A test of the renormalizable models of weak interactions in $e^+e^-$ collisions

O. P. Sushkov, V. V. Flambaum, and I. B. Khriplovich

Institute for Nuclear Physics, Siberian Division, USSR Academy of Sciences

(Novosibirsk March 1974)

Yad. Fiz. 28, 1016-1023 (November 1974)

The cross sections for the processes $e^+e^-\rightarrow \mu^+\mu^-$ (in particular near the Z resonance), $e^+e^-\rightarrow 2\gamma$, $e^+e^-\rightarrow W^+W^-$, and $e^+e^-\rightarrow e^+e^-W^+W^-$ are calculated within the frameworks of the Weizsäcker-Williams and Gell-Mann-Okubo models. The cross sections for the latter two processes become equal already at an energy $2E=14m_e$.

1. PROCESSES INVOLVING A NEUTRAL VECTOR BOSON

In the Weizsäcker model, the reaction $e^+e^-\rightarrow \mu^+\mu^-$ in second order in $\alpha$ is described by the two diagrams represented in Fig. 1. A simple calculation leads to the following expression for the reaction cross section:

$$
\sigma = \frac{\alpha^2}{2\pi} \int_0^1 \frac{d\xi}{\xi} \int_0^{1-\xi} \frac{d\eta}{\eta} \frac{\ln \left( \frac{1-\xi}{\xi} \right)}{\left( 1 - \eta - \xi + \eta \xi \right)^2} \left( \frac{1}{\xi \eta} \right)
$$

Here $s = 4E^2$ is the square of the total energy of the process, $\xi$ is the emission angle of the muon in the c.m., $m_\mu$ is the mass of the $\mu$ boson, and $\eta$ is the mixing angle which is characteristic for the Weinberg model.

The differences between this cross section and the purely electromagnetic one, in particular the asymmetry in the angular distribution of the muons, has been discussed in detail in other papers [2,3].

We discuss only the behavior of this cross section for $s \gg m_\mu^2$. Let us determine the width of the $W$ boson. It decays into leptons $e^\pm$, $\mu^\pm$, $\nu_{e\pm}$, $\nu_{\mu\pm}$ and hadrons.

According to the estimation of the hadronic decays of the $W$ we make use of a model of weak interactions of hadrons based on the SU(4) symmetry [4]. We also assume that for such high energies one may neglect the parton masses and that taking into account the strong interactions does not affect the total probability for the decay calculated in the free-gluon model. The total width of the $Z$-boson determined in this way is

$$
\Gamma_{Z} = \frac{m_Z}{8\pi} \alpha^2 \ln \left( 1 + \frac{m_Z^2}{m_W^2} \right)
$$

where $\Gamma_Z$ is the charge of the $\pi$-photon.

At resonance, the total cross section of the process $e^+e^-\rightarrow \mu^+\mu^-$ is

$$
\sigma_{\text{res}} = \frac{\pi}{\alpha^2} \left( 1 - \frac{m_W^2}{m_e^2} \right) \ln \left( 1 + \frac{m_W^2}{m_e^2} \right)
$$

The numerical values for $\Gamma_{Z}$ and $\sigma_{\text{res}}$ for $m_\mu = 0.3$ and $q = 2/3$ are: $\Gamma_{Z} = 0.6 GeV$, $\sigma_{\text{res}} = 3 \times 10^{-4} cm^2$.

As one sees, the cross section for the elastic $e^+e^-$ cross section takes the same value at the resonance (if we disregard the Rutherford increase in the small angle region), the resonance cross section for annihilation into hadrons, and also into $\nu_{e\pm}$ and $\nu_{\mu\pm}$, has the same order of magnitude.

The cross section of the process $e^+e^-\rightarrow \mu^+\mu^-$ decreases rapidly as one goes away from the resonance.
For completeness we indicate the asymptotic behavior as \( \theta \to \frac{\pi}{2} 
abla 2\pi 
abla R \).

The energy dependence of the cross section for various momenta is represented in Fig. 5.

