A SEMICLASSICAL THEORY FOR IONIZATION BY GRAND UNIFICATION MAGNETIC MONOPOLES

J.S. TREFIL

Physics Department, University of Virginia, Charlottesville, VA 22901, USA

Received 3 March 1982

The ionization loss by grand unification magnetic monopoles is studied. It is shown that for the velocities of interest this ionization is not large, so that monopoles of mass $10^{16}$ GeV could well have gone undetected until now.

1. Introduction

The idea that isolated magnetic poles might exist was first advanced over a half century ago [1], and since that time extensive theoretical and experimental work on this topic has been done [2]. There is now one candidate for the detection of a monopole [3]. Monopoles have been shown to arise in gauge theories [4], and in many versions of the grand unification theories very massive monopoles are predicted [5], typically of the order of $10^{-8}$ g ($10^{16}$ GeV). These massive monopoles cause difficulties in some cosmological models [6] because they are produced relatively copiously in the early stages of the Big Bang.

Recently, speculations on the effect of GUT monopoles on the earth [7] and their history in the early universe [8] have been published. This work marks the beginning of an attempt to develop a phenomenology for particles which are of microscopic size but macroscopic mass—an entirely new sort of entity for particle physics.

The essential fact that emerges from this literature is that it is hard to imagine any cosmological process which could accelerate a very massive monopole to relativistic energies. Typically, values of $\beta$ on the order of $10^{-3}$ to $10^{-2}$ are expected, which means that monopoles approaching the earth can be expected to have velocities comparable to those associated with the orbital motion of atomic electrons. (Even if monopoles are created with high velocities in the early universe, we shall see that ionization provides a mechanism whereby they can be slowed down quickly.) This, in turn, means that the conventional calculations used to predict the energy lost to ionization in the earth or in some apparatus must be examined with some care, since they are usually valid for relativistic monopoles only.
The purpose of this note is to establish two facts: First, we shall argue that the conventional formulation of the ionization loss for a magnetic monopole happens to break down in the middle of the range of velocities at which GUT monopoles might be expected. Second, we shall show how well known techniques can be used to extend the range of validity of the calculation to cover the expected values of $\beta$. We shall see that such a calculation predicts rather low ionization rates for monopoles in the region of interest. We shall then discuss some consequences of this result for monopole searches. We begin with a short review of the conventional formalism.

2. Monopole energy loss: the conventional approach

A magnetically charged object moving past an atom can transfer energy to the atomic electron. The easiest way to see this is to note that the purely radial magnetic field in the rest frame of the monopole becomes a mixed $E$ and $B$ field when transformed into the laboratory frame in the standard way [9]. The electrical field of a monopole of strength $g$ passing at an impact parameter $b$ with velocity $v$ is just

$$E = \frac{\gamma g b}{\left(b^2 + \gamma^2 v^2 t^2\right)^{3/2}}.$$  

If we integrate the electrical force on the atomic electron over time we can obtain the momentum (and hence the energy) transferred to the electron. In carrying out this integration, two assumptions are commonly made: (i) it is assumed that the monopole path is a straight line and (ii) it is assumed that the electron does not move appreciably while the monopole goes by. The momentum transferred to the electron is just $k = \frac{2eg}{cb}$, and the energy lost by the monopole to the electron is

$$dE(b) = \frac{k^2}{2m_e} = \frac{2e^2g^2}{m_e^2} \frac{1}{b^2}. \quad (2)$$

The total energy lost to all atomic electrons per unit path length is then given by integrating over the impact parameter [10]. The limits of integration are set by noting that the minimum value of $b$ consistent with the uncertainty principle is $\hbar/\gamma mv$, while the maximum value of $b$ consistent with the static approximation is $\gamma v/\omega$, where $\omega$ is the electron frequency. If we carry out the integral, we find for a medium with $N$ electrons per unit volume the energy loss per unit length is

$$\frac{dE}{dx} = \frac{4N e^2 g^2}{m_e^2} \ln \frac{2m_e v^2}{\hbar \omega}, \quad (3)$$

a formula first derived by Bauer [11] and Cole [12]. Several corrections are customarily applied to this result [13]. These include the Bloch correction [14], which takes
account of the fact that the incident beam may not completely overlap the target, and the effect of the screening produced by the polarization of atoms between the monopole and the electron being considered. Both of these effects reduce the theoretically predicted ionization. In addition, for the relativistic case, a correction can be made to take account of quantum mechanical effects [13, 15]. The net effect of all of these corrections, however, is to change the predicted result by less than 50%. Since we are interested in an order of magnitude calculation, we will neglect these corrections in this paper.

