It is clear that with mixtures of eigenfunctions of various $I$-values arbitrary $\tau/\tau'$ values can be obtained. However, if the experimental value 4 is confirmed, the most plausible explanation would be that the final pion state is a pure $I=1$ state of the special form $\pi \pi$. Besides supporting the selection rule (1), this would indicate that the interaction responsible for the $\pi$ decay is symmetric in the momenta, or coordinates, of the three pions. This implies no restriction for the spin and parity of the $\pi$ meson. (It may be mentioned, though, that for a $\pi$ of type $1^+$ the simplest interaction gives $u=0$ because of momentum conservation, but by inserting suitable relativistic invariants as factors, nonvanishing symmetric functions $u$ can be constructed even in this case. Particularly simple functions can be set up if the $\tau$ is $0^-$ or $3^+$.)

As a consequence of the symmetry of $u, \pi$ of all charges must have the same energy spectrum and the same angular correlations. The density distribution in the Dalitz circle should be invariant under rotations by $120^\circ$ (besides being by definition symmetric about the vertical axis). Insofar as the experimental data presently available indicate a more or less isotropic distribution, our conclusion is not contradicted by the observations, but with improved statistics a more sensitive test will become possible.

2. G. Takeda (to be published).
5. Introduce field operators: $\psi_\pi (\tau, \phi, \lambda)$ for the pseudovector $\tau$, and $\phi_\pi (\phi=1, 2, 3=\text{isotropic spin index})$ for the $\pi$. A simple interaction yielding a final state $I=1$ would be $\int \psi_\pi \psi_\pi (\phi_\pi, \phi_\pi) \times \sum_{\lambda} \phi_\pi \phi_\pi$, but this is not symmetric in the 3 pions; actually $\tau/\tau' = 1$ in this case. Straight symmetrization gives zero because $\partial_\tau \phi_\pi = 0$.
6. The corresponding interactions are:
   \[ 0^-: \int \psi_\pi (\phi_\pi, \phi_\pi) \sum_{\lambda} \phi_\pi \phi_\pi, \]
   \[ 3^+: \int \psi_\pi \psi_\pi (\phi_\pi, \phi_\pi) \sum_{\lambda} \phi_\pi \phi_\pi \phi_\pi \phi_\pi \]
   ($\psi$ symmetric in all 3 indices).

Method for Determining Spins of Hyperons*

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The purpose of this note is to point out an experimental method which may be used to determine the spins of the $\Lambda^0$ and $\Sigma$ hyperons. It consists in studying angular correlation effects in the decay of hyperons produced in the capture of negative $K$-mesons by protons. The capture reactions are

\[ K^- + p \rightarrow \Sigma^++\pi^0; \]
\[ K^- + p \rightarrow \Lambda^0+\pi^0; \]

and the angle in question, denoted by $\theta$, is the angle in the hyperon rest frame between the line of flight of the decay products and the line of flight of the hyperon.

Although the present remarks may be generalized to more complicated cases, we consider in detail here only the simplest possibilities: namely, that the spin of the $K$-meson is zero (there is some evidence that this is so for the $\pi$ meson). We also assume that the $K$-meson is captured from an $S$-state, as in the analogous case of capture of $\pi$ mesons by protons. The angular correlation function $f_2(\theta)$ (probability per unit solid angle) is then uniquely determined by the hyperon spin, denoted by $J$.

The theoretical analysis involved here is identical with that which is used to study angular correlation effects in nuclear reactions which proceed through a single compound-nucleus state of definite angular momentum and parity. Similar applications to the new unstable particles have been discussed by Adair, who considers angular correlation effects in the process $\pi^- + p \rightarrow Y + K^-$; and Gatto, who considers such effects in the chain of decay processes: $\Xi^+ \rightarrow \Lambda^0 + \pi^-; \Lambda^0 \rightarrow p + \pi^-$. Let $\psi_M^M'$ be the wave function of the system hyperon plus pion, where the index $\frac{1}{2}$ denotes the total angular momentum and $M' = \pm \frac{1}{2}$ is the $z$-component of angular momentum. We can decompose this into products of eigenfunctions representing the orbital angular momentum state of the reaction products and the spin state of the hyperon, $\psi_M^M$:

\[ \psi_M^M = \sum_{m=M} C \psi_L^M; \]

\[ \psi_L^M \psi_M = Y_L^m \psi_L^M. \]

The quantities $C$ are the usual Clebsch-Gordan coefficients; and $A_J$ is the amplitude of the orbital state $Y_L$. Note that no sum over $L$ is involved in the above expression. This is because in the example under consideration ($K$-meson of spin zero) the orbital angular momentum $L$ is fixed by the relative parities of the particles involved in the capture reaction; depending on the parities, $L$ may have either of the two values $L = J + \frac{1}{2}$, but not both. For $K$-mesons of spin greater than zero, more than one orbital state is possible and one must then know the relative values of the ampli-

<table>
<thead>
<tr>
<th>$J$</th>
<th>$f_2(\theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/2</td>
<td>$1 + 3 \cos^3 \theta$</td>
</tr>
<tr>
<td>5/2</td>
<td>$1 - 2 \cos^2 \theta + 5 \cos \theta$</td>
</tr>
<tr>
<td>7/2</td>
<td>$1 + 5 \cos^2 \theta - (55/3) \cos \theta + (175/9) \cos^3 \theta$</td>
</tr>
</tbody>
</table>
tudes $A_L$—something which could be obtained only from a detailed theory.