In the Georgi-Grishnov model\(^\text{10,11}\), the diagram of the process \( e^+ e^- \to W^+ W^- \) is of the form given in Fig. 4. Here the reaction cross section equals

\[
\sigma = \frac{e^2 m_W^4}{4 \pi} (1 + \frac{2}{3} \frac{1}{s}) \frac{s}{1 + \frac{4}{3} \frac{1}{s}} - \frac{1}{s} (1 - \frac{1}{s} - \frac{1}{s})
\]

where \( s = m_W^2 \), \( \theta \) is the scattering angle, and \( s = m_W^2 \). The expression for the cross section has been obtained under the assumption \( m_L \ll m_W \). Near the threshold

\[
\sigma = \frac{e^2 m_W^4}{4 \pi} (1 + \frac{2}{3} \frac{1}{s}) \frac{s}{1 + \frac{4}{3} \frac{1}{s}} - \frac{1}{s} (1 - \frac{1}{s} - \frac{1}{s})
\]

and the most interesting region \( s < m_W^2 \). For \( s < 10 m_W^2 \),

\[
\sigma = \frac{e^2 m_W^4}{4 \pi} (1 + \frac{2}{3} \frac{1}{s}) \frac{s}{1 + \frac{4}{3} \frac{1}{s}} - \frac{1}{s} (1 - \frac{1}{s} - \frac{1}{s})
\]

For \( s > 10 m_W^2 \),

\[
\sigma = \frac{e^2 m_W^4}{4 \pi} (1 + \frac{2}{3} \frac{1}{s}) \frac{s}{1 + \frac{4}{3} \frac{1}{s}} - \frac{1}{s} (1 - \frac{1}{s} - \frac{1}{s})
\]

We note that relations which are proportional to the masses of the electron and of the photon do not lead to a modification of the asymptotic behavior of the cross section.

We have learned that the reaction \( e^+ e^- \to W^+ W^- \) has also been considered by A. E. Katsanjan and M. A. R. Blaiznikov.

3. THE REACTION \( e^+ e^- \to W^+ W^- \)

Since the cross section of the reaction \( e^+ e^- \to W^+ W^- \) decreases with the energy, interest attaches to the reaction \( e^+ e^- \to W^+ W^- \). In spite of the fact that it is of higher order in \( e \), since it has a logarithmically increasing cross section.

For the calculations that follow we shall need the cross section for the process \( \gamma^* \to W^+ W^- \) (Fig. 5). A direct calculation yields for this cross section

\[
\sigma = \frac{e^2 m_W^4}{8 \pi} \frac{3}{2} \frac{1}{s} \left( \frac{1}{s} - \frac{1}{s} \right)
\]

where \( \theta = m_W \) is the mass of the \( W \) boson, \( x = s/m_W^2 \), and \( e^2 = 1/\alpha^2 = e^2 \) is the velocity of the \( W \) boson in the c.m.s. This expression agrees with the corresponding formulae obtained in Ref. 11. We note an interesting peculiarity of the process \( \gamma^* \to W^+ W^- \). As can be seen from Eq. (13), \( \sigma \) tends monotonically to a constant, differing essentially from the cross sections of two-photon production of vector and spinor particles, which fall off respectively like \( 1/K \) and \( \ln(\omega^2/\lambda^2) \). Such a monotonic decrease is easily understood if one notes that for \( \theta \to \frac{\pi}{2} \) the reaction \( \gamma^* \to W^+ W^- \) reduces in fact to elastic scattering of vector particles (the presence of charge exchange does not influence the structure of the various terms, as can be easily seen). Since the exchange of a vector particle contributes to the cross section, for \( \theta < \frac{\pi}{2} \) the usual Coulomb divergence at small angles will appear. The smaller mass of the exchanged particle makes the cross section finite, but subdominant with energy. The cross sections for the production of \( W \) bosons with helicities \( \pm 1 \) is constant. \( W \) bosons of zero helicity have cross sections which decrease with energy. In the zero mass \( \omega \) for the production of scalar and spinor particles, owing to the change of helicity of the real particles in the vertices of the exchange graphs. We note that a specific type of the renormalization group theory is the basis for the use of the \( g \) factor of the \( W \) boson in Eq. (8) (Ref. 9, 10), as this is the case where the cross section (13) does not increase with the energy.

