the components of \( E \). They describe, respectively, the linear and quadratic electroresistive effects. The term current-voltage equilibrium is here used since in many cases several hours are required for the current to approach a steady value following a change in the voltage.

The number of distinct or independent "electroresistive constants" \( R_{ik}, R_{ikn}, \) etc., in any crystal class can be deduced from Neumann's hypothesis. Thus in the form of carborundum which belongs to the symmetry group \( C_{6h} \) (Wyckoff's notation) or \( A_1 E_6 \) (Voigt's notation) the electroresistive constants for the linear and quadratic effects were found to be:

**Linear case**

- \( R_{111} = R_{111}; \quad R_{111} = R_{111}; \quad R_{111} = R_{111}; \quad R_{111} = R_{111}; \quad (4 \text{ constants}) \)  
  \[ \text{(3a)} \]

**Quadratic case**

- \( R_{111} = R_{111}; \quad R_{111} = R_{111}; \quad R_{111} = R_{111}; \quad R_{111} = R_{111}; \quad R_{111} = R_{111}; \quad R_{111} = R_{111}; \quad R_{111} = R_{111}; \quad R_{111} = R_{111}; \quad R_{111} = R_{111}; \quad (10 \text{ constants}) \)  
  \[ \text{(3b)} \]

All other constants are zero. The integers 1, 2, 3 pertain, respectively, to the \( X, Y \) and \( Z \) axes. The \( Z \) axis is chosen as the axis of sixfold symmetry and the plane \( x = 0 \) as one of the vertical planes of symmetry.

Assume that in (2) terms involving higher powers than the second of the components of \( E \) are negligible and suppose that a uniform voltage gradient \( E_z \) is established in a rectangular carborundum crystal such that \( E_x = E_y = 0 \). Since \( R_{11} = R_{22} \neq R_{33} \) and \( R_{ij} = 0 \) where \( i \neq j \), the component of current along \( Z \) should obey the equation

\[ E_z = R_{33} I_3 + R_{32} I_2 E_2 + R_{31} I_1 E_1. \]

(4)
The resistance of the specimen along the \( Z \) direction is given by

\[ R_z = E_z/I_3 = R_{33} E_z + R_{32} E_2 + R_{31} E_1. \]

(5)

If the sign of \( E_z \) is reversed, the magnitude of \( R_z \) changes by an amount depending upon \( R_{33} E_z \), the term which originates from the linear electroresistive effect. Hence this theory affords an explanation of the rectifying property of carborundum crystals. The portion of rectification due to a point and plane is, of course, not taken into account by this phenomenological theory. Any term involving odd powers of \( E \) in (2) will affect the rectifying property.

Carborundum crystals were found, as predicted, to rectify currents caused to flow along \( Z \) by applying uniform voltage gradients \( E_z \) sin \( \alpha \). Preliminary experiments indicate that in the case of voltage gradients which are greater than one volt/cm higher order tensors than the fourth are needed.

A later paper will contain a more complete account of the theory together with a list of the linear and quadratic constants for the various crystal classes and experimental data upon the intricate electroresistive phenomena in large, single crystals of carborundum and zincite.

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**Exchange Forces Between Elementary Particles**

The Fermi theory of beta-decay introduces an interaction term into the Hamiltonian of the elementary particles: proton, neutron, electron and neutrino. A consequence of this theory is the existence of an exchange force between protons and neutrons; this force is caused by the emission of light particles by one heavy particle and the absorption of these by the other particle, both processes occurring in one elementary act. It has been pointed out that only an interaction term containing the derivatives of the wave functions can give rise to a force of sufficient magnitude between heavy particles. The interaction term has the form

\[ \Omega = g \cdot \alpha^\mu \eta^\mu \eta^{\nu} \varepsilon^{\rho} \cdot (\partial/\partial x)^m \varepsilon^{\rho} \cdot (\partial/\partial x)^n \varepsilon^{\rho} \cdot \text{conj.} \]

(1)

\( \varepsilon, \Phi, \psi, \varphi \) are the wave functions of the proton, neutron, electron, and neutrino, respectively; \( x \) stands for the four space-time coordinates; the Greek letters are the spin indices; \( (\partial/\partial x)^m \) is a differential operator of degree \( m \) with respect to space and time; \( \alpha^\mu \eta^\mu \eta^{\nu} \) is a combination of Dirac matrices, making the term relativistically invariant; such a combination can be constructed in several ways.