We now choose the quantization axis along the line of flight of the hyperon, so that only the term $m=0$ enters into the sum of Eq. (1). The angular distribution function $f_x(\theta)$ is obtained then by averaging the expression $|\psi_x^{M'}|^2$ over all values of $M'=M$. Thus

$$f_x(\theta) \sim \sum_M |C(J_L, J, \frac{1}{2}; M, 0)|^2 |\psi_x^{M'}|^2. \quad (2)$$

But $M$ can take on only the values $\pm \frac{1}{2}$, which give identical contributions; so we obtain the result (choosing, say, $M=\frac{1}{2}$)

$$f_x(\theta) \sim |\psi_x^{\frac{1}{2}}|^2. \quad (3)$$

Finally, we decompose $\psi_x^{\frac{1}{2}}$ into products of eigenfunctions representing the orbital state of the hyperon decay products and the spin state of the nucleon. Again, because the hyperon has definite parity, only one orbital state $J$ is involved ($l=J+\frac{1}{2}$ or $J-\frac{1}{2}$). The angular correlation function is however independent of the parity.

Results for several low values of $J$ are given in Table I, where the correlation function $f_x(\theta)$ is normalized such that the constant term has the value unity. For $J=\frac{1}{2}$ the angular distribution is of course isotropic ($f_2=1$). It can be seen that the angular correlation effects are quite appreciable and so could be easily detected.

We may also note that quite generally the angular distribution in hyperon decay must have the form

$$f_x(\theta) = \sum_{n=0, \pm \frac{1}{2}, \pm \frac{3}{2}, \ldots} A_n \cos^n \theta. \quad (4)$$

This holds for hyperons produced in any manner, for example by $K$-meson capture in complex nuclei or by high-energy nuclear interactions. One may suspect, however, that the degree of anisotropy will in general be small in complicated interactions, so that simple reactions like $K$-meson capture by protons should provide the best possibility of determining the spins of hyperons. In particular, the coefficients in the above expression are uniquely determined when the spin of the $K$-meson is zero.*

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The general expression for $f_x(\theta)$ is most compactly written

$$f_x(\theta) \sim \sum_{K=\frac{1}{2}, \frac{3}{2}} P_K(\theta) C(J, J; \frac{1}{2}, \frac{3}{2}) \times \sum_{L=\frac{1}{2}, \frac{3}{2}} \sum_{n=0, \pm \frac{1}{2}, \pm \frac{3}{2}} A_L C(J, L, \frac{1}{2}; M, m) Y_L^n(\hat{\theta}) \psi_x^{M'}.$$ 

where the $P_K$ are Legendre polynomials and the quantities $W$ are Racah coefficients; the same results are obtained for both $L=\frac{1}{2}, \frac{3}{2}$. 5 In the case where the $K$-meson has spin $I$ greater than zero, the total angular momentum can have the values $J=I \pm \frac{1}{2}$. For given $J$ the wave function of the system hyperon plus pion can be decomposed:

$$\psi_x^{J'} = \sum_L \sum_{n=0, \pm \frac{1}{2}, \pm \frac{3}{2}} A_L C(J, L, \frac{1}{2}; M, m) Y_L^n(\hat{\theta}) \psi_x^{M'},$$

where the $A_L$ are amplitudes of the various orbital states. For given $J L$ ranges from $|J-\frac{1}{2}|$ to $J+\frac{1}{2}$ but with fixed parity. The angular correlation function corresponding to given $J$ can be written

$$f_x(\theta) \sim (-1)^{L+1} \sum_{J,J'} \sum_{L,L',n} A_{J,J'} C(J, J', L'; M, m) Y_L^n(\hat{\theta}) \times C(L, L', K; 0, 0) C(J, J', K; 0, 0) W(L, L', J'; J, K) \times W(L, J, J', \frac{1}{2}, K) P_K(\theta).$$

Since the particles involved in the capture reaction are assumed to be unpolarized, the net angular correlation function is given by

$$f_x(\theta) \sim (2J+1)\sum_{J=I+1, I-1}^{J=J+1} f_x^{J=I+1, I-1}(\theta).$$