All further calculations will be constructed to the energy region

\[
\omega = m_W^2 (1 + \frac{1}{s}) - \frac{1}{s} m_W^2
\]

Knowing \( \sigma \), it is easy to calculate the cross section in the present \( \gamma^* \to W^+ W^- \). A method of constructing the cross section cross section comes only from the diagram in Fig. 5. With the help of the diagram the Weiskopf-Williams method\(^\text{11,12}\), making use of Eq. (13) for \( \gamma^* \), we obtain

\[
\sigma (\gamma^* \to W^+ W^-) = \frac{\alpha^2}{\pi} \frac{3}{2} \frac{1}{s} \left( \frac{1}{s} - \frac{1}{s} \right)
\]

For a nonrelativistic electromagnetic interaction of the \( W \) boson the cross section for this process (which is assumed to be constant with energy) has been calculated by D. P. Gubarev et al.

Let us return to the process \( e^+ e^- \to W^+ W^- \). The main contribution to its cross section comes from the diagrams in Fig. 7. The two photon poles in this diagram lead to an enhancement of the contribution \( \omega \ln(\frac{\omega^2}{\lambda^2}) \). In addition one must take into account the diagrams of Fig. 8 which are enhanced doubly-logarithmically by the}

\[
\frac{\omega^2}{\lambda^2}
\]

The precession an term is not. At the assumption across these three diagrams do not intersect with each other in the c.m.s. of the incident particles in the coherent process in the e.m. the photon and positrons do not change their direction moment. At the same time the larger the process in the diagrams of Fig. 9, placed near the photon region, has the effect that the final e' are emitted within narrow cone along the direction of motion of the one which emits the exchanged photon. In addition the diagram of Fig. 7 and 8 there is a whole number of terms for the process under consideration, for we contributions do not decrease with energy (e.g. \( \alpha^2 \)). However, the one contains some powers of \( \ln(\frac{\omega^2}{\lambda^2}) \) and not the desired identity must not be into account.

We first calculate the cross section \( \gamma^* \to W^+ W^- \) in the two-photon mechanism (Fig. 7). Use of the \( 0 \) invariant Weiskopf-Williams method yields

\[
\omega = \frac{\omega^2}{\lambda^2} \frac{1}{\pi} \frac{3}{2} \frac{1}{s} \left( \frac{1}{s} - \frac{1}{s} \right)
\]

Here \( \omega = m_W^2 \left( 1 + \frac{1}{s} \right) - \frac{1}{s} m_W^2 \), the moments are distributed in the diagram,

\[
\omega = m_W^2 (1 - \frac{1}{s}) - \frac{1}{s} m_W^2
\]

In addition one must take into account the diagrams of Fig. 8 which are enhanced doubly-logarithmically by the

\[
\frac{\omega^2}{\lambda^2}
\]

We now go over to the calculation of \( \sigma \) (Fig. 9) against the coherent formulation of the Weiskopf-Williams method we obtain

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respectively like 1/\(m^5\) and \(\ln(m^4/m^3)\). Such a behavior of \(\sigma e\) is easily understood if one notes that for \(m^4 \to 0\) the function \(\gamma \to 2\pi^2 m^4\) reduces to a factor in elastic scattering \(-1\) of particles (the presence of charge exchange does not influence the structure of the matrix element). Thus the exchange of a vector particle contributes to this cross section, for \(m^4 = 0\) the usual Coulomb divergence at small angles will appear in \(\sigma e\).

The zero mass of the exchanged quantum means a cross section finite, but nondecreasing with energy, the cross section for the production of W bosons with velocities \(v \approx 1\) constant. W bosons of zero helicity have cross sections which decrease with energy, in the same manner as for the production of scalar and spinor particles, owing to the change of helicity of the real particles in the vertices of the exchange graphs. We note that specific traits of the renormalizable theory is the fact that the g factor of the W boson in its equals 2 (cf. (15)), so at the cross section (13) does not increase with the energy.

All further calculations will be restricted to the energy region

\[
\frac{\text{eV}}{\text{eV}} \ll \text{eV} \ll \text{eV},
\]

owing \(eW\), it is easy to calculate for cross section for a process \(e^+e^- \to e^-W^+\). A nondecreasing contribution to the cross section comes only from the diagram in Fig. 9. With the help of the covariant Weizsacker-Williams method [10, making use of Eq. (13) for \(\omega^\gamma\), we obtain

\[
\sigma_{\text{eV}} = \frac{\text{eV}}{\text{eV}} \ll \frac{\text{eV}}{\text{eV}} \ll \frac{\text{eV}}{\text{eV}},
\]

\[
\frac{eW}{eW} = \frac{A}{2} \ln \left(1 + \frac{1}{2} \frac{eW}{eW} \right).
\]

(14)

\(\sigma_{\text{eV}}\) is a renormalizable electromagnetic interaction of the bosons the cross section for this process (which linearly with the energy) has been calculated [11].