It can be shown that the interaction energy (1) also gives rise to exchange forces between other elementary particles. Let us consider a neutron at a place I and an electron at a place II. Let the neutron decay into a proton, an electron,
and a neutrino. In a subsequent process the proton can be transmuted back into a neutron at place II by absorbing the neutrino emitted and the initial electron, leaving the electron emitted in the first process at place I. The net result is that the initial neutron and electron have changed places. The two processes occurring in one elementary act produce an exchange force analogous to that between heavy particles.

In general, exchange forces between two elementary particles A and B occur whenever B can be created by decay of A, or A decay by a decay of B. Such forces are possible between: proton and neutron; proton and positive electron; proton and neutrino; neutron and neutrino; neutron and negative electron; neutrino and electron (positive and negative).

To get the last two of these exchange forces we must assume the existence of a transmutation of an electron into a neutrino or vice versa with the simultaneous creation of a pair of heavy particles. It should be possible for a positive electron, for instance, to decay into a neutron with emission of a proton and a neutron. It is easily shown that this kind of process is a consequence of the interaction term (1). Since we assume that the wave functions of all elementary particles follow the Dirac equation, we are forced to consider all negative energy states as occupied in the empty space. Then every interaction energy effecting a transmutation causes also a creation of two particles, since a transmutation of a particle in a negative state into another particle in a positive state must be considered as a creation of two particles.

A rough evaluation gives the following result: the potential of the exchange force between two elementary particles a and b in the distance r is of the following order of magnitude:

$$j_{ab}(r) \sim e^{2\hbar c} \left( \frac{\hbar c}{\epsilon} \right)^{(n+m+S_a-S_b)} \left( \frac{c_p b_p}{\epsilon} \right)^{S_a} \left( \frac{c_n b_n}{\epsilon} \right)^{S_b}$$

Here $\epsilon$ is the mean energy of $\beta$-ray particles and $b_0$ is a dimensionless constant $b_0 = \frac{\gamma}{h^2 c^2} \approx 10^{-11}$ the order of magnitude of which is independent of the values assumed for m and n. S_a and S_b are the degrees of differentiation of the wave function of a and b in (1): p_a and p_b are the corresponding momenta. We assume as usual that this expression only holds for distances $r > d$, d being of nuclear dimensions, and that $j_{ab}(r) \sim j_{ab}(d)$ for $r < d$. Now $m=1$, $n=2$ is the only combination which can account for the observed proton-neutron force and for the asymmetrical position of the maximum in the intensity curve of $\beta$-ray spectra. We obtain for the exchange forces at zero distances (in volts) between

- proton-neutron: $\sim 10^9$ neutron-neutrino electron: $\sim 0.5 \times 10^4$
- proton-electron: $\sim 0.5 \times 10^4$ neutron-neutrino: $\sim 2 \times 10^5$
- proton-neutrino: $\sim 2 \times 10^5$ electron-neutrino: $\sim 10^{-3}$

(The energies of the light particles are taken as about $10^6$ volts.) Thus the exchange forces, except in the case of two heavy particles, are too weak to be observed.

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**On the Hard Component of Cosmic Rays**

In a former paper it was shown that the depth-ionization curve of cosmic rays measured 80-240 m below sea level can easily be represented by an exponential function. The newer measurements of Weischefeld confirm this in principle. Therefore it is of interest to examine what range distribution is obtained by extrapolation of this exponential behavior of the curve due to rays coming in from all directions. Then:

$$j = J_0 e^{-\mu x}.$$  

The intensity curve for rays descending vertically through the atmosphere is converted by the transformation

$$\psi = J - \lambda dJ/dx$$

and the range distribution $\rho = -d\psi/\mu$ will be finally

$$\rho(x) dx = \mu^2 x e^{-\mu x} dx.$$  

For the average range one obtains

$$\bar{R} = \frac{2}{\mu}$$

and for the most frequent range

$$R_m = 1/\mu.$$  

Fig. 1 gives these relations with the value $\mu = 1.889 \times 10^{-3}$ per meter of water, formerly obtained for the hard component. A range phenomenon, curve II, is quite apparent after transformation to vertical incidence. The range distribution curve III is very like a Maxwellian one. A direct conclusion concerning the energy distribution is not yet available. In particular one cannot suppose a loss of energy exclusively produced by primary ionization, for this would require such small energy values of the incident rays that in the equatorial belt only a fraction could reach the earth's surface through the blocking effect of the earth's magnetic field. And then, contrary to all observations there should also result a strong latitude effect of this component.

It was pointed out by Lenz that the different "elementary functions" (exponential function, Gold’s integral, Kulenkampff-function) on which the analysis formerly was based are not adequate for describing the range distribution of cosmic rays.