3. Cosmological acceleration mechanisms

Because of their large mass, it is difficult to imagine processes by which a GUT monopole could be accelerated to relativistic velocities. We know that the highest energy cosmic rays at $10^{20}$ eV, have energies many orders of magnitude below that of the monopole mass [16]. It is safe to assume, then, that whatever processes accelerate normal particles to their highest energies will produce only non-relativistic monopoles.

Since GUT monopoles are likely to be relics of the Big Bang, the velocity that any monopole has will depend on its history. This means that we cannot define a "correct" value for the velocity, but will have to content ourselves with finding a range of velocities which seem reasonable. We list several possible monopole histories and the $\beta$ associated with each below:

A neutron star might have a field of $10^{12}$ G extending over $10^6$ cm. A monopole accelerated in such a field would reach $\beta = 6 \times 10^{-2}$ if we neglect gravity. However, some simple arithmetic shows that the force of gravity on a one solar mass star 10 km in radius exceeds the magnetic force on a GUT monopole by several orders of magnitude. Thus it is likely that monopoles in or near neutron stars will emerge with velocities significantly smaller than that quoted above, if they emerge at all.

In ref. [8], a simplified axially symmetric model for the galactic magnetic field is assumed. A monopole injected into the galactic center at zero velocity will have acquired a $\beta$ of $\approx 3 \times 10^{-2}$ by the time it leaves the galaxy. This provides one measure of the velocities one could expect to result from acceleration by magnetic fields.

The actual galactic field appears to be rather more chaotic than the one used above. Actually, the sparse data we have [17] indicates that the field is irregular on length scales of $10^4$ light years or less. Acceleration through a $5 \mu$G field of this extent would produce a monopole with $\beta = 5 \times 10^{-3}$. A monopole moving through a series of such fields would execute a random walk in velocity, and might reach $\beta \sim 10^{-2}$.

The escape velocity from the galaxy is $\beta \sim 10^{-3}$. Monopoles ejected from other galaxies and falling into our own would have velocities at least this high.
If we ignore magnetic effects and think only about how a monopole might act if its behavior were to be dominated by gravity, we would expect a $\beta$ between $7 \times 10^{-4}$ (the general galactic rotation velocity) and $10^{-4}$ (the local "noise" velocity).

4. Ionization by GUT monopoles

From the above discussion, we see that the assumption that the atomic electron does not move appreciably while the monopole passes, an assumption necessary in deriving eq. (3), is certainly not valid. At the most trivial level, we can see this by noting that if $\beta^2 < \hbar \omega_0/2 m_e c^2$ the argument of the logarithm becomes negative, yielding an unphysical negative energy loss. We therefore turn to a derivation of ionization loss which does not involve the static approximation.

If we take as our model of the atomic electron a simple harmonic oscillator of natural frequency $\omega_0$, then we can follow the development in ref. [10] to show that the energy loss in eq. [2] must be replaced by

$$dE(b) = \frac{\pi e^2}{m_e} |E(\omega_0)|^2,$$

where $E(\omega_0)$ is the Fourier component of the monopole-produced electric field at the electron frequency. Using the expression for the field in eq. (1), we find that

$$dE(b) = \frac{2 e^2 b^2}{m_e c^2 b^2} \left[ \frac{\omega_0^2 b^2}{\gamma^2 v^2} K_1(\frac{\omega_0 b}{\gamma v}) \right], \quad (4)$$

where $K_1$ is the modified Bessel function. If we integrate over impact parameters from the minimum value to infinity, we find the total energy loss to be

$$\frac{dE}{dx} = \frac{4 \pi N e^2 \beta^2}{m_e c^2} T(z), \quad (5)$$

where we have written

$$T(z) = \frac{1}{2} z^2 \left[ K_0(z) K_2(z) - K_1^2(z) \right], \quad (6)$$

and defined the variable such that

$$z = \frac{\omega_0 \hbar}{\gamma^2 m_e v^2}.$$
This represents the energy loss of a monopole for which the static approximation is not valid. We note that in the relativistic limits, where $z \ll 1$, it reduces to

$$\frac{dE}{dx} = \frac{4\pi Ne^2g^2}{m_e c^2} \left[ \ln \frac{2}{z} - \frac{1}{2} \right],$$

which is a slightly improved form of eq. (3). In the case of interest to us, however, we want the opposite limit. If we set $z \gg 1$ and look at the asymptotic forms of the modified Bessel functions for large arguments [1], we find

$$\frac{dE}{dx} = \frac{4\pi Ne^2g^2}{m_e c^2} \frac{\pi}{4} e^{-2z}.$$ (7)

This result gives an energy loss which approaches zero rapidly for small velocities, as it should. For cases intermediate between the two limits, the full result in eq. (5) must be used.