Let us return to the process \(e^+e^- \to e^-W^+\). The main contribution to its cross section comes from the diagram of Fig. 7. The two photon poles in this diagram lead to an enhancement of its contribution: \(-2\ln(m^2/m^3)\). In addition, one must take into account the diagrams of Fig. 8 which are enhanced doubly-logarithmically by the

\[
\sigma_{\text{eV}} = \frac{\text{eV}}{\text{eV}} \ll \frac{\text{eV}}{\text{eV}} \ll \frac{\text{eV}}{\text{eV}},
\]

\[
\frac{eW}{eW} = \frac{A}{2} \ln \left(1 + \frac{1}{2} \frac{eW}{eW} \right).
\]

(15)

Thus

\[
\sigma_{\text{eV}} = \frac{1}{2} \ln \left(1 + \frac{1}{2} \frac{eW}{eW} \right).
\]

(16)

The momenta entering the cross section are defined in the diagram.

\[
\sigma_{\text{eV}} = \frac{1}{2} \ln \left(1 + \frac{1}{2} \frac{eW}{eW} \right).
\]

(17)

Integrating the expression for \(\sigma_{\text{eV}}\) (13) into (16) and integrating with respect to \(s_1\), we obtain (retaining only terms which do not decrease with the energy)

\[
\sigma_{\text{eV}} = \frac{2}{\pi} s_1 \ln \left(1 + \frac{1}{2} \frac{s_1}{s_2} \right) \left[1 - \frac{1}{2} \frac{s_1}{s_2} + \frac{1}{2} \frac{s_1^2}{s_2^2} \right].
\]

(18)

where

\[
\sigma e = 2 \ln \left(1 + \frac{1}{2} \frac{s_1}{s_2} \right) \left[1 + \frac{1}{2} \frac{s_1}{s_2} - \frac{1}{2} \frac{s_1^2}{s_2^2} \right].
\]

\(\sigma e\) is the cross section for the photoprocesses of Figs. 3 and 9. We note that in Eq. (18) it is necessary to take into account the dependence of \(s_1\) on the photon mass \(s_2\), This is related to the fact that the fermion pole is close to the physical region. In the calculation of the cross section of the photoprocesses the covariant summation over the photon polarizations turns out cumbersome since it must be done with a gauge-invariant block, i.e., it requires taking into account simultaneously the diagrams of Figs. 3 and 9. For this reason we consider the three-dimensional longitudinal polarization of the photon which appears because \(s_1^2 \neq 0\). This polarization vector is of the form

\[
\sigma e = \frac{1}{2} \ln \left(1 + \frac{1}{2} \frac{s_1}{s_2} \right) \left[1 - \frac{1}{2} \frac{s_1}{s_2} - \frac{1}{2} \frac{s_1^2}{s_2^2} \right].
\]

(18)

Owing to gauge invariance, the first term will not contribute to the amplitude, the second one is proportional to the photon mass \(-s_1\), and therefore upon integration with respect to \(s_1\) in Eq. (18) we do not obtain a logarithmic enhancement. Thus, to the degree of accuracy desired here, one may neglect the contribution of the longitudinal polarization of the photon. In calculating the cross section with transversely polarized photons it is advisable to take into account only the diagram of Fig. 9, since it contains a fermion pole close to the physical region, and is therefore logarithmically enhanced.

The square of the amplitude of Fig. 10, averaged over the polarizations of the initial electron and summed over the polarizations of the final one, takes the following form after integration over the phase space of the W bosons:

\[
|\psi_{0}^{*}\psi_{1}|^{2} = \frac{1}{4} \rho \left[\frac{A}{2} \ln \left(1 + \frac{1}{2} \frac{s_1}{s_2} \right) \left[1 + \frac{1}{2} \frac{s_1}{s_2} - \frac{1}{2} \frac{s_1^2}{s_2^2} \right] \right]^{2}
\]

(19)

Here \(\rho = m_1 m_2 / m_1 m_2\) is the three-dimensionally transverse polarization vector of the photon, \((p_1, k)\) is the matrix describing the interaction in the upper block (the notation for the momenta is indicated in Fig. 10). The general form of the matrix \(\psi_{0}^{*}\psi_{1}\)

\[
|\psi_{0}^{*}\psi_{1}|^{2} = \frac{1}{4} \rho \left[\frac{A}{2} \ln \left(1 + \frac{1}{2} \frac{s_1}{s_2} \right) \left[1 + \frac{1}{2} \frac{s_1}{s_2} - \frac{1}{2} \frac{s_1^2}{s_2^2} \right] \right]^{2}
\]

(20)

This allows one to neglect the second term in Eq. (20), i.e.,

\[
|\psi_{0}^{*}\psi_{1}|^{2} = \frac{1}{4} \rho \left\{\frac{1}{2} \ln \left(1 + \frac{1}{2} \frac{s_1}{s_2} \right) \left[1 + \frac{1}{2} \frac{s_1}{s_2} - \frac{1}{2} \frac{s_1^2}{s_2^2} \right] \right\}^{2}
\]

(21)

For the cross section of the process \(e^-W^-\) one obtains easily

\[
|\psi_{0}^{*}\psi_{1}|^{2} = \frac{1}{4} \rho \left\{\frac{1}{2} \ln \left(1 + \frac{1}{2} \frac{s_1}{s_2} \right) \left[1 + \frac{1}{2} \frac{s_1}{s_2} - \frac{1}{2} \frac{s_1^2}{s_2^2} \right] \right\}^{2}
\]

(22)

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\[
\sigma_{12} = \frac{3}{4} \sigma_{11} \quad \text{(22)}
\]

Making use of Eqs. (20)–(22) and summing over the transverse photon polarizations we obtain for \( \gamma \)
\[
\alpha_{12} = \frac{i}{2} \sigma_{12} - \frac{i}{2} \sigma_{12} = \frac{1}{2} \gamma (1 - 2 \rho + \frac{3}{4} \rho^2).
\]

where
\[
- \sigma_{12} - \sigma_{12} = - \sigma_{12} - \sigma_{12}.
\]

We note that the described method for the calculation of the pole contribution to the cross section is, in fact, a covariant formulation of the so-called method of quantized real electrons, described in (38, 39).

Substitution of the expression (23) into Eq. (18) and subsequent integration with respect to \( k^\prime, q^\prime, \) and \( s_2 \) to logarithmic accuracy yields
\[
\alpha_{12} = \frac{1}{2} \gamma (1 + \frac{3}{2} \rho) \int_{1/2}^{1} \frac{1}{s_2} \langle \phi_2 | \langle \phi_2 | \frac{1}{2} \gamma (1 - 2 \rho + \frac{3}{4} \rho^2).
\]

The integration over \( s_2 \) (we retain only terms which do not decrease with the energy) leads to the following expressions for the cross section: in the Georgi-Glashow model \((m^2 < s < m^2/2m)\) we have
\[
\alpha_{12} = \frac{1}{2} \gamma (1 + \frac{3}{2} \rho) \int_{1/2}^{1} \frac{1}{s_2} \langle \phi_2 | \langle \phi_2 | \frac{1}{2} \gamma (1 - 2 \rho + \frac{3}{4} \rho^2).
\]

and in the Weinberg model
\[
\alpha_{12} = \frac{1}{2} \gamma (1 + \frac{3}{2} \rho) \int_{1/2}^{1} \frac{1}{s_2} \langle \phi_2 | \langle \phi_2 | \frac{1}{2} \gamma (1 - 2 \rho + \frac{3}{4} \rho^2).
\]

where \( f = 1, 1.1, 1.15, \) and \( 1.25 \) for \( \sin^2 \theta \) equal respectively to \( 0.2, 0.3, \) and \( 0.4. \) The total cross section for the process \( e^- e^- W^- W^+ \) is obviously equal to
\[
\alpha_{12} = \frac{1}{2} \gamma (1 + \frac{3}{2} \rho) \int_{1/2}^{1} \frac{1}{s_2} \langle \phi_2 | \langle \phi_2 | \frac{1}{2} \gamma (1 - 2 \rho + \frac{3}{4} \rho^2).
\]

In this formula the second term is smaller by one to two orders of magnitude than the first one, i.e., the contributions of the diagrams of Fig. 8 is numerically small. By comparing Eqs. (6) or (14) and (17) it can be seen that the process \( e^- e^- W^- W^+ \) dominates over \( e^- e^- W^- W^- \) starting with an energy \( E \) \( e^{-1.9} = 14 \text{eV}. \) In this region the cross section obtained from Eq. (17) differs from the result of a numerical calculation based on Eq. (16) by at most a few percent.