The result in eq. (5) is subject, of course, to the usual restrictions placed on semiclassical models. We cannot, for example, take account of the shell structure of the atomic electrons. However, in keeping with our quest for order of magnitude accuracy, we do not feel that such considerations are important in our case. A further approximation is implicit in the expression for the energy loss which leads to eq. (5) in that we have, in effect, ignored the fact that the electric field is not quite constant over the atom. This effect has been estimated for the case of an electrically charged projectile [19], where relative increases in stopping power for $^{27}$Al of roughly a factor of 2 were attributed to this effect.

5. Numerical results

In gaussian units, $g$ has the numerical value of $3.3 \times 10^{-8}$. In order to specify the electron density $N$ we must decide on a typical target with which the monopole has to interact. A reasonable choice would be a material like the earth, with a density of 5.5 g/cm$^3$ and atomic number around 14. This gives $N \approx 1.6 \times 10^{24}/\text{cm}^3$. Eq. (5) then becomes

$$\frac{dE}{dx} \text{(MeV/cm)} = 3700T(z).$$

A typical electron energy for atoms of this type might be a few hundred eV. If we set $\hbar \omega_0 = 200$ eV, we have

$$z = \left( \frac{2 \times 10^{-2}}{\beta} \right)^2.$$
For completeness, we show in fig. 1 the function $T(z)$ from eq. (6) as well as the small-$z$ limit which corresponds to the usual ionization loss calculation. The graph shows quite dramatically both the breakdown of the conventional result in the region of interest and the rapid falloff of the ionization as the velocity decreases.

In table 1 we show the expected ionization rates for the monopole velocities discussed above. In addition, we show the percentage of its initial energy that a monopole would lose in traversing the entire earth, taking the earth to be a uniform sphere $10^4$ km across.

The ionization losses in table 1 are quite small. In fact, except for the first two entries in table 1, which probably represents rather unrealistic acceleration mechanisms, there is no value of $dE/dx$ which is larger than that associated with a normal electrically charged particle.
6. Discussion

Since the numerical results predicted in the previous section are so contrary to the traditional thinking on monopole energy loss, it is worthwhile to take a moment to discuss the basic formulation we have presented. Several standard corrections have already been mentioned, but it is unlikely that any of these will produce the kind of order-of-magnitude effects which would be needed to negate our conclusions. One argument which can be made to back up this assertion follows from fig. 1. At the upper end of the probable monopole distribution, the conventional and extended formulations are identical. In a sense, the only thing that eq. (5) does is allow us to extrapolate through the region of interest in spite of the fact that the conventional approach breaks down there. Such relatively short-range extrapolations have an intuitive appeal, particularly since the most likely values of $\beta$ are close to the point at which the extrapolation begins.

On the other hand, eq. (5) predicts a very rapid fall in $dE/dx$. How far can this be trusted? Clearly, at some point other effects, not included in this work, will become important. For example, in the case of very slowly moving monopoles, we might expect forces associated with image charges to play a role in trapping monopoles (at least in magnetic materials) [20]. At velocities near the lower end of our expected range, energy losses associated with the interaction of the electron magnetic moment and the gradient of the monopole's magnetic field become important. A semiclassical development like that leading to eq. (3) leads to an energy loss through this mechanism of

$$\frac{dE}{dx} = \frac{2\pi N\mu^2 g^2}{9mV^2} \frac{1}{b_{\text{min}}^2},$$

where $\mu$ is the electron dipole moment. For the same numerical example used in the previous section, this leads to an energy loss of $10^{-27}$ MeV/cm, which is comparable to the prediction of eq. (5) for velocities around $\beta = 10^{-3}$. This verifies a speculation by Ullman [21].
An accurate determination of the range of $\beta$ for which eq. (5) can be used reliably is clearly an important goal for those interested in monopole searches.

7. Conclusions

(1) The most important conclusion we have reached is that GUT monopoles accelerated in the galaxy will approach the earth at velocities low enough so that they will not lose an appreciable fraction of their energy through ionization, even if they pass through an equatorial plane. This means that geological searches which depend on the trapping of monopoles in surface material exposed to cosmic ray flux are not sensitive to monopoles at the most likely velocities.

(2) Similarly, direct monopole searches often involve the assumption that monopoles will ionize heavily. It is unlikely that tracks associated with energy losses in the MeV/cm range, an energy loss typical of ordinary charged particles, would be noticed in such searches. Thus, they would detect monopoles only in the very upper range of expected velocities, and we have already argued that these upper values are somewhat unrealistic.

Thus we conclude that indirect limits of the density of GUT magnetic monopoles of the type proposed by Longo [22] are the best that can be placed at the moment.

The author wishes to thank Ed Whitten, Bob Rood, and Paul Fishbane for useful and stimulating discussions.

References

   R.E. Craven, W.P. Trower, and R.A. Carrigan, Jr., Fermilab 81/37 (1981);
 [22] M.J. Longo, University of Michigan preprint Um He 81-83