The cross section for the process \( e^- e^- W^- W^+ \) has been calculated in (28); the result for the correction of the diagram of Fig. 7 differs markedly from our result. We have no way of indicating the reason for this disagreement, since \( \gamma \) has been obtained numerically in the cited papers. Moreover, in [28] the contribution of the diagrams of Fig. 8 has not been taken into account.

In the relativizable electrodynamics of the W bosons with a g-factor equal to one, the cross-section for the process \( e^- e^- W^- W^+ \) has been calculated in (29, 30). The authors are indebted to A. I. Vainshtein and A. A. Siposnqsn for valuable discussions.

14. Translated by M. E. Mayer
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Spin effects in elastic scattering

V. V. Flambaum

Nuclear Physics Institute, Soviet Division, USSR Academy

Submitted March 25, 1974

Vol. 28, 1024–1028 (November 1974)

An expression is derived for the amplitude of elastic scattering. This expression makes it possible to calculate the spin effects for small scattering angles. The momenta transfer can lie in the nucleus, i.e., the nucleus is not assumed to be pointlike.

The well-known approximation solution of the problem of elastic scattering of a fast electron by a nucleus (13) does not make it possible to obtain the spin effects. In this paper we obtain for this solution a correction that describes the correlation of the muon spin with the normal to the scattering plane. All the derived expressions are valid also for electron scattering. However, in the region where the fundamental results are applicable, the value of \( g \) [up to \( g = 1 - \rho^2 \)] for electrons is large. Since the discussed spin correction to the cross sections contains, relative to the principal part, only an extra power of \( \rho^2 \) (as a result of helicity conservation in elastic scattering in the ultrarelativistic case), the effect of the electron turns out to be small.

The Dirac equation for the motion in a nucleus in the field of a nucleus can be written in the form

\[
(i\gamma^\mu \partial_\mu - m) \psi = 0,
\]

where \( p, c, \) and \( \mu(\theta, \phi, \varphi) \) are the momentum, energy, and potential energy of the muon. We represent \( \psi \) in the form of the sum

\[
\psi = \psi_0 + \psi_1,
\]

where \( \psi_0 + \psi_1 \) is the Furry-Sommerfeld-Maxwell solution (1, 11), which satisfies the following relations:

\[
(i\gamma^\mu \partial_\mu - m) \psi_0 = 0,
\]

\[
(i\gamma^\mu \partial_\mu - m) \psi_1 = - \mu(\theta, \phi, \varphi) \psi_0,
\]

where \( p_0 \) and \( p_1 \) are the initial and final moments of the muon, and \( \mu(\theta, \phi, \varphi) \) is the constant isotropic amplitude of the plane wave. The correction \( \psi_1 \) makes no contribution to the elastic-scattering amputations, reaching as \( \rho^2 \to \infty \) we have

\[
\int |\mu(\theta, \phi, \varphi)|^2 d\Omega = \int |\mu(\theta, \phi, \varphi)|^2 d\Omega = (\mu^2 - m^2) d\Omega = 0.
\]

For \( \mu_1 \) we obtain the equation

\[
(i\gamma^\mu \partial_\mu - m) \psi_1 = 0,
\]

where

\[
\psi_1 = \int \psi_1 d\Omega = \psi_0 - \frac{m}{\mu(\theta, \phi, \varphi)}
\]

and

\[
\int \psi_1 d\Omega = \psi_0 - \frac{m}{\mu(\theta, \phi, \varphi)} = \psi_0 - \frac{m}{\mu(\theta, \phi, \varphi)}.
\]

From (1) and (2) we obtain

\[
\psi_1 = \frac{m}{\mu(\theta, \phi, \varphi)}
\]

Let us find the Green's function of Eq. (4). It coincides with the equations that describe the propagation of the waves from the source to the medium. We assume